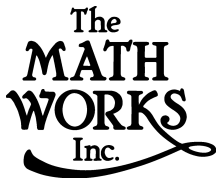
Control System Toolbox

For Use with MATLAB®

Computation

Visualization

Programming



User's Guide

Version 4.2

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Control System Toolbox User's Guide

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Preface

MATLAB® has a rich collection of functions immediately useful to the control engineer or system theorist. Complex arithmetic, eigenvalues, root-finding, matrix inversion, and FFTs are just a few examples of MATLAB's important numerical tools. More generally, MATLAB's linear algebra, matrix computation, and numerical analysis capabilities provide a reliable foundation for control system engineering as well as many other disciplines.

The Control System Toolbox uses MATLAB matrix structures and builds upon the foundations of MATLAB to provide functions specialized to control engineering. The Control System Toolbox is a collection of algorithms, expressed mostly in M-files, which implements common control system design, analysis, and modeling techniques.

Control systems can be modeled as transfer functions or in zero-pole-gain or state-space form, allowing you to use both classical and modern techniques. You can manipulate both continuous-time and discrete-time systems. Conversions between various model representations are provided. Time responses, frequency responses, and root loci can be computed and graphed. Other functions allow pole placement, optimal control, and estimation. Finally, and most importantly, tools that are not found in the toolbox can be created by writing new M-files.

Installation

Instructions for installing the Control System Toolbox can be found in the *MATLAB Installation Guide* for your platform. We recommend that you store the files from this toolbox in a directory named `control` off the main `matlab` directory. To determine if the Control System Toolbox is already installed on your system, check for a subdirectory named `control` within the main toolbox directory or folder.

Five demonstration files are available. The demonstration M-file called `ctrldemo.m` runs through some basic control design and analysis functions and the demonstration files `jetdemo.m`, `diskdemo.m`, `milldemo.m`, and `kalmdemo.m` go through the design case studies described in Chapter 9. To start a demo, type `ctrldemo`, for example, at the MATLAB prompt.

Getting Started

If you are a new user, begin with Chapters 2 through 5 to learn:

- How to specify and manipulate linear time-invariant models
- How to analyze such models and plot their time and frequency response

If you are an experienced toolbox user, see:

- *The New Features Guide* for details on the latest release
- Chapter 1 for an overview of some product features
- Chapter 4 to learn about LTI arrays
- Chapter 6 for an introduction to the LTI Viewer GUI
- Chapter 8 for an introduction to the Root Locus Design GUI

All toolbox users should use Chapter 11 for reference information on functions and tools. For functions, reference descriptions include a synopsis of the function's syntax, as well as a complete explanation of options and operation. Many reference descriptions also include helpful examples, a description of the function's algorithm, and references to additional reading material. For GUI-based tools, the descriptions include options for invoking the tool.

Typographic Conventions

To Indicate	This Guide Uses	Example
Example code	`Monospace type`	To assign the value 5 to A, enter `A = 5`
Function names	`Monospace type`	The `cos` function finds the cosine of each array element.
Function syntax	`Monospace type` for text that must appear as shown. *`Monospace italics`* for components you can replace with any variable.	The `magic` function uses the syntax *`M`* = `magic(`*`n`*`)`
Keys	**Boldface**	Press the **Return** key.
Mathematical expressions	Variables in *italics*. Functions, operators, and constants in standard type.	This vector represents the polynomial $p = x^2 + 2x + 3$

MATLAB output	`Monospace type`	MATLAB responds with
		`A =`
		`5`

Menu names, menu items, and controls	**Boldface**	Choose the **File** menu.
New terms	*italics*	An *array* is an ordered collection of information.

1

Quick Start

This chapter provides a quick overview of some features of the Control Systems Toolbox.

LTI Models

You can specify linear time-invariant (LTI) systems as transfer function (TF) models, zero/pole/gain (ZPK) models, state-space (SS) models, or *frequency response data* (FRD) model. You can construct the corresponding models using the constructor functions.

```
sys = tf(num,den)               % transfer function
sys = zpk(z,p,k)                % zero/pole/gain
sys = ss(a,b,c,d)               % state space
sys = frd(response,frequencies) % frequency response data
```

To find out information about LTI models, type

```
ltimodels
```

See also “Creating LTI Models” on page 2-8.

The output `sys` is a model-specific data structure called a TF, ZPK, SS, or FRD object, respectively. These objects store the model data and enable you to manipulate the LTI model as a single entity; see “LTI Objects” on page 2-3 for more information. For example, type

```
h = tf(1,[1 1])    % creates transfer function 1/(s+1)
```

and MATLAB responds with

```
Transfer function:
  1
-----
s + 1
```

Type

```
1+h
```

MATLAB responds with

```
Transfer function:
s + 2
-----
s + 1
```

To create discrete-time models, append the sample time `Ts` to the previous calling sequences.

```
sys = tf(num,den,Ts)
sys = zpk(z,p,k,Ts)
sys = ss(a,b,c,d,Ts)
sys = frd(response,frequency,Ts)
```

For more information, see "Discrete-Time Models" on page 2-20.

For example, type

```
sys = zpk(0.5,[-0.1 0.3],1,0.05)
```

and MATLAB responds with

```
Zero/pole/gain:
    (z-0.5)
---------------
(z+0.1) (z-0.3)

Sampling time: 0.05
```

You can retrieve the model data stored in the LTI object `sys` with the following commands (see "Data Retrieval" on page 2-24 for more information).

```
[num,den,Ts] = tfdata(sys)
[z,p,k,Ts] = zpkdata(sys)
[a,b,c,d,Ts] = ssdata(sys)
[response,frequency,Ts] = frdata(sys)
```

Alternately, you can access this data by direct structure-like referencing (see "Direct Property Referencing" on page 2-33 for more information), as in this example.

```
sys.num
sys.a
sys.Ts
```

MIMO Systems

You can also create multi-input/multi-output (MIMO) models, including arbitrary MIMO transfer functions and zero/pole/gain models. MIMO transfer functions are arrays of single-input/single-output (SISO) transfer functions

where each SISO entry is characterized by its numerator and denominator. Cell arrays provide an ideal means to specify the resulting arrays of numerators and denominators; see, "MIMO Transfer Function Models" on page 2-10 for more information. For example,

```
num = {0.5,[1 1]}     % 1-by-2 cell array of numerators
den = {[1 0],[1 2]}   % 1-by-2 cell array of denominators
H = tf(num,den)
```

creates the one-output/two-input transfer function

$$H(s) = \begin{bmatrix} \frac{0.5}{s} & \frac{s+1}{s+2} \end{bmatrix}$$

Alternatively, you can create the same transfer function by matrix-like concatenation of its SISO entries

```
h11 = tf(0.5,[1 0])     % 0.5/s
h12 = tf([1 1],[1 2])   % (s+1)/(s+2)
H = [h11,h12]
```

MIMO zero/pole/gain systems are defined in a similar fashion. For example, the following commands specify $H(s)$ above as a zero/pole/gain model

```
Zeros = {[],-1}    % Note: use [] when no zero
Poles = {0 ,-2}
Gains = [0.5,1]    % Note: use regular matrix for gains
H = zpk(Zeros,Poles,Gains)
```

Model Conversion

The functions `tf`, `zpk`, `frd`, and `ss` also perform model conversion; see "Model Conversion" on page 2-42 for more information. For example,

```
sys_ss = ss(sys)
```

converts some `tf` or `zpk` model `sys` to state space. Similarly, if you type

```
h = tf(1,[1 2 1])  % transfer function 1/(s^2+2s+1)
zpk(h)
```

MATLAB derives the zero/pole/gain representation of the transfer function h

```
Zero/pole/gain:
   1
-------
(s+1)^2
```

LTI Arrays

You can now create multidimensional arrays of LTI models and manipulate them as a single entity. LTI arrays are useful to perform batch analysis on an entire set of models. For more information, see Chapter 4, “Arrays of LTI Models.”

LTI Properties

In addition to the model data, the TF, ZPK, FRD, and SS objects can store extra information, such as the system sample time, time delays, and input or output names. The various pieces of information that can be attached to an LTI object are called the *LTI properties*. For information on LTI properties, type

```
ltiprops
```

See also "LTI Properties" on page 2-26 and "Time Delays" on page 2-45.

Use `set` to list all LTI properties and their assignable values, and `get` to display the current properties of the system. For example, type

```
sys = ss(-1,1,1,0,0.5) ;% 1/(z+1), sample time = 0.5 sec
set(sys)
```

```
   a:  Nx-by-Nx matrix (Nx = no. of states)
   b:  Nx-by-Nu matrix (Nu = no. of inputs)
   c:  Ny-by-Nx matrix (Ny = no. of outputs)
   d:  Ny-by-Nu matrix
   e:  Nx-by-Nx matrix (or empty)
   StateName:  Nx-by-1 cell array of strings
   Ts:  scalar
   InputDelay:  Nu-by-1 vector
   OutputDelay:  Ny-by-1 vector
   ioDelayMatrix:  Ny-by-Nu array (I/O delays)
   InputName:  Nu-by-1 cell array of strings
   OutputName:  Ny-by-1 cell array of strings
   InputGroup:  M-by-2 cell array if M input groups
   OutputGroup:  P-by-2 cell array if P output groups
   Notes:  array or cell array of strings
   UserData:  arbitrary
```

Type:

```
get(sys)
```

```
    a: -1
    b: 1
    c: 1
    d: 0
    e: []
    StateName: {''}
    Ts: 0.5
    InputDelay: 0
    OutputDelay: 0
    ioDelayMatrix: 0
    InputName: {''}
    OutputName: {''}
    InputGroup: {0x2 cell}
    OutputGroup: {0x2 cell}
    Notes: {}
    UserData: []
```

You can also use `set` and `get` to access/modify LTI properties in a Handle Graphics® fashion; see "Setting LTI Properties" on page 2-30 for more information. For example, give names to the input and output of the SISO state-space model `sys`. Type

```
set(sys,'inputname','thrust','outputname','velocity')
get(sys,'inputn')
```

MATLAB responds with

```
ans =

'thrust'
```

Finally, you can also use a structure-like syntax for accessing or modifying a single property. For example, type

```
sys.Ts = 0.3;  % Set sample time to 0.3 sec.
sys.Ts% Get sample time value
```

MATLAB returns

```
ans =
   3.0000e–01
```

Model Characteristics

The Control System Toolbox contains commands to query such model characteristics as the I/O dimensions, poles, zeros, and DC gain. See “General Model Characteristics” on page 5-2 for more information. These commands apply to both continuous- and discrete-time model. Their LTI-based syntax is summarized below (with sys denoting an arbitrary LTI model).

```
size(sys)     % number of inputs, outputs, and array dimensions
ndims(sys)    % number of dimensions
isct(sys)     % returns 1 for continuous systems
isdt(sys)     % returns 1 for discrete systems
hasdelay(sys) % true if system has delays
pole(sys)     % system poles
zero(sys)     % system (transmission) zeros
dcgain(sys)   % DC gain
norm(sys)     % system norms (H2 and Linfinity)
covar(sys,W)  % covariance of response to white noise
pade(sys)     % Pade approximation of input delays
```

Operations on LTI Models

You can perform simple matrix operations, such as addition, multiplication, or concatenation on LTI models. See Chapter 3, “Operations on LTI Models” for more information. Thanks to MATLAB object-oriented programming capabilities, these operations assume appropriate functionalities when applied to LTI models. For example, addition performs a parallel interconnection. Type

```
tf(1,[1 0]) + tf([1 1],[1 2])% 1/s + (s+1)/(s+2)
```

and MATLAB responds:

```
Transfer function
s^2 + 2 s + 2
-------------
  s^2 + 2 s
```

Multiplication performs a series interconnection. Type

```
2 * tf(1,[1 0])*tf([1 1],[1 2])% 2*1/s*(s+1)/(s+2)
```

and MATLAB responds

```
Transfer function:
2 s + 2
---------
s^2 + 2 s
```

If the operands are models of different types, the resulting model type is determined by precedence rules; see “Precedence Rules” on page 2-5 for more information. State-space models have highest precedence while transfer functions have lowest precedence. Hence the sum of a transfer function and a state-space model is always a state-space model.

Other available operations include system inversion, transposition, and pertransposition; see “Inversion and Related Operations” on page 3-13. Matrix-like indexing for extracting subsystems is also supported; see “Extracting and Modifying Subsystems” on page 3-5 for more information. For instance, if sys is a MIMO system with two inputs and three outputs,

```
sys(3,1)
```

extracts the subsystem mapping the first input to the third output. Note that row indices select the outputs while column indices select the inputs. Similarly,

```
sys(3,1) = tf(1,[1 0])
```

redefines the relation between the first input and third output as an integrator.

Continuous/Discrete Conversions

The commands `c2d`, `d2c`, and `d2d` perform continuous to discrete, discrete to continuous, and discrete to discrete (resampling) conversions, respectively.

```
sysd = c2d(sysc,Ts) % discretization w/ sample period Ts
sysc = d2c(sysd)    % equivalent continuous-time model
sysd1= d2d(sysd,Ts) % resampling at the period Ts
```

See “Continuous/Discrete Conversions of LTI Models” on page 3-20 for more information.

Various discretization/interpolation methods are available, including zero-order hold (default), first-order hold, Tustin approximation with or without prewarping, and matched zero-pole. For example,

```
sysd = c2d(sysc,Ts,'foh')% uses first-order hold
sysc = d2c(sysd,'tustin')% uses Tustin approx.
```

Time and Frequency Response

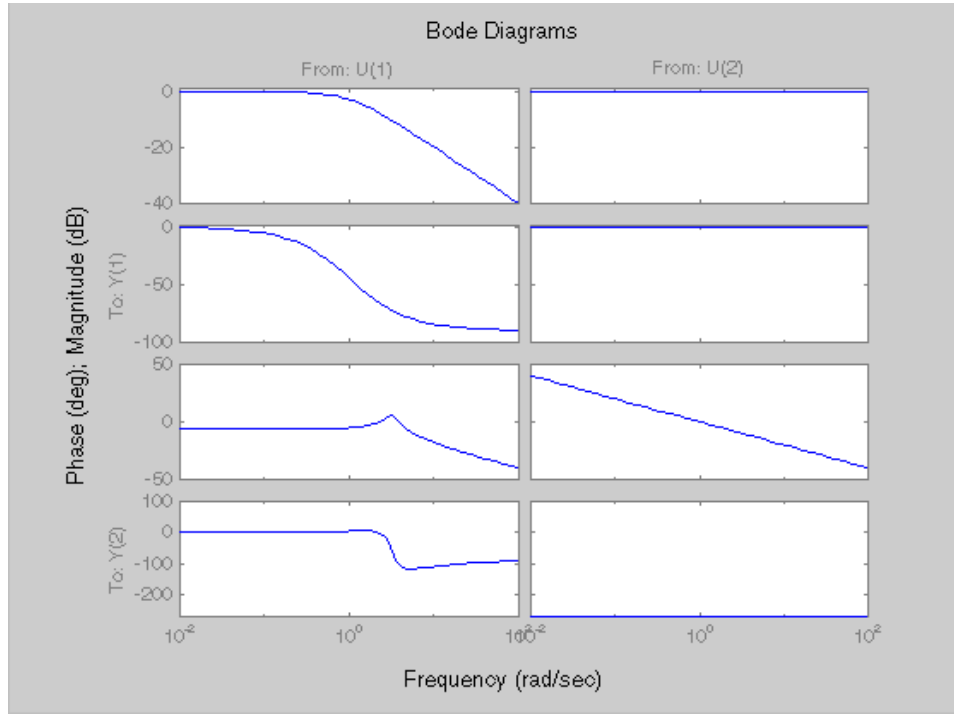
The following commands produce various time and frequency response plots for LTI models (see “Time and Frequency Response” on page 5-9 for more information).

```
step(sys)        % step response
impulse(sys)     % impulse response
initial(sys,x0)  % undriven response to initial condition
lsim(sys,u,t,x0) % response to input u

bode(sys)        % Bode plot
nyquist(sys)     % Nyquist plot
nichols(sys)     % Nichols plot
sigma(sys)       % singular value plot
freqresp(sys,w)  % complex frequency response
```

These commands work for both continuous- and discrete-time LTI models sys without restriction on the number of inputs or outputs. For MIMO systems, they produce an array of plots with one plot per I/O channel. For example,

```
sys = [tf(1,[1 1]) 1 ; tf([1 5],[1 1 10]) tf(-1,[1 0])];
bode(sys)
```

produces the Bode plot shown below.



To superimpose and compare the responses of several LTI systems, use the syntax

```
bode(sys1,sys2,sys3,...)
```

You can also control the plot style by specifying a color/linestyle/marker for each system, much as with the `plot` command; see “Plotting and Comparing Multiple Systems” on page 5-13 for more information. For example,

```
bode(sys1,'r',sys2,'b--')
```

draws the response of `sys1` with a red solid line and the response of `sys2` with a dashed blue line.

These commands automatically determine an appropriate simulation horizon or frequency range based on the system dynamics. To override the default range, type

```
step(sys,tfinal)       % final time = tfinal
bode(sys,{wmin,wmax})  % freq. range = [wmin,wmax]
```

The LTI Viewer

You can also analyze time and frequency domain responses using the LTI Viewer; see “The LTI Viewer” on page 6-1 for more information. The LTI Viewer is an interactive user interface that assists you with the analysis of LTI model responses by facilitating such functions as:

- Toggling between types of response plots
- Plotting responses of several LTI models
- Zooming into regions of the response plots
- Calculating response characteristics, such as settling time
- Displaying different I/O channels
- Changing the plot styles of the response plots

To initialize an LTI Viewer, type

```
ltiview
```

`ltiview` can also be called with additional input arguments that allow you to specify the type of LTI model response displayed when the window is first opened. The generic syntax is

```
ltiview(plottype,sys1,...,sysn)
```

where `sys1,...,sysn` are names of LTI models in the MATLAB workspace and *`plottype`* is either a string for one of the following plot types, or a cell array, containing up to six of the following strings

```
'step'
'impulse'
'initial'
'lsim'
'pzmap'
'bode'
'nyquist'
'nichols'
'sigma'
```

For example, you can initialize an LTI Viewer showing the step response of the LTI model `sys` by

```
ltiview('step',sys)
```

For more detail on the use of the LTI Viewer and how it can be integrated into a Simulink diagram, see Chapter 6, “The LTI Viewer.”

System Interconnections

You can derive LTI models for various system interconnections ranging from simple series connections to more complex block diagrams; see “Model Interconnection Functions” on page 3-16 for more information. Related commands include

```
append(sys1,sys2,...) % appends systems inputs and outputs
parallel(sys1,sys2)   % general parallel connection
series(sys1,sys2)     % general series connection
feedback(sys1,sys2)   % feedback loop
lft(sys1,sys2)        % LFT interconnection (star product)
connect(sys,q)        % state-space model of block diagram
```

Note that simple parallel and series interconnections can be performed by direct addition and multiplication, respectively.

When combining LTI models of different types (for example, state-space `sys1` and transfer function `sys2`), the type of the resultant model is determined by the same precedence rules as for arithmetic operations. See “Precedence Rules” on page 2-5 for more information. Specifically, the ranking of the different types of LTI models from highest to lowest precedence is FRD, SS, ZPK, and TF.

Control Design Tools

The Control System Toolbox supports three mainstream control design methodologies: gain selection from root locus, pole placement, and linear-quadratic-Gaussian (LQG) regulation. The first two techniques are covered by the `rlocus` and `place` commands. The LQG design tools include commands to compute the LQ-optimal state-feedback gain (`lqr`, `dlqr`, and `lqry`), to design the Kalman filter (`kalman`), and to form the resulting LQG regulator (`lqgreg`). See "LQG Design" on page 7-8 for more information.

As an example of LQG design, consider the regulation problem illustrated by Figure 1-1. The goal is to regulate the plant output y around zero. The system is driven by the white noise disturbance d, there is some measurement noise n, and the noise intensities are given by

$$E(d^2) = 1, \qquad E(n^2) = 0.01$$

The cost function

$$J(u) = \int_0^\infty (10y^2 + u^2)dt$$

is used to specify the trade-off between regulation performance and cost of control. Note that an open-loop state-space model is

$$\dot{x} = Ax + Bu + Bd \qquad \text{(state equations)}$$
$$y_n = Cx + n \qquad \text{(measurements)}$$

where (A, B, C) is a state-space realization of $100/(s^2 + s + 100)$.

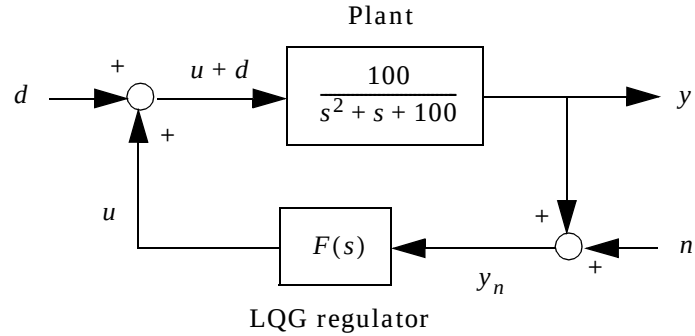


Figure 1-1: Simple Regulation Loop

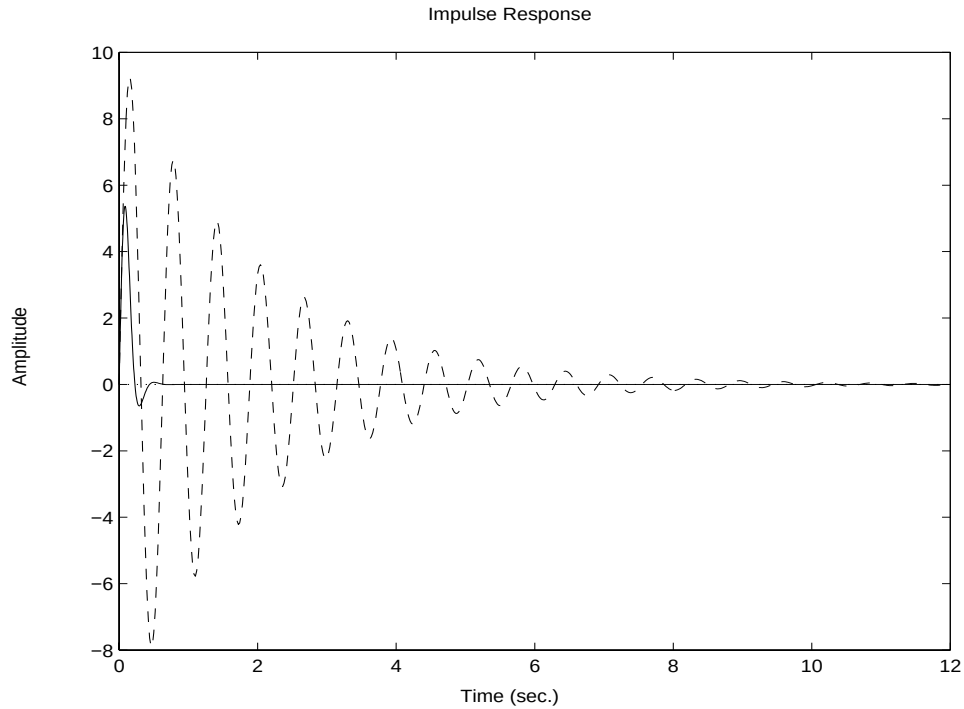
The following commands design the optimal LQG regulator $F(s)$ for this problem.

```
sys = ss(tf(100,[1 1 100]))% state-space plant model

% Design LQ-optimal gain K
K = lqry(sys,10,1) % u = -Kx minimizes J(u)

% Separate control input u and disturbance input d
P = sys(:,[^ ^]);
% input [u;d], output y

% Design Kalman state estimator KEST
Kest = kalman(P,1,0.01)

% Form LQG regulator = LQ gain + Kalman filter
F = lqgreg(Kest,K)
```

The last command returns a state-space model `F` of the LQG regulator $F(s)$. Note that `lqry`, `kalman`, and `lqgreg` perform discrete-time LQG design when applied to discrete plants.

To validate the design, close the loop with `feedback` and compare the open- and closed-loop impulse responses with `impulse`.

```
% Close loop
clsys = feedback(sys,F,+1)   % Note positive feedback

% Open- vs. closed-loop impulse responses
impulse(sys,'r--',clsys,'b-')
```



The Root Locus Design GUI

You can also design a compensator using the Root Locus Design Graphical User Interface (GUI). See Chapter 8, “The Root Locus Design GUI” for more information. The Root Locus Design GUI is an interactive graphical user interface that assists you in designing a compensator by providing the following features:

- Tuning the value of the feedback or compensator gain
- Adding/removing compensator poles and zeros directly on the root locus plot
- Dragging compensator poles and zeros around in the root locus plane
- Examining changes in the closed-loop response whenever the compensator is changed
- Drawing boundaries on the root locus plane for parameters such as minimum damping ratio, etc.
- Zooming in to regions of the root locus

To initialize the Root Locus Design GUI, simply type

```
rltool
```

`rltool` can also be called with additional input arguments that allow you to initialize the plant and compensator used in the Root Locus Design GUI. For example,

```
rltool(sys)
```

initializes a Root Locus Design GUI with the linear time invariant (LTI) object `sys` as the plant. Adding a second input argument, as in

```
rltool(sys,comp)
```

also initializes the LTI object `comp` as the root locus compensator.

When one or two input arguments are provided, the root locus of the closed-loop poles and their locations for the current compensator gain are drawn. The closed-loop model is generated by placing the compensator and design model in the forward path of a negative unity feedback system.

2

LTI Models

Introduction

The Control System Toolbox offers extensive tools to manipulate and analyze linear time-invariant (LTI) models. It supports both continuous- and discrete-time systems. Systems can be single-input/single-output (SISO) or multiple-input/multiple-output (MIMO). In addition, you can store several LTI models in an array under a single variable name. See Chapter 4, "Arrays of LTI Models" for information on LTI arrays.

This section introduces key concepts about the MATLAB representation of LTI models, including LTI objects, precedence rules for operations, and an analogy between LTI systems and matrices. In addition, it summarizes the basic commands you can use on LTI objects.

LTI Models

You can specify LTI models as:

- Transfer functions (TF), for example,

$$P(s) = \frac{s+2}{s^2+s+10}$$

- Zero-pole-gain models (ZPK), for example,

$$H(z) = \begin{bmatrix} \frac{2(z-0.5)}{z(z+0.1)} & \frac{(z^2+z+1)}{(z+0.2)(z+0.1)} \end{bmatrix}$$

- State-space models (SS), for example,

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

  where A, B, C, and D are matrices of appropriate dimensions, x is the state vector, and u and y are the input and output vectors.

- Frequency response data (FRD) models

  FRD models consist of sampled measurements of a system's frequency response. For example, you can store experimentally collected frequency response data in an FRD.

Using LTI Models in the Control System Toolbox

You can manipulate TF, SS, and ZPK models using the arithmetic and model interconnection operations described in Chapter 3, "Operations on LTI Models," and analyze them using the model analysis functions, such as `bode` and `step`, described in Chapter 5, "Model Analysis Tools." FRD models can be manipulated and analyzed in much the same way you analyze the other model types, but analysis is restricted to frequency-domain methods.

Using a variety of design techniques, you can design compensators for systems specified with TF, ZPK, SS, and FRD models. These techniques include root locus analysis, pole placement, LQG optimal control, and frequency domain loop-shaping. For FRD models, you can either:

- Obtain an identified TF, SS, or ZPK model using system identification techniques.
- Use frequency-domain analysis techniques.

Other Uses of FRD Models

FRD models are unique model types available in the Control System Toolbox collection of LTI model types, in that they don't have a parametric representation. In addition to the standard operations you may perform on FRD models, you can also use them to:

- Perform frequency-domain analysis on systems with nonlinearities using describing functions.
- Validate identified models against experimental frequency response data.

LTI Objects

Depending on the type of model you use, the data for your model may consist of a simple numerator/denominator pair for SISO transfer functions, four matrices for state-space models, and multiple sets of zeros and poles for MIMO zero-pole-gain models or frequency and response vectors for FRD models. For convenience, the Control System Toolbox provides customized data structures (*LTI objects*) for each type of model. These are called the TF, ZPK, SS, and FRD objects. These four LTI objects encapsulate the model data and enable you to manipulate LTI systems as single entities rather than collections of data vectors or matrices.

Creating an LTI Object: An Example

An LTI object of the type TF, ZPK, SS, or FRD is created whenever you invoke the corresponding constructor function, `tf`, `zpk`, `ss`, or `frd`. For example,

```
P = tf([1 2],[1 1 10])
```

creates a TF object, `P`, that stores the numerator and denominator coefficients of the transfer function

$$P(s) = \frac{s+2}{s^2+s+10}$$

See "Creating LTI Models" on page 2-8 for methods for creating all of the LTI object types.

LTI Properties and Methods

The LTI object implementation relies on MATLAB object-oriented programming capabilities. Objects are MATLAB structures with an additional flag indicating their class (TF, ZPK, SS, or FRD for LTI objects) and have pre-defined fields called *object properties*. For LTI objects, these properties include the model data, sample time, delay times, input or output names, and input or output groups (see "LTI Properties" on page 2-26 for details). The functions that operate on a particular object are called the object *methods*. These may include customized versions of simple operations such as addition or multiplication. For example,

```
P = tf([1 2],[1 1 10])
Q = 2 + P
```

performs transfer function addition.

$$Q(s) = 2 + P(s) = \frac{2s^2+3s+22}{s^2+s+10}$$

The object-specific versions of such standard operations are called *overloaded* operations. For more details on objects, methods, and object-oriented programming, see Chapter 14, "Classes and Objects" in *Using MATLAB*. For details on operations on LTI objects, see Chapter 3, "Operations on LTI Models."

Precedence Rules

Operations like addition and commands like `feedback` operate on more than one LTI model at a time. If these LTI models are represented as LTI objects of different types (for example, the first operand is TF and the second operand is SS), it is not obvious what type (for example, TF or SS) the resulting model should be. Such type conflicts are resolved by *precedence rules*. Specifically, TF, ZPK, SS, and FRD objects are ranked according to the precedence hierarchy.

FRD > SS > ZPK > TF

Thus ZPK takes precedence over TF, SS takes precedence over both TF and ZPK, and FRD takes precedence over all three. In other words, any operation involving two or more LTI models produces:

- An FRD object if at least one operand is an FRD object
- An SS object if no operand is an FRD object and at least one operand is an SS object
- A ZPK object if no operand is an FRD or SS object and at least one is an ZPK object
- A TF object only if all operands are TF objects

Operations on systems of different types work as follows: the resulting type is determined by the precedence rules, and all operands are first converted to this type before performing the operation.

Viewing LTI Systems As Matrices

In the frequency domain, an LTI system is represented by the linear input/output map

$$y = Hu$$

This map is characterized by its transfer matrix H, a function of either the Laplace or Z-transform variable. The transfer matrix H maps inputs to outputs, so there are as many columns as inputs and as many rows as outputs.

If you think of LTI systems in terms of (transfer) matrices, certain basic operations on LTI systems are naturally expressed with a matrix-like syntax.

For example, the parallel connection of two LTI systems `sys1` and `sys2` can be expressed as

```
sys = sys1 + sys2
```

because parallel connection amounts to adding the transfer matrices. Similarly, subsystems of a given LTI model `sys` can be extracted using matrix-like subscripting. For instance,

```
sys(3,1:2)
```

provides the I/O relation between the first two inputs (column indices) and the third output (row index), which is consistent with

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H(1,1) & H(2,1) \\ H(2,1) & H(2,2) \\ H(3,1) & H(3,2) \end{bmatrix} \begin{bmatrix} u_1 & u_2 \end{bmatrix}$$

for $y = Hu$.

Command Summary

The next two tables give an overview of the main commands you can apply to LTI models.

Table 2-1: Creating LTI Models or Getting Data From Them

Command	Description
`drss`	Generate random discrete state-space model.
`dss`	Create descriptor state-space model.
`filt`	Create discrete filter with DSP convention.
`frd`	Create an FRD model.
`frdata`	Retrieve FRD model data.
`get`	Query LTI model properties.
`rss`	Generate random continuous state-space model.

Table 2-1: Creating LTI Models or Getting Data From Them (Continued)

Command	Description
`set`	Set LTI model properties.
`size`	Get output/input/array dimensions or model order.
`ss`	Create a state-space model.
`ssdata`, `dssdata`	Retrieve state-space data (respectively, descriptor state-space data) or convert it to cell array format.
`tf`	Create a transfer function.
`tfdata`	Retrieve transfer function data.
`zpk`	Create a zero-pole-gain model.
`zpkdata`	Retrieve zero-pole-gain data.

Table 2-2: Converting LTI Models

Command	Description
`c2d`	Continuous- to discrete-time conversion.
`d2c`	Discrete- to continuous-time conversion.
`d2d`	Resampling of discrete-time models.
`frd`	Conversion to an FRD model.
`pade`	Padé approximation of input delays.
`ss`	Conversion to state space.
`tf`	Conversion to transfer function.
`zpk`	Conversion to zero-pole-gain.

Creating LTI Models

The functions `tf`, `zpk`, `ss`, and `frd` create transfer function models, zero-pole-gain models, state-space models, and frequency response data models, respectively. These functions take the model data as input and produce TF, ZPK, SS, or FRD objects that store this data in a single MATLAB variable. This section shows how to create continuous or discrete, SISO or MIMO LTI models with `tf`, `zpk`, `ss`, and `frd`.

Note: You can only specify TF, ZPK, and SS models for systems whose transfer matrices have real-valued coefficients.

Transfer Function Models

This section explains how to specify continuous-time SISO and MIMO transfer function models. The specification of discrete-time transfer function models is a simple extension of the continuous-time case (see "Discrete-Time Models" on page 2-20). In this section you can also read about how to specify transfer functions consisting of pure gains.

SISO Transfer Function Models

A continuous-time SISO transfer function

$$h(s) = \frac{n(s)}{d(s)}$$

is characterized by its numerator $n(s)$ and denominator $d(s)$, both polynomials of the Laplace variable s.

There are two ways to specify SISO transfer functions:

- Using the `tf` command
- As rational expressions in the Laplace variable s

To specify a SISO transfer function model $h(s) = n(s)/d(s)$ using the `tf` command, type

```
h = tf(num,den)
```

where num and den are row vectors listing the coefficients of the polynomials $n(s)$ and $d(s)$, respectively, when these polynomials are ordered in *descending* powers of s. The resulting variable h is a TF object containing the numerator and denominator data.

For example, you can create the transfer function $h(s) = s/(s^2 + 2s + 10)$ by typing

```
h = tf([1 0],[1 2 10])
```

MATLAB responds with

```
Transfer function:
      s
--------------
s^2 + 2 s + 10
```

Note the customized display used for TF objects.

You can also specify transfer functions as rational expressions in the Laplace variable s by:

1. Defining the variable s as a special TF model

   ```
   s = tf('s');
   ```

2. Entering your transfer function as a rational expression in s

For example, once s is defined with tf as in 1,

```
H = s/(s^2 + 2*s +10);
```

produces the same transfer function as

```
h = tf([1 0],[1 2 10]);
```

Note: You need only define the variable s as a TF model once. All of the subsequent models you create using rational expressions of s are specified as TF objects, unless you convert the variable s to ZPK. See "Model Conversion" on page 2-42 for more information.

MIMO Transfer Function Models

MIMO transfer functions are two-dimensional arrays of elementary SISO transfer functions. There are several ways to specify MIMO transfer function models, including:

- Concatenation of SISO transfer function models
- Using `tf` with cell array arguments

Consider the rational transfer matrix

$$H(s) = \begin{bmatrix} \dfrac{s-1}{s+1} \\ \dfrac{s+2}{s^2+4s+5} \end{bmatrix}$$

You can specify $H(s)$ by concatenation of its SISO entries. For instance,

```
h11 = tf([1 –1],[1 1]);
h21 = tf([1 2],[1 4 5]);
```

or, equivalently,

```
s = tf('s')
h11 = (s–1)/(s+1);
h21 = (s+2)/(s^2+4*s+5);
```

can be concatenated to form $H(s)$.

```
H = [h11; h21]
```

This syntax mimics standard matrix concatenation and tends to be easier and more readable for MIMO systems with many inputs and/or outputs. See "Model Interconnection Functions" on page 3-16 for more details on concatenation operations for LTI systems.

Alternatively, to define MIMO transfer functions using `tf`, you need two cell arrays (say, `N` and `D`) to represent the sets of numerator and denominator polynomials, respectively. See Chapter 13, "Structures and Cell Arrays" in *Using MATLAB* for more details on cell arrays.

For example, for the rational transfer matrix $H(s)$, the two cell arrays `N` and `D` should contain the row-vector representations of the polynomial entries of

$$N(s) = \begin{bmatrix} s-1 \\ s+2 \end{bmatrix} \qquad D(s) = \begin{bmatrix} s+1 \\ s^2+4s+5 \end{bmatrix}$$

You can specify this MIMO transfer matrix $H(s)$ by typing

```
N = {[1 -1];[1 2]}; % cell array for N(s)
D = {[1 1];[1 4 5]}; % cell array for D(s)
H = tf(N,D)
```

MATLAB responds with

```
Transfer function from input to output...
      s - 1
 #1:  -----
      s + 1

          s + 2
 #2:  -------------
      s^2 + 4 s + 5
```

Notice that both `N` and `D` have the same dimensions as H. For a general MIMO transfer matrix $H(s)$, the cell array entries `N{i,j}` and `D{i,j}` should be row-vector representations of the numerator and denominator of $H_{ij}(s)$, the ijth entry of the transfer matrix $H(s)$.

Pure Gains

You can use `tf` with only one argument to specify simple gains or gain matrices as TF objects. For example,

```
G = tf([1 0;2 1])
```

produces the gain matrix

$$G = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

while

```
E = tf
```

creates an empty transfer function.

Zero-Pole-Gain Models

This section explains how to specify continuous-time SISO and MIMO zero-pole-gain models. The specification for discrete-time zero-pole-gain models is a simple extension of the continuous-time case. See "Discrete-Time Models" on page 2-20.

SISO Zero-Pole-Gain Models

Continuous-time SISO zero-pole-gain models are of the form

$$h(s) = k\frac{(s-z_1)\ldots(s-z_m)}{(s-p_1)\ldots(s-p_n)}$$

where k is a real-valued scalar (the *gain*), and $z_1,\ldots,z_m$ and $p_1,\ldots,p_n$ are the real or complex conjugate pairs of zeros and poles of the transfer function $h(s)$. This model is closely related to the transfer function representation: the zeros are simply the numerator roots, and the poles, the denominator roots.

There are two ways to specify SISO zero-pole-gain models:

- Using the `zpk` command
- As rational expressions in the Laplace variable `s`

The syntax to specify ZPK models directly using `zpk` is

```
h = zpk(z,p,k)
```

where `z` and `p` are the vectors of zeros and poles, and `k` is the gain. This produces a ZPK object `h` that encapsulates the `z`, `p`, and `k` data. For example, typing

```
h = zpk(0, [1-i 1+i 2], -2)
```

produces

```
Zero/pole/gain:
        -2 s
--------------------
(s-2) (s^2 - 2s + 2)
```

You can also specify zero-pole-gain models as rational expressions in the Laplace variable *s* by:

1 Defining the variable `s` as a ZPK model

```
s = zpk('s')
```

2 Entering the transfer function as a rational expression in `s`.

For example, once `s` is defined with `zpk`,

```
H = -2s/((s - 2)*(s^2 + 2*s + 2));
```

returns the same ZPK model as

```
h = zpk([0], [2 -1-i -1+i ], -2);
```

Note: You need only define the ZPK variable `s` once. All subsequent rational expressions of `s` will be ZPK models, unless you convert the variable `s` to TF. See "Model Conversion" on page 2-42 for more information on conversion to other model types.

MIMO Zero-Pole-Gain Models

Just as with TF models, you can also specify a MIMO ZPK model by concatenation of its SISO entries (see "Model Interconnection Functions" on page 3-16).

You can also use the command `zpk` to specify MIMO ZPK models. The syntax to create a *p*-by-*m* MIMO zero-pole-gain model using `zpk` is

```
H = zpk(Z,P,K)
```

where

- `Z` is the *p*-by-*m* cell array of zeros (`Z{i,j}` = zeros of $H_{ij}(s)$)
- `P` is the *p*-by-*m* cell array of poles (`P{i,j}` = poles of $H_{ij}(s)$)
- `K` is the *p*-by-*m* matrix of gains (`K(i,j)` = gain of $H_{ij}(s)$)

For example, typing

```
Z = {[],-5;[1-i 1+i] []};

P = {0,[-1 -1];[1 2 3],[]};

K = [-1  3;2  0];

H = zpk(Z,P,K)
```

creates the two-input/two-output zero-pole-gain model

$$H(s) = \begin{bmatrix} \dfrac{-1}{s} & \dfrac{3(s+5)}{(s+1)^2} \\ \dfrac{2(s^2-2s+2)}{(s-1)(s-2)(s-3)} & 0 \end{bmatrix}$$

Notice that you use `[]` as a place-holder in `Z` (or `P`) when the corresponding entry of $H(s)$ has no zeros (or poles).

State-Space Models

State-space models rely on linear differential or difference equations to describe the system dynamics. Continuous-time models are of the form

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

where x is the state vector and u and y are the input and output vectors. Such models may arise from the equations of physics, from state-space identification, or by state-space realization of the system transfer function.

Use the command `ss` to create state-space models

```
sys = ss(A,B,C,D)
```

For a model with `Nx` states, `Ny` outputs, and `Nu` inputs

- `A` is an `Nx`-by-`Nx` real-valued matrix.
- `B` is an `Nx`-by-`Nu` real-valued matrix.
- `C` is an `Ny`-by-`Nx` real-valued matrix.
- `D` is an `Ny`-by-`Nu` real-valued matrix.

This produces an SS object `sys` that stores the state-space matrices A, B, C, and D. For models with a zero D matrix, you can use `D = 0` (zero) as a shorthand for a zero matrix of the appropriate dimensions.

As an illustration, consider the following simple model of an electric motor.

$$\frac{d^2\theta}{dt^2} + 2\frac{d\theta}{dt} + 5\theta = 3I$$

where θ is the angular displacement of the rotor and I the driving current.

The relation between the input current $u = I$ and the angular velocity $y = d\theta/dt$ is described by the state-space equations

$$\frac{dx}{dt} = Ax + Bu$$
$$y = Cx$$

where

$$x = \begin{bmatrix} \theta \\ \frac{d\theta}{dt} \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

This model is specified by typing

```
sys = ss([0 1;–5 –2],[0;3],[0 1],0)
```

to which MATLAB responds

```
a =
                    x1          x2
        x1           0     1.00000
        x2    −5.00000    −2.00000

b =
                    u1
        x1           0
        x2     3.00000

c =
                    x1          x2
        y1           0     1.00000

d =
                    u1
        y1           0
```

In addition to the *A*, *B*, *C*, and *D* matrices, the display of state-space models includes state names, input names, and output names. Default names (here, x1, x2, u1, and y1) are displayed whenever you leave these unspecified. See "LTI Properties" on page 2-26 for more information on how to specify state, input, or output names.

Descriptor State-Space Models

Descriptor state-space (DSS) models are a generalization of the standard state-space models discussed above. They are of the form

$$E\frac{dx}{dt} = Ax + Bu$$
$$y = Cx + Du$$

The Control System Toolbox supports only descriptor systems with a nonsingular E matrix. While such models have an equivalent explicit form

$$\frac{dx}{dt} = (E^{-1}A)x + (E^{-1}B)u$$
$$y = Cx + Du$$

it is often desirable to work with the descriptor form when the E matrix is poorly conditioned with respect to inversion.

The function `dss` is the counterpart of `ss` for descriptor state-space models. Specifically,

```
sys = dss(A,B,C,D,E)
```

creates a continuous-time DSS model with matrix data `A,B,C,D,E`. For example, consider the dynamical model

$$J\frac{d\omega}{dt} + F\omega = T$$
$$y = \omega$$

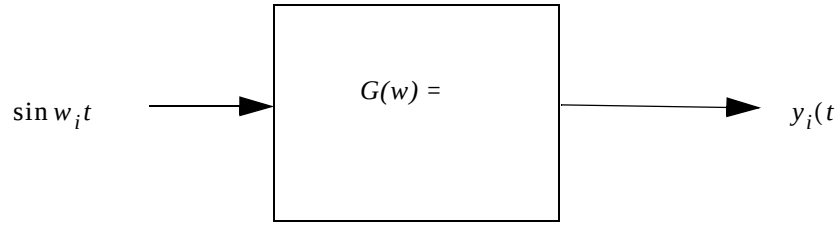
with vector ω of angular velocities. If the inertia matrix J is poorly conditioned with respect to inversion, you can specify this system as a descriptor model by

```
sys = dss(-F,eye(n),eye(n),0,J)   % n = length of vector ω
```

Frequency Response Data (FRD) Models

In some instances, you may only have sampled frequency response data, rather than a transfer function or state-space model for the system you want to analyze or control. For information on frequency response analysis of linear systems, see Chapter 8 of [1].

For example, suppose the frequency response function for the SISO system you want to model is *G(w)*. Suppose, in addition, that you perform an experiment to evaluate *G(w)* at a fixed set of frequencies, $w_1, w_2, \ldots, w_n$. You can do this by driving the system with a sequence of sinusoids at each of these frequencies, as depicted below.



Here w_i is the input frequency of each sinusoid, $i = 1 \ldots n$, and $G(w) = |G(w)|\exp(j\angle G(w))$. The steady state output response of this system satisfies

$$y_i(t) = |G(w_i)|\sin(w_i t + \angle G(w_i)); \quad i = 1 \ldots n$$

A *frequency response data* (FRD) *object* is a model form you can use to store frequency response data (complex frequency response, along with a corresponding vector of frequency points) that you obtain either through simulations or experimentally. In this example, the frequency response data is obtained from the set of response pairs: $\{(G(w_i), w_i)\}$, $i = 1 \ldots n$.

Once you store your data in an FRD model, you can treat it as an LTI model, and manipulate an FRD model in most of the same ways you manipulate TF, SS, and ZPK models.

The basic syntax for creating a SISO FRD model is

```
sys = frd(response,frequencies,units)
```

where

- `frequencies` is a real vector of length `Nf`.
- `response` is a vector of length `Nf` of complex frequency response values for these frequencies.
- `units` is an optional string for the units of frequency: either `'rad/s'` (default) or `'Hz'`

For example, the MAT-file `LTIexamples.mat` contains a frequency vector `freq`, and a corresponding complex frequency response data vector `respG`. To load this frequency-domain data and construct an FRD model, type

```
load LTIexamples
sys = frd(respG,freq)

Continuous-time frequency response with 1 output and 1 input
at 5 frequency points.

From input 1 to:
Frequency(rad/s)        output 1
----------------        --------
        1        -0.812505 -0.000312i
        2        -0.092593 -0.462963i
        4        -0.075781 -0.001625i
        5        -0.043735 -0.000390i
```

The syntax for creating a MIMO FRD model is the same as for the SISO case, except that `response` is a p-by-m-by-N_f multidimensional array, where p is the number of outputs, m is the number of inputs, and N_f is the number of frequency data points (the length of `frequency`).

The following table summarizes the complex-valued response data format for FRD models.

Table 2-3: Data Format for the Argument response in FRD Models

Model Form	Response Data Format
SISO model	Vector of length `Nf` for which `response(i)` is the frequency response at the frequency `frequency(i)`
MIMO model with `Ny` outputs and `Nu` inputs	`Ny`-by-`Nu`-by-`Nf` multidimensional array for which `response(i,j,k)` specifies the frequency response from input `j` to output `i` at frequency `frequency(k)`
`S1`-by-...-by-`Sn` array of models with `Ny` outputs and `Nu` inputs	`Ny`-by-`Nu`-by-`S1`-by-...-by-`Sn` multidimensional array, for which `response(i,j,k,:)` specifies the array of frequency response data from input `j` to output `i` at frequency `frequency(k)`

Discrete-Time Models

Creating discrete-time models is very much like creating continuous-time models, except that you must also specify a sampling period or *sample time* for discrete-time models. The sample time value should be scalar and expressed in seconds. You can also use the value –1 to leave the sample time unspecified.

To specify discrete-time LTI models using `tf`, `zpk`, `ss`, or `frd`, simply append the desired sample time value `Ts` to the list of inputs.

```
sys1 = tf(num,den,Ts)
sys2 = zpk(z,p,k,Ts)
sys3 = ss(a,b,c,d,Ts)
sys4 = frd(response,frequency,Ts)
```

For example,

```
h = tf([1 –1],[1 –0.5],0.1)
```

creates the discrete-time transfer function $h(z) = (z-1)/(z-0.5)$ with sample time 0.1 seconds, and

```
sys = ss(A,B,C,D,0.5)
```

specifies the discrete-time state-space model

$$x[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sampling period 0.5 second. The vectors $x[n]$, $u[n]$, $y[n]$ denote the values of the state, input, and output vectors at the nth sample.

By convention, the sample time of continuous-time models is `Ts = 0`. Setting `Ts = –1` leaves the sample time of a discrete-time model unspecified. For example,

```
h = tf([1 –0.2],[1 0.3],–1)
```

produces

```
Transfer function:
z - 0.2
-------
z + 0.3

Sampling time: unspecified
```

Note: Do not simply omit `Ts` in this case. This would make `h` a continuous-time transfer function.

If you forget to specify the sample time when creating your model, you can still set it to the correct value by reassigning the LTI property `Ts`. See "Sample Time" on page 2-34 for more information on setting this property.

Discrete-Time TF and ZPK Models

You can specify discrete-time TF and ZPK models using `tf` and `zpk` as indicated above. Alternatively, it is often convenient to specify such models by:

1. Defining the variable *z* as a particular discrete-time TF or ZPK model with the appropriate sample time
2. Entering your TF or ZPK model directly as a rational expression in *z*.

This approach parallels the procedure for specifying continuous-time TF or ZPK models using rational expressions. This procedure is described in "SISO Transfer Function Models" on page 2-8 and "SISO Zero-Pole-Gain Models" on page 2-12.

For example,

```
z = tf('z', 0.1);
H = (z+2)/(z^2 + 0.6*z + 0.9);
```

creates the same TF model as

```
H = tf([1 2], [1 0.6 0.9], 0.1);
```

Similarly,

```
z = zpk('z', 0.1);
H = [z/(z+0.1)/(z+0.2) ; (z^2+0.2*z+0.1)/(z^2+0.2*z+0.01)]
```

produces the single-input, two-output ZPK model

```
Zero/pole/gain from input to output...
              z
 #1:  ---------------
      (z+0.1) (z+0.2)

      (z^2 + 0.2z + 0.1)
 #2:  ------------------
           (z+0.1)^2

Sampling time: 0.1
```

Note that:

- The syntax `z = tf('z')` is equivalent to `z = tf('z',-1)` and leaves the sample time unspecified. The same applies to `z = zpk('z')`.
- Once you have defined `z` as indicated above, any rational expressions in `z` creates a discrete-time model of the same type and with the same sample time as `z`.

Discrete Transfer Functions in DSP Format

In digital signal processing (DSP), it is customary to write discrete transfer functions as rational expressions in z^{-1} and to order the numerator and denominator coefficients in *ascending powers of* z^{-1}. For example, the numerator and denominator of

$$H(z^{-1}) = \frac{1 + 0.5z^{-1}}{1 + 2z^{-1} + 3z^{-2}}$$

would be specified as the row vectors `[1 0.5]` and `[1 2 3]`, respectively. When the numerator and denominator have different degrees, this convention clashes with the "*descending powers of z*" convention assumed by `tf` (see "Transfer Function Models" on page 2-8, or `tf` on page 11-224). For example,

```
h = tf([1 0.5],[1 2 3])
```

produces the transfer function

$$\frac{z + 0.5}{z^2 + 2z + 3}$$

which differs from $H(z^{-1})$ by a factor z.

To avoid such convention clashes, the Control System Toolbox offers a separate function `filt` dedicated to the DSP-like specification of transfer functions. Its syntax is

```
h = filt(num,den)
```

for discrete transfer functions with unspecified sample time, and

```
h = filt(num,den,Ts)
```

to further specify the sample time `Ts`. This function creates TF objects just like `tf`, but expects `num` and `den` to list the numerator and denominator coefficients in *ascending powers of* z^{-1}. For example, typing

```
h = filt([1 0.5],[1 2 3])
```

produces

```
Transfer function:
   1 + 0.5 z^-1
-------------------
1 + 2 z^-1 + 3 z^-2

Sampling time: unspecified
```

You can also use `filt` to specify MIMO transfer functions in z^{-1}. Just as for `tf`, the input arguments `num` and `den` are then cell arrays of row vectors of appropriate dimensions (see "Transfer Function Models" on page 2-8 for details). Note that each row vector should comply with the "ascending powers of z^{-1}" convention.

Data Retrieval

The functions `tf`, `zpk`, `ss`, and `frd` pack the model data and sample time in a single LTI object. Conversely, the following commands provide convenient data retrieval for any type of TF, SS, or ZPK model `sys`, or FRD model `sysfr`.

```
[num,den,Ts] = tfdata(sys)     % Ts = sample time
[z,p,k,Ts] = zpkdata(sys)
[a,b,c,d,Ts] = ssdata(sys)
[a,b,c,d,e,Ts] = dssdata(sys)
[response,frequency,Ts] = frdata(sysfr)
```

Note that:

- `sys` can be any type of LTI object, *except* an FRD model
- `sysfr`, the input argument to `frdata`, can only be an FRD model

You can use any variable names you want in the output argument list of any of these functions. The ones listed here correspond to the model property names described in Tables 2-2 – 2.5.

The output arguments `num` and `den` assigned to `tfdata`, and `z` and `p` assigned to `zpkdata`, are cell arrays, even in the SISO case. These cell arrays have as many rows as outputs, as many columns as inputs, and their *ij*th entry specifies the transfer function from the *j*th input to the *i*th output. For example,

```
H = [tf([1 -1],[1 2 10]) , tf(1,[1 0])]
```

creates the one-output/two-input transfer function

$$H(s) = \left[\frac{s-1}{s^2+2s+10} \quad \frac{1}{s}\right]$$

Typing

```
[num,den] = tfdata(H);
num{1,1}, den{1,1}
```

displays the coefficients of the numerator and denominator of the first input channel.

```
ans =
     0     1    -1
ans =
     1     2    10
```

Note that the same result is obtained using

```
H.num{1,1}, H.den{1,1}
```

See "Direct Property Referencing" on page 2-33 for more information about this syntax.

To obtain the numerator and denominator of SISO systems directly as row vectors, use the syntax

```
[num,den,Ts] = tfdata(sys,'v')
```

For example, typing

```
sys = tf([1 3],[1 2 5]);
[num,den] = tfdata(sys,'v')
```

produces

```
num =

     0     1     3

den =

     1     2     5
```

Similarly,

```
[z,p,k,Ts] = zpkdata(sys,'v')
```

returns the zeros, `z`, and the poles, `p`, as *vectors* for SISO systems.

LTI Properties

The previous section shows how to create LTI objects that encapsulate the model data and sample time. You also have the option to attribute additional information, such as the input names or notes on the model history, to LTI objects. This section gives a complete overview of the *LTI properties*, i.e., the various pieces of information that can be attached to the TF, ZPK, SS, and FRD objects. Type `help ltiprops` for online help on available LTI properties.

From a data structure standpoint, the LTI properties are the various fields in the TF, ZPK, SS, and FRD objects. These fields have names (the *property names*) and are assigned values (the *property values*). We distinguish between *generic properties*, common to all four types of LTI objects, and *model-specific properties* that pertain only to one particular type of model.

Generic Properties

The generic properties are those shared by all four types of LTI models (TF, ZPK, SS, and FRD objects). They are listed in the table below.

Table 2-4: Generic LTI Properties

Property Name	Description	Property Value
`IoDelayMatrix`	I/O delay(s)	Matrix
`InputDelay`	Input delay(s)	Vector
`InputGroup`	Input channel groups	Cell array
`InputName`	Input channel names	Cell vector of strings
`Notes`	Notes on the model history	Text
`OutputDelay`	Output delay(s)	Vector
`OutputGroup`	Output channel groups	Cell array
`OutputName`	Output channel names	Cell vector of strings
`Ts`	Sample time	Scalar
`Userdata`	Additional data	Arbitrary

The sample time property `Ts` keeps track of the sample time (in seconds) of discrete-time systems. By convention, `Ts` is `0` (zero) for continuous-time systems, and `Ts` is `–1` for discrete-time systems with unspecified sample time. `Ts` is always a scalar, even for MIMO systems.

The `InputDelay`, `OutputDelay`, and `ioDelayMatrix` properties allow you to specify time delays in the input or output channels, or for each input/output pair. Their default value is zero (no delay). See “Time Delays” on page 2-45 for details on modeling delays.

The `InputName` and `OutputName` properties enable you to give names to the *individual* input and output channels. The value of each of these properties is a cell vector of strings with as many cells as inputs or outputs. For example, the `OutputName` property is set to

```
{ 'temperature' ; 'pressure' }
```

for a system with two outputs labeled `temperature` and `pressure`. The default value is a cell of empty strings.

Using the `InputGroup` and `OutputGroup` properties of LTI objects, you can create different groups of input or output channels, and assign names to the groups. For example, you may want to designate the first four inputs of a five-input model as `controls`, and the last input as `noise`. See “Input Groups and Output Groups” on page 2-37 for more information.

Finally, `Notes` and `Userdata` are available to store additional information on the model. The `Notes` property is dedicated to any text you want to supply with your model, while the `Userdata` property can accommodate arbitrary user-supplied data. They are both empty by default.

For more detailed information on how to use LTI properties, see “Additional Insight into LTI Properties” on page 2-34.

Model-Specific Properties

The remaining LTI properties are specific to one of the four model types (TF, ZPK, SS, or FRD). For single LTI models, these are summarized in the following four tables. The property values differ for LTI arrays. See set on page 11-193 for more information on these values.

Table 2-5: TF-Specific Properties

Property Name	Description	Property Value
den	Denominator(s)	Real cell array of row vectors
num	Numerator(s)	Real cell array of row vectors
Variable	Transfer function variable	String 's', 'p', 'z', 'q', or 'z^-1'

Table 2-6: ZPK-Specific Properties

Property Name	Description	Property Value
k	Gains	Two-dimensional real matrix
p	Poles	Cell array of column vectors
Variable	Transfer function variable	String 's', 'p', 'z', 'q', or 'z^-1'
z	Zeros	Cell array of column vectors

Table 2-7: SS-Specific Properties

Property Name	Description	Property Value
a	State matrix A	2-D real matrix
b	Input-to-state matrix B	2-D real matrix
c	State-to-output matrix C	2-D real matrix
d	Feedthrough matrix D	2-D real matrix

Table 2-7: SS-Specific Properties (Continued)

Property Name	Description	Property Value
`e`	Descriptor E matrix	2-D real matrix
`StateName`	State names	Cell vector of strings

Table 2-8: FRD-Specific Properties

Property Name	Description	Property Value
`Frequency`	Frequency data points	Real-valued vector
`ResponseData`	Frequency response	Complex-valued multidimensional array
`Units`	Units for frequency	String `'rad/s'` or `'Hz'`

Most of these properties are dedicated to storing the model data. Note that the E matrix is set to `[]` (the empty matrix) for standard state-space models, a storage-efficient shorthand for the true value $E = I$.

The `Variable` property is only an attribute of TF and ZPK objects. This property defines the frequency variable of transfer functions. The default values are `'s'` (Laplace variable s) in continuous time and `'z'` (Z-transform variable z) in discrete time. Alternative choices include `'p'` (equivalent to s) and `'q'` or `'z^-1'` for the reciprocal $q = z^{-1}$ of the z variable. The influence of the variable choice is mostly limited to the display of TF or ZPK models. One exception is the specification of discrete-time transfer functions with `tf` (see `tf` on page 11-224 for details).

Note that `tf` produces the same result as `filt` when the `Variable` property is set to `'z^-1'` or `'q'`.

Finally, the `StateName` property is analogous to the `InputName` and `OutputName` properties and keeps track of the state names in state-space models.

Setting LTI Properties

There are three ways to specify LTI property values:

- You can set properties when creating LTI models with `tf`, `zpk`, `ss`, or `frd`.
- You can set or modify the properties of an existing LTI model with `set`.
- You can also set property values using structure-like assignments.

This section discusses the first two options. See "Direct Property Referencing" on page 2-33 for details on the third option.

The function `set` for LTI objects follows the same syntax as its Handle Graphics counterpart. Specifically, each property is updated by a pair of arguments

```
PropertyName,PropertyValue
```

where

- *`PropertyName`* is a string specifying the property name. You can type the property name without regard for the case (upper or lower) of the letters in the name. Actually, you need only type any abbreviation of the property name that uniquely identifies the property. For example, `'user'` is sufficient to refer to the `Userdata` property.
- `PropertyValue` is the value to assign to the property (see `set` on page 11-193 for details on admissible property values).

As an illustration, consider the following simple SISO model for a heating system with an input delay of 0.3 seconds, an input called "energy," and an output called "temperature."

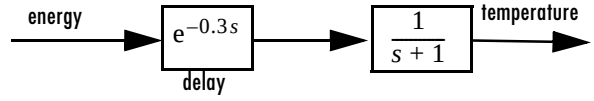


Figure 2-1: A Simple Heater Model

You can use a TF object to represent this delay system, and specify the time delay, the input and output names, and the model history by setting the corresponding LTI properties. You can either set these properties directly when you create the LTI model with `tf`, or by using the `set` command.

For example, you can specify the delay directly when you create the model, and then use the `set` command to assign `InputName`, `OutputName`, and `Notes` to `sys`.

```
sys = tf(1,[1 1],'Inputdelay',0.3);
set(sys,'inputname','energy','outputname','temperature',...
        'notes','A simple heater model')
```

Finally, you can also use the `set` command to obtain a listing of all settable properties for a given LTI model type, along with valid values for these properties. For the transfer function `sys` created above

```
set(sys)
```

produces

```
num:  Ny-by-Nu cell of row vectors (Nu = no. of inputs)
den:  Ny-by-Nu cell of row vectors (Ny = no. of outputs)
Variable:  [ 's' | 'p' | 'z' | 'z^-1' | 'q' ]
Ts:  scalar
InputDelay:  Nu-by-1 vector
OutputDelay:  Ny-by-1 vector
ioDelayMatrix:  Ny-by-Nu array (I/O delays)
InputName:  Nu-by-1 cell array of strings
OutputName:  Ny-by-1 cell array of strings
InputGroup:  M-by-2 cell array if M input groups
OutputGroup:  P-by-2 cell array if P output groups
Notes:  array or cell array of strings
UserData:  arbitrary
```

Accessing Property Values Using get

You access the property values of an LTI model `sys` with `get`. The syntax is

```
PropertyValue = get(sys,PropertyName)
```

where the string *PropertyName* is either the full property name, or any abbreviation with enough characters to identify the property uniquely. For example, typing

```
h = tf(100,[1 5 100],'inputname','voltage',...
                        'outputn','current',...
                        'notes','A simple circuit')
get(h,'notes')
```

produces

```
ans =

    'A simple circuit'
```

To display all of the properties of an LTI model `sys` (and their values), use the syntax `get(sys)`. In this example,

```
get(h)
```

produces

```
num: {[0 0 100]}
den: {[1 5 100]}
Variable: 's'
Ts: 0
InputDelay: 0
OutputDelay: 0
ioDelayMatrix: 0
InputName: {'voltage'}
OutputName: {'current'}
InputGroup: {0x2 cell}
OutputGroup: {0x2 cell}
Notes: {'A simple circuit'}
UserData: []
```

Notice that default (output) values have been assigned to any LTI properties in this list that you have not specified.

Finally, you can also access property values using direct structure-like referencing. This topic is explained in the next section.

Direct Property Referencing

An alternative way to query/modify property values is by structure-like referencing. Recall that LTI objects are basic MATLAB structures except for the additional flag that marks them as TF, ZPK, SS, or FRD objects (see page 2-3). The field names for LTI objects are the property names, so you can retrieve or modify property values with the structure-like syntax.

```
PropertyValue = sys.PropertyName% gets property value
sys.PropertyName = PropertyValue% sets property value
```

These commands are respectively equivalent to

```
PropertyValue = get(sys,'PropertyName')
set(sys,'PropertyName',PropertyValue)
```

For example, type

```
sys = ss(1,2,3,4,'InputName','u');
sys.a
```

and you get the value of the property "a" for the state-space model sys.

```
ans =
     1
```

Similarly,

```
sys.a = -1;
```

resets the state transition matrix for sys to –1. See "LTI Arrays of SS Models with Differing Numbers of States" on page 4-23 for information on setting the properties of LTI arrays of state-space models with different numbers of states in each model.

Unlike standard MATLAB structures, you do not need to type the entire field name or use upper-case characters. You only need to type the minimum number of characters sufficient to identify the property name uniquely. Thus either of the commands

```
sys.InputName
sys.inputn
```

produces

```
ans =

    'u'
```

Any valid syntax for structures extends to LTI objects. For example, given the TF model $h(p) = 1/p$

```
h = tf(1,[1,0],'variable','p');
```

you can reset the numerator to $p + 2$ by typing

```
h.num{1} = [1 2];
```

or equivalently, with

```
h.num{1}(2) = 2;
```

Additional Insight into LTI Properties

By reading this section, you can learn more about using the `Ts`, `InputName`, `OutputName`, `InputGroup`, and `OutputGroup` LTI properties through a set of examples. For basic information on `Notes` and `Userdata`, see "Generic Properties" on page 2-26. For detailed information on the use of `InputDelay`, `OutputDelay`, and `ioDelayMatrix`, see "Time Delays" on page 2-45.

Sample Time

The sample time property `Ts` is used to specify the sampling period (in seconds) for either discrete-time or discretized continuous-time LTI models. Suppose you want to specify

$$H(z) = \frac{z}{2z^2 + z + 1}$$

as a discrete-time transfer function model with a sampling period of 0.5 seconds. To do this, type

```
h = tf([1 0],[2 1 1],0.5);
```

This sets the `Ts` property to the value 0.5, as is confirmed by

```
h.Ts

ans =
    0.5000
```

For continuous-time models, the sample time property `Ts` is 0 by convention. For example, type

```
h = tf(1,[1 0]);
get(h,'Ts')

ans =
     0
```

To leave the sample time of a discrete-time LTI model unspecified, set `Ts` to –1. For example,

```
h = tf(1,[1 –1],–1)
```

produces

```
Transfer function:
  1
-----
z – 1

Sampling time: unspecified
```

The same result is obtained by using the `Variable` property.

```
h = tf(1,[1 –1],'var','z')
```

In operations that combine several discrete-time models, all *specified* sample times must be identical, and the resulting discrete-time model inherits this common sample time. The sample time of the resultant model is unspecified if all operands have unspecified sample times. With this inheritance rule for `Ts`, the following two models are equivalent.

```
tf(0.1,[1 –1],0.1) + tf(1,[1 0.5],–1)
```

and

```
tf(0.1,[1 –1],0.1) + tf(1,[1 0.5],0.1)
```

Note that

```
tf(0.1,[1 -1],0.1) + tf(1,[1 0.5],0.5)
```

returns an error message.

```
??? Error using ==> lti/plus
In SYS1+SYS2, both models must have the same sample time.
```

Caution: Resetting the sample time of a continuous-time LTI model `sys` from zero to a nonzero value does *not* discretize the original model `sys`. The command

```
set(sys,'Ts',0.1)
```

only affects the `Ts` property and does not alter the remaining model data. Use `c2d` and `d2c` to perform continuous-to-discrete and discrete-to-continuous conversions. For example, use

```
sysd = c2d(sys,0.1)
```

to discretize a continuous system `sys` at a 10Hz sampling rate.

Use `d2d` to change the sample time of a discrete-time system and resample it.

Input Names and Output Names

You can use the `InputName` and `OutputName` properties (in short, I/O names) to assign names to any or all of the input and output channels in your LTI model.

For example, you can create a SISO model with input `thrust`, output `velocity`, and transfer function $H(p) = 1/(p + 10)$ by typing

```
h = tf(1,[1 10]);
set(h,'inputname','thrust','outputname','velocity',...
                            'variable','p')
```

Equivalently, you can set these properties directly by typing

```
h = tf(1,[1 10],'inputname','thrust',...
                'outputname','velocity',...
                'variable','p')
```

This produces

```
Transfer function from input "thrust" to output "velocity":
  1
------
p + 10
```

Note how the display reflects the input and output names and the variable selection.

In the MIMO case, use cell vectors of strings to specify input or output channel names. For example, type

```
num = {3 , [1 2]};
den = {[1 10] , [1 0]};
H = tf(num,den);        % H(s) has one output and two inputs

set(H,'inputname',{'temperature' ; 'pressure'})
```

The specified input names appear in the display of `H`.

```
Transfer function from input "temperature" to output:
  3
------
s + 10

Transfer function from input "pressure" to output:
s + 2
-----
  s
```

To leave certain names undefined, use the empty string `''` as in

```
H = tf(num,den,'inputname',{ 'temperature' ; '' })
```

Input Groups and Output Groups

In many applications, you may want to create several (distinct or intersecting) groups of input or output channels and name these groups. For example, you may want to label one set of input channels as `noise` and another set as `controls`.

To see how input and output groups (I/O groups) work:

1. Create a random state-space model with one state, three inputs, and three outputs.

2. Assign the first two inputs to a group named `controls`, the first output to a group named `temperature`, and the last two outputs to a group named `measurements`.

To do this, type

```
h = rss(1,3,3);
set(h, 'InputGroup',{[1 2] 'controls'})
set(h, 'OutputGroup', {[1] 'temperature'; [2 3] 'measurements'})
h
```

and MATLAB returns a state-space model of the following form.

```
a =
                      x1
           x1     -0.64884


b =
                      u1           u2           u3
           x1      0.12533            0            0


c =
                      x1
           y1       1.1909
           y2       1.1892
           y3            0


d =
                      u1           u2           u3
           y1      0.32729            0      -0.1364
           y2            0            0            0
           y3            0       2.1832            0

I/O Groups:
   Group Name     I/O    Channel(s)
    controls       I        1,2
  temperature      O         1
  measurements     O        2,3

Continuous-time model.
```

Notice that the middle column of the I/O group listing indicates whether the group is an input group (`I`) or an output group (`O`).

In general, to specify M input groups (or output groups), you need an M-by-2 cell array organized as follows.

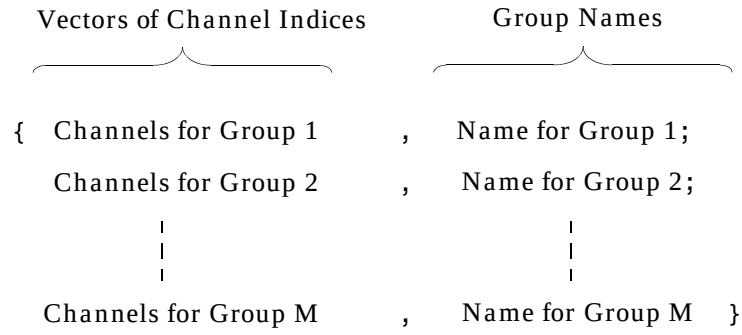


Figure 2-2: Two Column Cell Array

When you specify the cell array for input (or output) groups, keep in mind:

- Each row of this cell array designates a different input (output) group.
- You can add input (or output) groups by appending rows to the cell array.
- You can choose not to assign any of the group names when you assign the groups, and leave off the second column of this array. In that case,
  - Empty strings are assigned to the group names by default.
  - If you append rows to a cell array with no group names assigned, you have to assign empty strings (`''`) to the group names.

For example,

```
h.InputGroup = [h.InputGroup; {[3] 'disturbance'}];
```

adds another input group called `disturbance` to `h`.

You can use regular cell array syntax for accessing or modifying I/O group components. For example, to delete the first output group, `temperature`, type

```
h.OutputGroup(1,:) = []

ans =
    [1x2 double]    'measurements'
```

Similarly, you can add or delete channels from an existing input or output group. Recalling that input group channels are stored in the first column of the corresponding cell array, to add channel three to the input group `controls`, type

```
h.inputgroup{1,1} = [h.inputgroup{1,1} 3]
```

or, equivalently,

```
h.inputgroup{1,1} = [1 2 3]
```

Model Conversion

There are four LTI model types you can use with the Control System Toolbox: TF, ZPK, SS, and FRD. This section shows how to convert models from one type to the other.

Explicit Conversion

Model conversions are performed by `tf`, `ss`, `zpk`, and `frd`. Given any TF, SS, or ZPK model `sys`, the syntax for conversion to another model type is

```
sys = tf(sys)                    % Conversion to TF
sys = zpk(sys)                   % Conversion to ZPK
sys = ss(sys)                    % Conversion to SS
sys = frd(sys,frequency)         % Conversion to FRD
```

Notice that FRD models can't be converted to the other model types. In addition, you must also include a vector of frequencies (`frequency`) as an input argument when converting to an FRD model.

For example, you can convert the state-space model

```
sys = ss(-2,1,1,3)
```

to a zero-pole-gain model by typing

```
zpk(sys)
```

to which MATLAB responds

```
Zero/pole/gain:
3 (s+2.333)
-----------
   (s+2)
```

Note that the transfer function of a state-space model with data (A, B, C, D) is

$$H(s) = D + C(sI - A)^{-1}B$$

for continuous-time models, and

$$H(z) = D + C(zI - A)^{-1}B$$

for discrete-time models.

Automatic Conversion

Some algorithms operate only on one type of LTI model. For example, the algorithm for zero-order-hold discretization with `c2d` can only be performed on state-space models. Similarly, commands like `tfdata` expect one particular type of LTI models (TF). For convenience, such commands automatically convert LTI models to the appropriate or required model type. For example, in

```
sys = ss(0,1,1,0)
[num,den] = tfdata(sys)
```

`tfdata` first converts the state-space model `sys` to an equivalent transfer function in order to return numerator and denominator data.

Note that conversions to state-space models are not uniquely defined. For this reason, automatic conversions to state space are disabled when the result depends on the choice of state coordinates, for example, in commands like `initial` or `kalman`.

Caution About Model Conversions

When manipulating or converting LTI models, keep in mind that:

- The three LTI model types TF, ZPK, and SS, are not equally well-suited for numerical computations. In particular, the accuracy of computations using high-order transfer functions is often poor. Therefore, it is often preferable to work with the state-space representation. In addition, it is often beneficial to balance and scale state-space models using `ssbal`. You get this type of

balancing automatically when you convert any TF or ZPK model to state space using `ss`.

- Conversions to the transfer function representation using `tf` may incur a loss of accuracy. As a result, the transfer function poles may noticeably differ from the poles of the original zero-pole-gain or state-space model.
- Conversions to state space are not uniquely defined in the SISO case, nor are they guaranteed to produce a minimal realization in the MIMO case. For a given state-space model `sys`,

  ```
  ss(tf(sys))
  ```

  may return a model with different state-space matrices, or even a different number of states in the MIMO case. Therefore, if possible, it is best to avoid converting back and forth between state-space and other model types.

Time Delays

Using the `ioDelayMatrix`, `InputDelay`, and `OutputDelay` properties of LTI objects, you can specify delays in both continuous- and discrete-time LTI models. With these properties, you can, for example, represent:

- LTI models with independent delays for each input/output pair. For example, the continuous-time model with transfer function

$$H(s) = \begin{bmatrix} e^{-0.1s}\,\dfrac{2}{s} & e^{-0.3s}\,\dfrac{s+1}{s+10} \\ 10 & e^{-0.2s}\,\dfrac{s-1}{s+5} \end{bmatrix}$$

- State-space models with delayed inputs and/or delayed outputs. For example,

$$\dot{x}(t) = Ax(t) + Bu(t-\tau)$$
$$y(t) = Cx(t-\theta) + Du(t-(\theta+\tau))$$

  where τ is the time delay between the input $u(t)$ and the state vector $x(t)$, and θ is the time delay between $x(t)$ and the output $y(t)$.

You can assign the delay properties `ioDelayMatrix`, `InputDelay`, and `OutputDelay` either when first creating your model with the `tf`, `zpk`, `ss`, or `frd` constructors, or later with the `set` command (see "LTI Properties and Methods" on page 2-4 for details).

Supported Functionality

Most analysis commands support time delays, including:

- All time and frequency response commands
- Conversions between model types
- Continuous-to-discrete conversions (`c2d`)
- Horizontal and vertical concatenation
- Series, parallel, and feedback interconnections of discrete-time models with delays

- Interconnections of continuous-time delay systems as long as the resulting transfer function from input j to output i is of the form $\exp(-s\tau_{ij})\ h_{ij}(s)$ where $h_{ij}(s)$ is a rational function of s
- Padé approximation of time delays (`pade`)

Specifying Input/Output Delays

Using the `ioDelayMatrix` property, you can specify frequency-domain models with independent delays in each entry of the transfer function. In continuous time, such models have a transfer function of the form

$$H(s) = \begin{bmatrix} e^{-s\tau_{11}}h_{11}(s) & \dots & e^{-s\tau_{1m}}h_{1m}(s) \\ \vdots & & \vdots \\ e^{-s\tau_{p1}}h_{p1}(s) & \dots & e^{-s\tau_{pm}}h_{pm}(s) \end{bmatrix} = [\exp(-s\tau_{ij})\ h_{ij}(s)]$$

where the h_{ij}'s are rational functions of s, and τ_{ij} is the time delay between input j and output i. See "Specifying Delays in Discrete-Time Models" on page 2-52 for details on the discrete-time counterpart. We collectively refer to the scalars τ_{ij} as the *I/O delays*.

The syntax to create $H(s)$ above is

```
H = tf(num,den,'iodelaymatrix',Tau)
```

or

```
H = zpk(z,p,k,'iodelaymatrix',Tau)
```

where

- `num`, `den` (respectively, `z`, `p`, `k`) specify the rational part $[h_{ij}(s)]$ of the transfer function $H(s)$
- `Tau` is the matrix of time delays for each I/O pair. That is, `Tau(i,j)` specifies the I/O delay τ_{ij} in seconds. Note that `Tau` and $H(s)$ should have the same row and column dimensions.

You can also use the `ioDelayMatrix` property in conjunction with state-space models, as in

```
sys = ss(A,B,C,D,'iodelaymatrix',Tau)
```

This creates the LTI model with the following transfer function.

$$H(s) = \left[\exp(-s\tau_{ij})\ r_{ij}(s)\right]$$

Here $r_{ij}(s)$ is the (i, j) entry of

$$R(s) = D + C(sI - A)^{-1}B$$

Note: State-space models with I/O delays have only a frequency-domain interpretation. They cannot, in general, be described by state-space equations with delayed inputs and outputs.

Distillation Column Example

This example is adapted from [2] and illustrates the use of I/O delays in process modeling. The process of interest is the distillation column depicted by the figure below. This column is used to separate a mix of methanol and water (the *feed*) into *bottom products* (mostly water) and a methanol-saturated *distillate*.

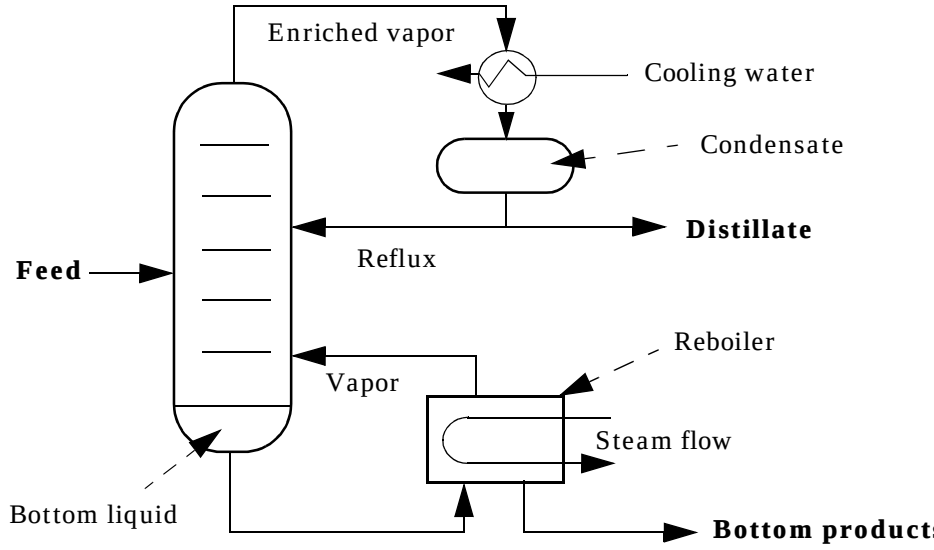


Figure 2-3: Distillation Column

Schematically, the distillation process functions as follows:

- Steam flows into the reboiler and vaporizes the bottom liquid. This vapor is reinjected into the column and mixes with the feed
- Methanol, being more volatile than water, tends to concentrate in the vapor moving upward. Meanwhile, water tends to flow downward and accumulate as the bottom liquid
- The vapor exiting at the top of the column is condensed by a flow of cooling water. Part of this condensed vapor is extracted as the distillate, and the rest of the condensate (the *reflux*) is sent back to the column.
- Part of the bottom liquid is collected from the reboiler as bottom products (waste).

The regulated output variables are:

- Percentage X_D of methanol in the distillate
- Percentage X_B of methanol in the bottom products.

The goal is to maximize X_D by adjusting the reflux flow rate R and the steam flow rate S in the reboiler.

To obtain a linearized model around the steady-state operating conditions, the transient responses to pulses in steam and reflux flow are fitted by first-order plus delay models. The resulting transfer function model is

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-1s}}{16.7e+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} R(s) \\ S(s) \end{bmatrix}$$

Note the different time delays for each input/output pair.

You can specify this MIMO transfer function by typing

```
H = tf({12.8 -18.9;6.6 -19.4},...
       {[16.7 1] [21 1];[10.9 1] [14.4 1]},...
       'iodelay',[1 3;7 3],...
       'inputname',{'R' , 'S'},...
       'outputname',{'Xd' , 'Xb'})
```

The resulting TF model is displayed as

```
Transfer function from input "R" to output...
                     12.8
 Xd:  exp(-1*s) * ----------
                  16.7 s + 1

                     6.6
 Xb:  exp(-7*s) * ----------
                  10.9 s + 1

Transfer function from input "S" to output...
                  -18.9
 Xd:  exp(-3*s) * --------
                  21 s + 1

                    -19.4
 Xb:  exp(-3*s) * ----------
                  14.4 s + 1
```

Specifying Delays on the Inputs or Outputs

While ideal for frequency-domain models with I/O delays, the `ioDelayMatrix` property is inadequate to capture delayed inputs or outputs in state-space models. For example, the two models

$$(M_1)\begin{cases}\dot{x}(t) = -x(t) + u(t-0.1)\\ y(t) = x(t)\end{cases} \qquad (M_2)\begin{cases}\dot{z}(t) = -z(t) + u(t)\\ y(t) = z(t-0.1)\end{cases}$$

share the same transfer function

$$h(s) = \frac{e^{-0.1s}}{s+1}$$

As a result, they cannot be distinguished using the `ioDelayMatrix` property (the I/O delay value is 0.1 seconds in both cases). Yet, these two models have different state trajectories since $x(t)$ and $z(t)$ are related by

$$z(t) = x(t-0.1)$$

Note that the 0.1 second delay is on the *input* in the first model, and on the *output* in the second model.

InputDelay and OutputDelay Properties

When the state trajectory is of interest, you should use the `InputDelay` and `OutputDelay` properties to distinguish between delays on the inputs and delays on the outputs in state-space models. For example, you can accurately specify the two models above by

```
M1 = ss(-1,1,1,0,'inputdelay',0.1)
M2 = ss(-1,1,1,0,'outputdelay',0.1)
```

In the MIMO case, you can specify a different delay for each input (or output) channel by assigning a vector value to `InputDelay` (or `OutputDelay`). For example,

```
sys = ss(A,[B1 B2],[C1;C2],[D11 D12;D21 D22])
sys.inputdelay = [0.1 0]
sys.outputdelay = [0.2 0.3]
```

creates the two-input, two-output model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 u_1(t-0.1) + B_2 u_2(t) \\ y_1(t+0.2) &= C_1 x(t) + D_{11} u_1(t-0.1) + D_{12} u_2(t) \\ y_2(t+0.3) &= C_2 x(t) + D_{21} u_1(t-0.1) + D_{22} u_2(t) \end{aligned}$$

You can also use the `InputDelay` and `OutputDelay` properties to conveniently specify input or output delays in TF, ZPK, or FRD models. For example, you can create the transfer function

$$H(s) = \begin{bmatrix} \frac{1}{s} \\ \frac{2}{s+1} \end{bmatrix} e^{-0.1s}$$

by typing

```
s = tf('s');
H = [1/s ; 2/(s+1)];   % rational part
H.inputdelay = 0.1
```

The resulting model is displayed as

```
Transfer function from input to output...
                    1
 #1:  exp(-0.1*s) * -
                    s

                      2
 #2:  exp(-0.1*s) * -----
                    s + 1
```

By comparison, to produce an equivalent transfer function using the `ioDelayMatrix` property, you would need to type

```
H = [1/s ; 2/(s+1)];
H.iodelay = [0.1 ; 0.1];
```

Notice that the 0.1 second delay is repeated twice in the I/O delay matrix. More generally, for a TF, ZPK, or FRD model with input delays $[\alpha_1, ..., \alpha_m]$ and output delays $[\beta_1, ..., \beta_p]$, the equivalent I/O delay matrix is

$$\begin{bmatrix} \alpha_1+\beta_1 & \alpha_2+\beta_1 & \dots & \alpha_m+\beta_1 \\ \alpha_1+\beta_2 & \alpha_2+\beta_2 & & \alpha_m+\beta_2 \\ \vdots & \vdots & & \vdots \\ \alpha_1+\beta_p & \alpha_2+\beta_p & \dots & \alpha_m+\beta_p \end{bmatrix}$$

Specifying Delays in Discrete-Time Models

You can also use the `ioDelayMatrix`, `InputDelay`, and `OutputDelay` properties to specify delays in discrete-time LTI models. You specify time delays in discrete-time models with integer multiples of the sampling period. The integer `k` you supply for the time delay of a discrete-time model specifies a time delay of k sampling periods. Such a delay contributes a factor z^{-k} to the transfer function.

For example,

```
h = tf(1,[1 0.5 0.2],0.1,'inputdelay',3)
```

produces the discrete-time transfer function

```
Transfer function:
                  1
z^(-3) * -----------------
         z^2 + 0.5 z + 0.2

Sampling time: 0.1
```

Notice the `z^(-3)` factor reflecting the three-sampling-period delay on the input.

Mapping Discrete-Time Delays to Poles at the Origin

Since discrete-time delays are equivalent to additional poles at $z = 0$, they can be easily absorbed into the transfer function denominator or the state-space equations. For example, the transfer function of the delayed integrator

$$y[k+1] = y[k] + u[k-2]$$

is

$$H(z) = \frac{z^{-2}}{z-1}$$

You can specify this model either as the first-order transfer function $1/(z-1)$ with a delay of two sampling periods on the input

```
Ts = 1;    % sampling period
H1 = tf(1,[1 -1],Ts,'inputdelay',2)
```

or directly as a third-order transfer function:

```
H2 = tf(1,[1 -1 0 0],Ts)    % 1/(z^3-z^2)
```

While these two models are mathematically equivalent, `H1` is a more efficient representation both in terms of storage and subsequent computations.

When necessary, you can map all discrete-time delays to poles at the origin using the command `delay2z`. For example,

```
H2 = delay2z(H1)
```

absorbs the input delay in H1 into the transfer function denominator to produce the third-order transfer function

```
Transfer function:
    1
---------
z^3 - z^2

Sampling time: 1
```

Note that

```
H2.inputdelay
```

now returns 0 (zero).

Retrieving Information About Delays

There are several ways to retrieve time delay information from a given LTI model sys:

- Use property display commands to inspect the values of the ioDelayMatrix, InputDelay, and OutputDelay properties. For example,

  ```
  sys.iodelay
  get(sys,'inputdelay')
  ```

- Use the helper function hasdelay to determine if sys has any delay at all. The syntax is

  ```
  hasdelay(sys)
  ```

  which returns 1 (true) if sys has any delay, and 0 (false) otherwise
- Use the function totaldelay to determine the total delay between each input and each output (cumulative contribution of the ioDelayMatrix, InputDelay, and OutputDelay properties). Type help totaldelay or see the Reference pages for details.

Conversion of Models with Delays to State Space

When you use ss to convert TF or ZPK models to state-space form, the delays in the resulting state-space model are redistributed when it is possible to reduce the overall number of I/O delays, input channel delays, and output

channel delays. The resulting model has a minimum number of delays. When this minimization takes place:

- All or part of the I/O delay matrix is absorbed into the input and output delay vectors. This minimizes the total number of I/O delays.
- The input delays are transferred to output delays (or vice-versa), so as to minimize the overall number of input and output channel delays.

Padé Approximation of Time Delays

The function `pade` computes rational approximations of time delays in continuous-time LTI models. The syntax is

```
sysx = pade(sys,n)
```

where `sys` is a continuous-time model with delays, and the integer `n` specifies the Padé approximation order. The resulting LTI model `sysx` is of the same type as `sys`, but is delay free.

For models with multiple delays or a mix of input, output, and I/O delays, you can use the syntax

```
sysx = pade(sys,ni,no,nio)
```

where the vectors `ni` and `no`, and the matrix `nio` specify independent approximation orders for each input, output, and I/O delay, respectively. Set `ni=[]` if there are no input delays, and similarly for `no` and `nio`.

For example, consider the “Distillation Column Example” on page 2-47. The two-input, two-output transfer function in this example is

$$H(s) = \begin{bmatrix} \frac{12.8e^{-1s}}{16.7e+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}$$

To compute a Padé approximation of $H(s)$ using:

- A first-order approximation for the 1 second and 3 second delays
- A second-order approximation for the 7 second delay,

type

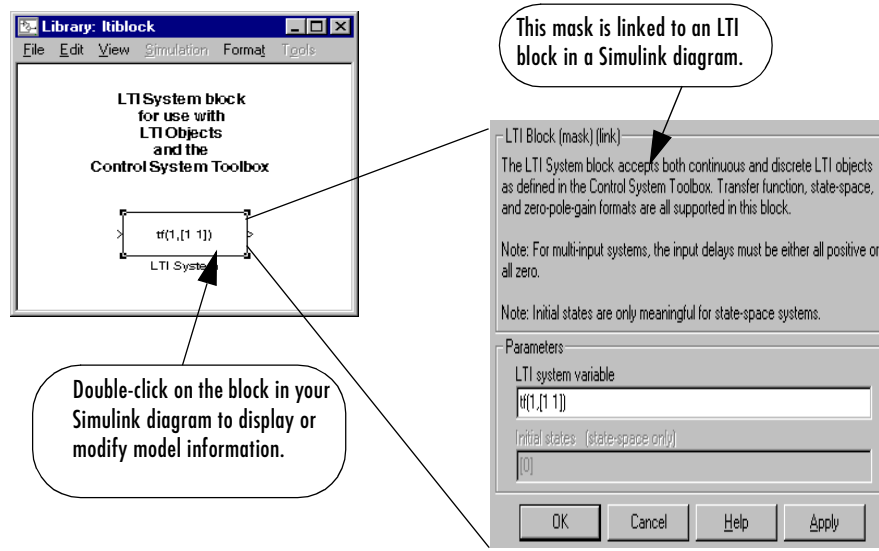
```
pade(H,[],[],[1 1;2 1])
```

where H is the TF representation of $H(s)$ defined in the "Distillation Column Example" on page 2-47. This command produces a rational transfer function.

```
Transfer function from input "R" to output...
        -12.8 s + 25.6
 Xd:  ---------------------
      16.7 s^2 + 34.4 s + 2

             6.6 s^2 - 5.657 s + 1.616
 Xb:  -------------------------------------
      10.9 s^3 + 10.34 s^2 + 3.527 s + 0.2449

Transfer function from input "S" to output...
          18.9 s - 12.6
 Xd:  ----------------------
      21 s^2 + 15 s + 0.6667

            19.4 s - 12.93
 Xb:  -------------------------
      14.4 s^2 + 10.6 s + 0.6667
```

Simulink Block for LTI Systems

You can incorporate LTI objects into Simulink diagrams using the LTI System block shown below.



The LTI System block can be accessed either by typing

```
ltiblock
```

at the MATLAB prompt or by selecting **Control System Toolbox** from the **Blocksets and Toolboxes** section of the main Simulink library.

The LTI System block consists of the dialog box shown on the right in the figure above. In the editable text box labeled **LTI system variable**, enter either the variable name of an LTI object located in the MATLAB workspace (for example, `sys`) or a MATLAB expression that evaluates to an LTI object (for example, `tf(1,[1 1])`). The LTI System block accepts both continuous and discrete LTI objects in either transfer function, zero-pole-gain, or state-space form. Simulink converts the model to its state-space equivalent prior to initializing the simulation.

Use the editable text box labeled **Initial states** to enter an initial state vector for state-space models. The concept of “initial state” is not well-defined for

transfer functions or zero-pole-gain models, as it depends on the choice of state coordinates used by the realization algorithm. As a result, you cannot enter nonzero initial states when you supply TF or ZPK models to LTI blocks in a Simulink diagram.

Note:

- For MIMO systems, the input delays stored in the LTI object must be either all positive or all zero.
- LTI blocks in a Simulink diagram cannot be used for FRD models or LTI arrays.

References

[1] Dorf, R.C. and R.H. Bishop, *Modern Control Systems*, Addison-Wesley, Menlo Park, CA, 1998.

[2] Wood, R.K. and M.W. Berry, "Terminal Composition Control of a Binary Distillation Column," *Chemical Engineering Science*, 28 (1973), pp. 1707-1717.

3

Operations on LTI Models

Introduction

You can perform basic matrix operations such as addition, multiplication, or concatenation on LTI models. Such operations are "overloaded," which means that they use the same syntax as they do for matrices, but are adapted so as to apply to the LTI model context. These overloaded operations and their interpretation in this context are discussed in this chapter. You can read about discretization methods in this chapter as well. The following topics and operations on LTI models are covered in this chapter:

- Precedence and Property Inheritance
- Extracting and Modifying Subsystems
- Arithmetic Operations
- Model Interconnection Functions
- Continuous/Discrete-Time Conversions of LTI Models
- Resampling of Discrete-Time Models

These operations can be applied to LTI models of different types. As a result, before discussing operations on LTI models, we discuss model type precedence and how LTI model properties are inherited when models are combined using these operations. To read about how you can apply these operations to arrays of LTI models, see "Operations on LTI Arrays" on page 4-25. To read about the available functions with which you can analyze LTI models, see Chapter 5, "Model Analysis Tools,"

Precedence and Property Inheritance

You can apply operations to LTI models of different types. The resulting type is then determined by the rules discussed in "Precedence Rules" on page 2-5. For example, if `sys1` is a transfer function and `sys2` is a state-space model, then the result of their addition

```
sys = sys1 + sys2
```

is a state-space model, since state-space models have precedence over transfer function models.

To supersede the precedence rules and force the result of an operation to be a given type, for example, a transfer function (TF), you can either:

- Convert all operands to TF *before* performing the operation
- Convert the result to TF *after* performing the operation

Suppose, in the above example, you want to compute the transfer function of `sys`. You can either use *a priori* conversion of the second operand

```
sys = sys1 + tf(sys2);
```

or *a posteriori* conversion of the result

```
sys = tf(sys1 + sys2)
```

Note: These alternatives are not equivalent numerically; computations are carried out on transfer functions in the first case, and on state-space models in the second case.

Another issue is property inheritance, that is, how the operand property values are passed on to the result of the operation. While inheritance is partly operation-dependent, some general rules are summarized below:

- In operations combining discrete-time LTI models, all models must have identical or unspecified (`sys.Ts = –1`) sample times. Models resulting from such operations inherit the specified sample time, if there is one.
- Most operations ignore the `Notes` and `Userdata` properties.

- In general, when two LTI models `sys1` and `sys2` are combined using operations such as `+`, `*`, `[,]`, `[;]`, `append`, and `feedback`, the resulting model inherits its I/O names and I/O groups from `sys1` and `sys2`. However, conflicting I/O names or I/O groups are not inherited. For example, the `InputName` property for `sys1 + sys2` is left unspecified if `sys1` and `sys2` have different `InputName` property values.
- A model resulting from operations on TF or ZPK models inherits its `Variable` property value from the operands. Conflicts are resolved according the following rules:
  - For continuous-time models, `'p'` has precedence over `'s'`.
  - For discrete-time models, `'z^-1'` has precedence over `'q'` and `'z'`, while `'q'` has precedence over `'z'`.

Extracting and Modifying Subsystems

Subsystems relate subsets of the inputs and outputs of a system. The transfer matrix of a subsystem is a submatrix of the system transfer matrix. For example, if `sys` is a system with two inputs, three outputs, and I/O relation

$$y = Hu$$

then $H(3, 1)$ gives the relation between the first input and third output.

$$y_3 = H(3,1)\ u_1$$

Accordingly, we use matrix-like subindexing to extract this subsystem.

```
SubSys = sys(3,1)
```

The resulting subsystem `SubSys` is an LTI model of the same type as `sys`, with its sample time, time delay, I/O name, and I/O group property values inherited from `sys`.

For example, if `sys` has an input group named `controls` consisting of channels one, two, and three, then `SubSys` also has an input group named `controls` with the first channel of `SubSys` assigned to it.

If `sys` is a state-space model with matrices `a`, `b`, `c`, `d`, the subsystem `sys(3,1)` is a state-space model with data `a`, `b(:,1)`, `c(3,:)`, `d(3,1)`.

Note:

- In the expression `sys(3,1)`, the first index selects the output channel while the second index selects the input channel.
- When extracting a subsystem from a given state-space model, the resulting state-space model may not be minimal. Use the command `sminreal` to eliminate unnecessary states in the subsystem.

You can use similar syntax to modify the LTI model `sys`. For example,

```
sys(3,1) = NewSubSys
```

redefines the I/O relation between the first input and third output, provided `NewSubSys` is a SISO LTI model.

Note:

- sys, the LTI model that has had a portion reassigned, retains its original model type (TF, ZPK, SS, or FRD) regardless of the model type of NewSubSys.
- If NewSubSys is an FRD model, then sys must also be an FRD model. Furthermore, their frequencies must match.
- Subsystem assignment does not reassign any I/O names or I/O group names of NewSubSys that are already assigned to NewSubSys.
- Reassigning parts of a MIMO state-space model generally increases its order.

Other standard matrix subindexing extends to LTI objects as well. For example,

```
sys(3,1:2)
```

extracts the subsystem mapping the first two inputs to the third output.

```
sys(:,1)
```

selects the first input and all outputs, and

```
sys([1 3],:)
```

extracts a subsystem with the same inputs, but only the first and third outputs.

For example, consider the two-input/two-output transfer function

$$T(s) = \begin{bmatrix} \frac{1}{s+0.1} & 0 \\ \frac{s-1}{s^2+2s+2} & \frac{1}{s} \end{bmatrix}$$

To extract the transfer function $T_{11}(s)$ from the first input to the first output, type

```
T(1,1)

Transfer function:
   1
-------
s + 0.1
```

Next reassign $T_{11}(s)$ to $1/(s+0.5)$ and modify the second input channel of `T` by typing

```
T(1,1) = tf(1,[1 0.5]);
T(:,2) = [ 1 ; tf(0.4,[1 0]) ]

Transfer function from input 1 to output...
         1
 #1:  -------
      s + 0.5

          s - 1
 #2:  -------------
      s^2 + 2 s + 2

Transfer function from input 2 to output...
 #1:  1

      0.4
 #2:  ---
       s
```

Referencing FRD Models Through Frequencies

You can extract subsystems from FRD models, as you do with other LTI model types, by indexing into input and output (I/O) dimensions. You can also extract subsystems by indexing into the frequencies of an FRD model.

To index into the frequencies of an FRD model, use the string `'Frequency'` (or any abbreviation, such as, `'freq'`, as long as it does not conflict with existing

I/O channel or group names) as a keyword. There are two ways you can specify FRD models using frequencies:

- Using integers to index into the frequency vector of the FRD model
- Using a Boolean (logical) expression to specify desired frequency points in an FRD model

For example, if `sys` is an FRD model with five frequencies, (e.g., `sys.Frequency=[1 1.1 1.2 1.3 1.4]`), then you can create a new FRD model `sys2` by indexing into the frequencies of `sys` as follows.

```
sys2 = sys('frequency', 2:3);
sys2.Frequency

ans =
    1.1000
    1.2000
```

displays the second and third entries in the frequency vector.

Similarly, you can use logical indexing into the frequencies.

```
sys2 = sys('frequency',sys.Frequency >1.0 & sys.Frequency <1.15);
sys2.freq

ans =
    1.1000
```

You can also combine model extraction through frequencies with indexing into the I/O dimensions. For example, if `sys` is an FRD model with two inputs, two outputs, and frequency vector `[2.1 4.2 5.3]`, with `sys.Units` specified in rad/s, then

```
sys2 = sys(1,2,'freq',1)
```

specifies `sys2` as a SISO FRD model, with one frequency data point, 2.1 rad/s.

Referencing Channels by Name

You can also extract subsystems using I/O group or channel names. For example, if `sys` has an input group named `noise`, consisting of channels two, four, and five, then

```
sys(1,'noise')
```

is equivalent to

```
sys(1,[2 4 5]
```

Similarly, if `pressure` is the name assigned to an output channel of the LTI model `sys`, then

```
sys('pressure',1) = tf(1, [1 1])
```

reassigns the subsystem from the first input of `sys` to the output labeled `pressure`.

You can reference a set of channels by input or output name by using a cell array of strings for the names. For example, if `sys` has one output channel named `pressure` and one named `temperature`, then these two output channels can be referenced using

```
sys({'pressure','temperature'})
```

Resizing LTI Systems

Resizing a system consists of adding or deleting inputs and/or outputs. To delete the first two inputs, simply type

```
sys(:,1:2) = []
```

In deletions, at least one of the row/column indexes should be the colon (`:`) selector.

To perform input/output augmentation, you can proceed by concatenation or subassignment. Given a system `sys` with a single input, you can add a second input using

```
sys = [sys,h];
```

or, equivalently, using

```
sys(:,2) = h;
```

where `h` is any LTI model with one input, and the same number of outputs as `sys`. There is an important difference between these two options: while concatenation obeys the precedence rules (see page 2-5), subsystem assignment does not alter the model type. So, if `sys` and `h` are TF and SS objects, respectively, the first statement produces a state-space model, and the second statement produces a transfer function.

For state-space models, both concatenation and subsystem assignment increase the model order because they assume that `sys` and `h` have independent states. If you intend to keep the same state matrix and merely update the input-to-state or state-to-output relations, use `set` instead and modify the corresponding state-space data directly. For example,

```
sys = ss(a,b1,c,d1)
set(sys,'b',[b1 b2],'d',[d1 d2])
```

adds a second input to the state-space model `sys` by appending the *B* and *D* matrices. You should *simultaneously* modify both matrices with a single `set` command. Indeed, the statements

```
sys.b = [b1 b2]
```

and

```
set(sys,'b',[b1 b2])
```

cause an error because they create invalid intermediate models in which the *B* and *D* matrices have inconsistent column dimensions.

Arithmetic Operations

You can apply almost all arithmetic operations to LTI models, including those shown below.

Operation	Description
`+`	Addition
`–`	Subtraction
`*`	Multiplication
`/`	Right matrix divide
`\`	Left matrix divide
`inv`	Matrix inversion
`'`	Pertransposition
`.'`	Transposition
`^`	Powers of an LTI model (as in `s^2`)

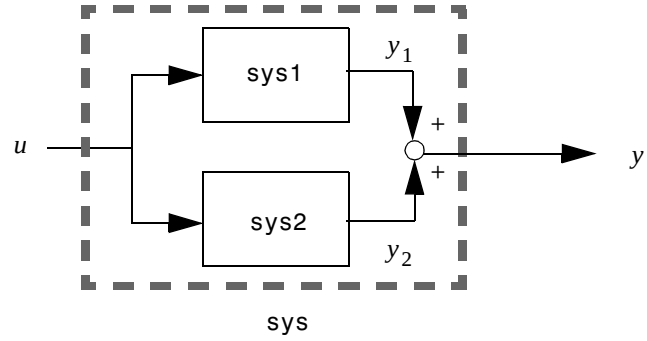
To understand how these operations work on LTI models, it's useful to keep in mind the matrix analogy for systems discussed in "Viewing LTI Systems As Matrices" on page 2-5.

Addition and Subtraction

As mentioned in "Viewing LTI Systems As Matrices" on page 2-5, adding LTI models is equivalent to connecting them in parallel. Specifically, the LTI model

```
sys = sys1 + sys2
```

represents the parallel interconnection shown below.



If sys1 and sys2 are two state-space models with data A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2, the state-space data associated with sys1 + sys2 is

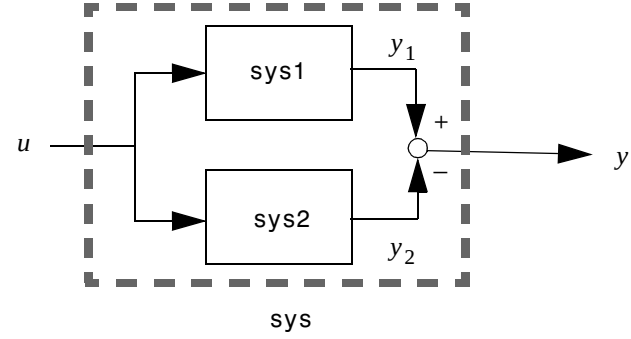
$$\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \qquad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \qquad D_1 + D_2$$

Scalar addition is also supported and behaves as follows: if sys1 is MIMO and sys2 is SISO, sys1 + sys2 produces a system with the same dimensions as sys1 whose *ij*th entry is sys1(i,j) + sys2.

Similarly, the subtraction of two LTI models

```
sys = sys1 − sys2
```

is depicted by the following block diagram.

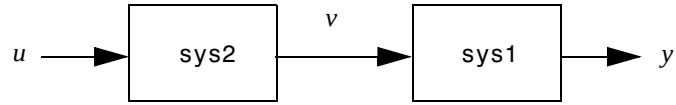


Multiplication

Multiplication of two LTI models connects them in series. Specifically,

```
sys = sys1 * sys2
```

returns an LTI model sys for the series interconnection shown below.



Notice the reverse orders of sys1 and sys2 in the multiplication and block diagram. This is consistent with the way transfer matrices are combined in a series connection: if sys1 and sys2 have transfer matrices H_1 and H_2, then

$$y = H_1 v = H_1(H_2 u) = (H_1 \times H_2)\, u$$

For state-space models sys1 and sys2 with data A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2, the state-space data associated with sys1*sys2 is

$$\begin{bmatrix} A_1 & B_1C_2 \\ 0 & A_2 \end{bmatrix}, \qquad \begin{bmatrix} B_1D_2 \\ B_2 \end{bmatrix}, \qquad \begin{bmatrix} C_1 & D_1C_2 \end{bmatrix}, \qquad D_1D_2$$

Finally, if sys1 is MIMO and sys2 is SISO, then sys1*sys2 or sys2*sys1 is interpreted as an entry-by-entry scalar multiplication and produces a system with the same dimensions as sys1, whose *ij*th entry is sys1(i,j)*sys2.

Inversion and Related Operations

Inversion of LTI models amounts to inverting the following input/output relationship.

$$y = H\, u \qquad \rightarrow \qquad u = H^{-1} y$$

This operation is defined only for square systems (that is, systems with as many inputs as outputs) and is performed using

```
inv(sys)
```

The resulting inverse model is of the same type as sys. Related operations include:

- Left division sys1\sys2, which is equivalent to inv(sys1)*sys2
- Right division sys1/sys2, which is equivalent to sys1*inv(sys2)

For a state-space model sys with data A, B, C, D, inv(sys) is defined only when D is a square invertible matrix, in which case its state-space data is

$$A - BD^{-1}C, \qquad BD^{-1}, \qquad -D^{-1}C, \qquad D^{-1}$$

Transposition

You can transpose an LTI model sys using

```
sys.'
```

This is a literal operation with the following effect:

- For TF models (with input arguments, num and den), the cell arrays num and den are transposed.
- For ZPK models (with input arguments, z, p, and k), the cell arrays, z and p, and the matrix k are transposed.
- For SS models (with model data A, B, C, D), transposition produces the state-space model A^T, C^T, B^T, D^T.
- For FRD models (with complex frequency response matrix Response), the matrix of frequency response data at each frequency is transposed.

Pertransposition

For a continuous-time system with transfer function $H(s)$, the *pertransposed* system has the transfer function

$$G(s) = [H(-s)]^T$$

The discrete-time counterpart is

$$G(z) = [H(z^{-1})]^T$$

Pertransposition of an LTI model sys is performed using

```
sys'
```

You can use pertransposition to obtain the Hermitian (conjugate) transpose of the frequency response of a given system. The frequency response of the pertranspose of $H(s)$, $G(s) = [H(-s)]^T$, is the Hermitian transpose of the frequency response of $H(s)$: $G(jw) = H(jw)^H$.

To obtain the Hermitian transpose of the frequency response of a system `sys` over a frequency range specified by the vector `w`, type

```
freqresp(sys', w);
```

Operations on State-Space Models with Delays

When you apply operations such as `+`, `*`, `\`, `/`, `[]`, to state-space models, all or part of the I/O delay matrix of the resulting model is absorbed into the input and output delay vectors when it is possible to reduce the total number of I/O delays. The resulting model has a minimum number of such delays.

Model Interconnection Functions

The Control System Toolbox provides a number of functions to help with the model building process. These include model interconnection functions to perform I/O concatenation (`[,]`, `[;]`, and `append`), general parallel and series connections (`parallel` and `series`), and feedback connections (`feedback` and `lft`). These functions are useful to model open- and closed-loop systems.

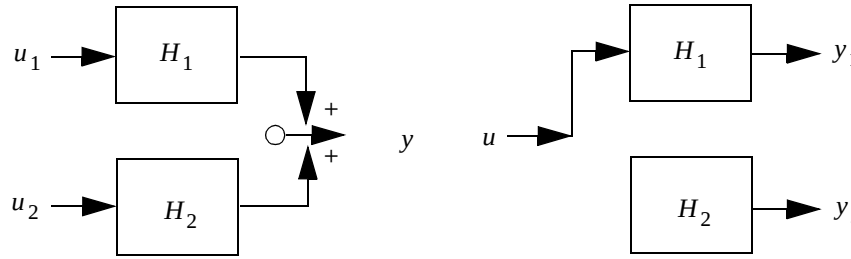
Interconnection Operator	Description
`[,]`	Concatenates horizontally
`[;]`	Concatenates vertically
`append`	Appends models in a block diagonal configuration
`augstate`	Augments the output by appending states
`connect`	Forms an SS model from a block diagonal LTI object for an arbitrary interconnection matrix
`feedback`	Forms the feedback interconnection of two models
`lft`	Produces the LFT interconnection (Redheffer Star product) of two models
`parallel`	Forms the generalized parallel connection of two models
`series`	Forms the generalized series connection of two models

Concatenation of LTI Models

LTI model concatenation is done in a manner similar to the way you concatenate matrices in MATLAB, using

```
sys = [sys1 , sys2]    % horizontal concatenation
sys = [sys1 ; sys2]    % vertical concatenation
sys = append(sys1,sys2)% block diagonal appending
```

In I/O terms, horizontal and vertical concatenation have the following block-diagram interpretations (with H_1 and H_2 denoting the transfer matrices of sys1 and sys2).



$$y = \begin{bmatrix} H_1 , H_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u$$

Horizontal Concatenation **Vertical Concatenation**

You can use concatenation as an easy way to create MIMO transfer functions or zero-pole-gain models. For example,

```
H = [ tf(1,[1 0])  1 ; 0  tf([1 -1],[1 1]) ]
```

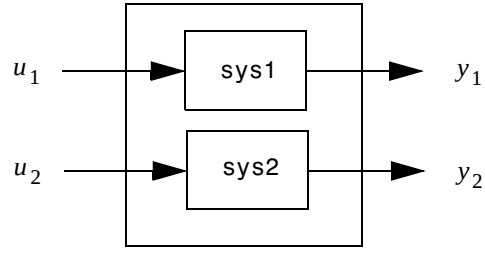
specifies

$$H(s) = \begin{bmatrix} \frac{1}{s} & 1 \\ 0 & \frac{s-1}{s+1} \end{bmatrix}$$

Use

```
append(sys1,sys2)
```

to specify the block-decoupled LTI model interconnection.



Appended Models

$$\begin{bmatrix} sys1 & 0 \\ 0 & sys2 \end{bmatrix}$$

Transfer Function

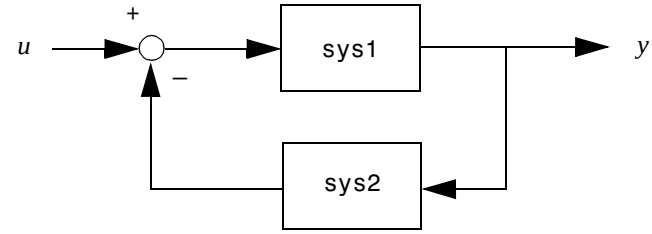
See append on page 11-12 for more information on this command.

Feedback and Other Interconnection Functions

The following LTI model interconnection functions are useful for specifying closed- and open-loop model configurations:

- `feedback` puts two LTI models with compatible dimensions in a feedback configuration.
- `series` connects two LTI models in series.
- `parallel` connects two LTI models in parallel.
- `lft` performs the Redheffer star product on two LTI models.
- `connect` works with `append` to apply an arbitrary interconnection scheme to a set of LTI models.

For example, if `sys1` has *m* inputs and *p* outputs, while `sys2` has *p* inputs and *m* outputs, then the negative feedback configuration of these two LTI models



is realized with

```
feedback(sys1,sys2)
```

This specifies the LTI model with *m* inputs and *p* outputs whose I/O map is

$$(I + sys1 \cdot sys2)^{-1} sys1$$

See Chapter 11, "Reference," for more information on `feedback`, `series`, `parallel`, `lft`, and `connect`.

Continuous/Discrete Conversions of LTI Models

The function `c2d` discretizes continuous-time TF, SS, or ZPK models. Conversely, `d2c` converts discrete-time TF, SS, or ZPK models to continuous time. Several discretization/interpolation methods are supported, including zero-order hold (ZOH), first-order hold (FOH), Tustin approximation with or without frequency prewarping, and matched poles and zeros.

The syntax

```
sysd = c2d(sysc,Ts);     % Ts = sampling period in seconds
sysc = d2c(sysd);
```

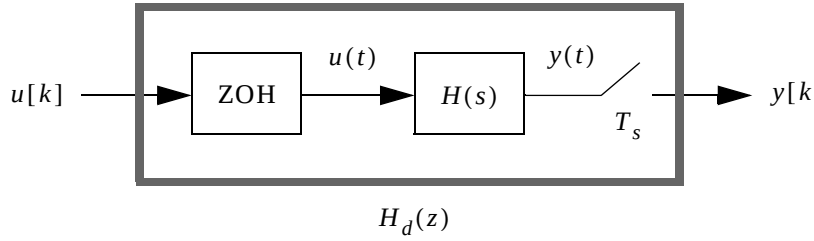
performs ZOH conversions by default. To use alternative conversion schemes, specify the desired method as an extra string input:

```
sysd = c2d(sysc,Ts,'foh'); % use first-order hold
sysc = d2c(sysd,'tustin'); % use Tustin approximation
```

The conversion methods and their limitations are discussed next.

Zero-Order Hold

Zero-order hold (ZOH) devices convert sampled signals to continuous-time signals for analyzing sampled continuous-time systems. The zero-order-hold discretization $H_d(z)$ of a continuous-time LTI model $H(s)$ is depicted in the following block diagram.



The ZOH device generates a continuous input signal $u(t)$ by holding each sample value $u[k]$ constant over one sample period.

$$u(t) = u[k], \qquad kT_s \le t \le (k+1)T_s$$

The signal $u(t)$ is then fed to the continuous system $H(s)$, and the resulting output $y(t)$ is sampled every T_s seconds to produce $y[k]$.

Conversely, given a discrete system $H_d(z)$, the `d2c` conversion produces a continuous system $H(s)$ whose ZOH discretization coincides with $H_d(z)$. This inverse operation has the following limitations:

- `d2c` cannot operate on LTI models with poles at $z = 0$ when the ZOH is used.
- Negative real poles in the z domain are mapped to *pairs* of complex poles in the s domain. As a result, the `d2c` conversion of a discrete system with negative real poles produces a continuous system with higher order.

The next example illustrates the behavior of `d2c` with real negative poles. Consider the following discrete-time ZPK model.

```
hd = zpk([],-0.5,1,0.1)

Zero/pole/gain:
   1
-------
(z+0.5)

Sampling time: 0.1
```

Use `d2c` to convert this model to continuous-time

```
hc = d2c(hd)
```

and you get a second-order model.

```
Zero/pole/gain:
   4.621 (s+149.3)
----------------------
(s^2 + 13.86s + 1035)
```

Discretize the model again

```
c2d(hc,0.1)
```

and you get back the original discrete-time system (up to canceling the pole/zero pair at z=–0.5):

```
Zero/pole/gain:
 (z+0.5)
---------
(z+0.5)^2

Sampling time: 0.1
```

First-Order Hold

First-order hold (FOH) differs from ZOH by the underlying hold mechanism. To turn the input samples $u[k]$ into a continuous input $u(t)$, FOH uses linear interpolation between samples.

$$u(t) = u[k] + \frac{t - kT_s}{T_s}(u[k+1] - u[k]), \qquad kT_s \le t \le (k+1)T_s$$

This method is generally more accurate than ZOH for systems driven by smooth inputs. Due to causality constraints, this option is only available for `c2d` conversions, and not `d2c` conversions.

Note: This FOH method differs from standard causal FOH and is more appropriately called *triangle approximation* (see [2], p. 151). It is also known as *ramp-invariant* approximation because it is distortion-free for ramp inputs.

Tustin Approximation

The Tustin or bilinear approximation uses the approximation

$$z = e^{sT_s} \approx \frac{1 + sT_s/2}{1 - sT_s/2}$$

to relate s-domain and z-domain transfer functions. In `c2d` conversions, the discretization $H_d(z)$ of a continuous transfer function $H(s)$ is derived by

$$H_d(z) = H(s'), \text{ where } \quad s' = \frac{2}{T_s}\frac{z-1}{z+1}$$

Similarly, the `d2c` conversion relies on the inverse correspondence

$$H(s) = H_d(z'), \text{ where } \quad z' = \frac{1+sT_s/2}{1-sT_s/2}$$

Tustin with Frequency Prewarping

This variation of the Tustin approximation uses the correspondence

$$H_d(z) = H(s'), \quad s' = \frac{\omega}{\tan(\omega T_s/2)}\frac{z-1}{z+1}$$

This change of variable ensures the matching of the continuous- and discrete-time frequency responses at the frequency ω.

$$H(j\omega) = H_d(e^{j\omega T_s})$$

Matched Poles and Zeros

The matched pole-zero method applies only to SISO systems. The continuous and discretized systems have matching DC gains and their poles and zeros correspond in the transformation

$$z = e^{sT_s}$$

See [2], p. 147 for more details.

Discretization of Systems with Delays

You can also use `c2d` to discretize SISO or MIMO continuous-time models with time delays. If `Ts` is the sampling period used for discretization:

- A delay of `tau` seconds in the continuous-time model is mapped to a delay of `k` sampling periods in the discretized model, where `k = fix(tau/Ts)`.
- The residual *fractional delay* `tau – k*Ts` is absorbed into the coefficients of the discretized model (for the zero-order-hold and first-order-hold methods only).

For example, to discretize the transfer function

$$H(s) = e^{-0.25s}\frac{10}{s^2 + 3s + 10}$$

using zero-order hold on the input, and a 10 Hz sampling rate, type

```
h = tf(10,[1 3 10],'inputdelay',0.25);
hd = c2d(h,0.1)
```

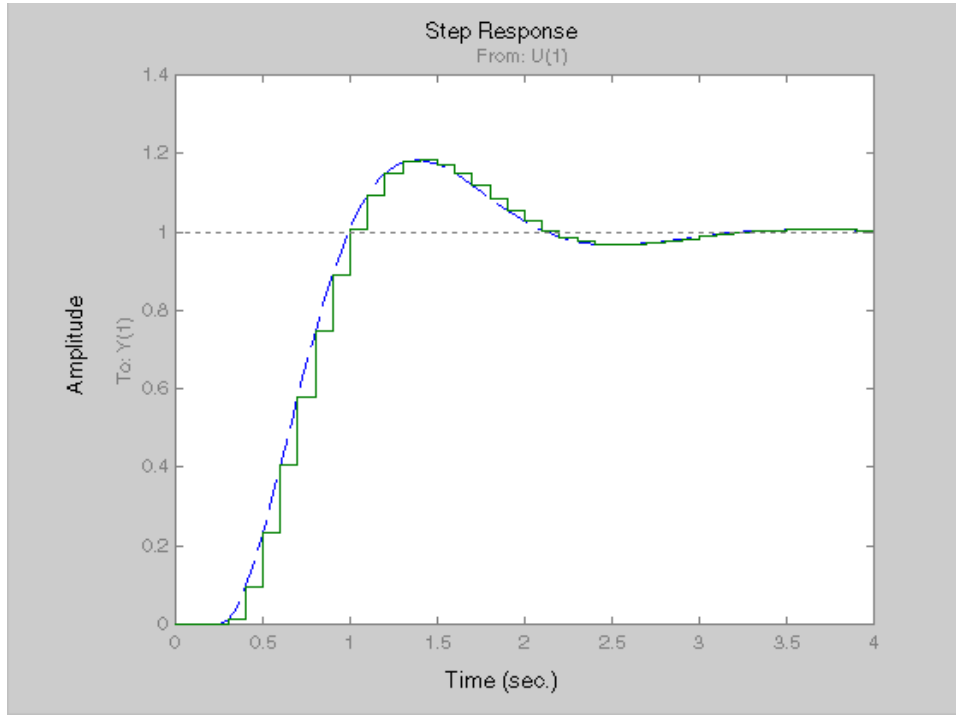
This produces the discrete-time transfer function

```
Transfer function:
         0.01187 z^2 + 0.06408 z + 0.009721
z^(-2) * ----------------------------------
             z^3 - 1.655 z^2 + 0.7408 z

Sampling time: 0.1
```

Here the input delay in $H(s)$ amounts to 2.5 times the sampling period of 0.1 seconds. Accordingly, the discretized model `hd` inherits an input delay of two sampling periods, as confirmed by the value of `hd.inputdelay`. The residual half-period delay is factored into the coefficients of `hd` by the discretization algorithm.

The step responses of the continuous and discretized models are compared in the figure below. This plot was produced by the command

```
step(h,'--',hd,'-')
```



Note: The Tustin and matched pole/zero methods are accurate only for delays that are integer multiples of the sampling period. It is therefore preferable to use the `zoh` and `foh` discretization methods for models with delays.

Delays and Continuous/Discrete Model Conversions

When you apply `c2d` and `d2c` to state-space models, all or part of the I/O delay matrix of the resulting model is absorbed into the input and output delay

vectors when it is possible to reduce the total number of I/O delays. The resulting model has a minimum number of such delays.

Resampling of Discrete-Time Models

You can resample a discrete-time TF, SS, or ZPK model `sys1` by typing

```
sys2 = d2d(sys1,Ts)
```

The new sampling period `Ts` does not have to be an integer multiple of the original sampling period. For example, typing

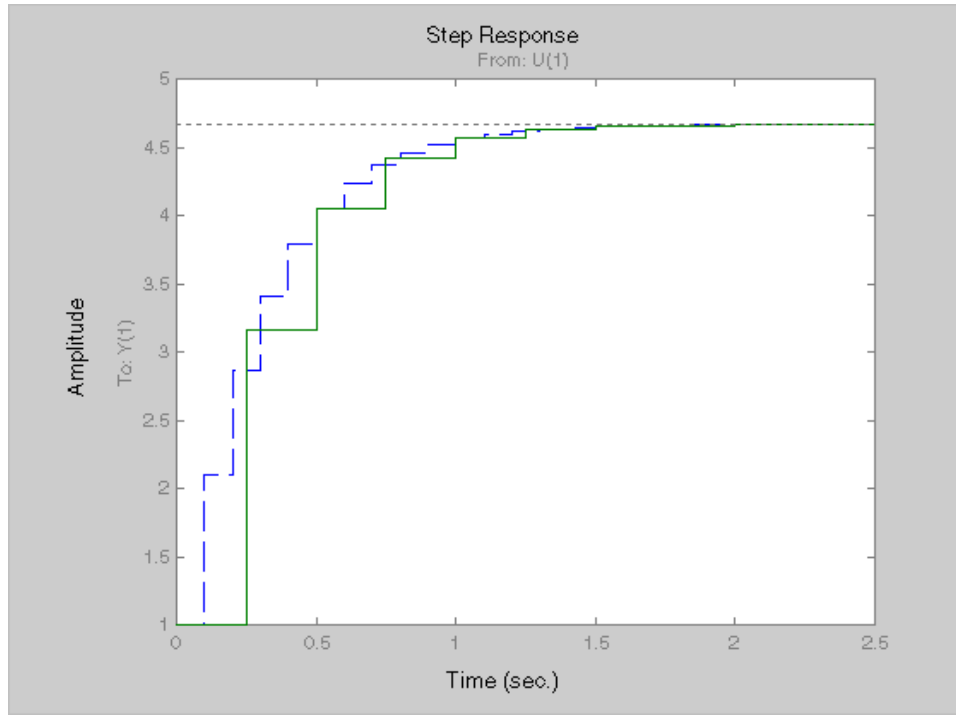
```
h1 = tf([1 0.4],[1 –0.7],0.1);
h2 = d2d(h1,0.25);
```

resamples `h1` at the sampling period of 0.25 seconds, rather than 0.1 seconds.

You can compare the step responses of `h1` and `h2` by typing

```
step(h1,'--',h2,'-')
```

The resulting plot is shown on the figure below (`h1` is the dashed line).



References

[1] Åström, K.J. and B. Wittenmark, *Computer-Controlled Systems: Theory and Design*, Prentice-Hall, 1990, pp. 48–52.

[2] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

4

Arrays of LTI Models

Introduction

You can use LTI arrays to collect a set of LTI models into a single MATLAB variable. You then use this variable to manipulate or analyze the entire collection of models in a vectorized fashion. You access the individual models in the collection through indexing rather than by individual model names.

LTI arrays extend the concept of single LTI models in a similar way to how multidimensional arrays extend two-dimensional matrices in MATLAB (see Chapter 12, "Multidimensional Arrays" in *Using MATLAB*). Additionally, when you think about LTI arrays, it is useful to keep in mind the matrix interpretation of LTI models developed in "Viewing LTI Systems As Matrices" on page 2-5.

When to Collect a Set of Models in an LTI Array

You can use LTI arrays to represent:

- A set of LTI models arising from the linearization of a nonlinear system at several operating points
- A collection of transfer functions that depend on one or more parameters
- A set of LTI models arising from several system identification experiments applied to one plant
- A set of gain-scheduled LTI controllers
- A list of LTI models you want to collect together under the same name

Restrictions for LTI Models Collected in an Array

For each model in an LTI array, the following properties must be the same:

- The number of inputs and outputs
- The sample time, for discrete-time models
- The I/O names and I/O groups

Note: You cannot specify Simulink LTI blocks with LTI arrays.

Where to Find Information on LTI Arrays

The next two sections give examples that illustrate the concept of an LTI array, its dimensions, and size. To read about how to build an LTI array, go to “Building LTI Arrays” on page 4-12. The remainder of the chapter is devoted to indexing and operations on LTI Arrays. You can also apply the analysis functions in the Control System Toolbox to LTI arrays. See Chapter 5, “Model Analysis Tools,” for more information on these functions. You can also view response plots of LTI arrays with the LTI Viewer. For information, see Chapter 6, “The LTI Viewer.”

The Concept of an LTI Array

To visualize the concept of an LTI array, consider the set of five transfer function models shown below. In this example, each model has two inputs and two outputs. They differ by parameter variations in the individual model components.

$$\begin{bmatrix} \frac{1.1}{s+1} & 0 \\ 0 & \frac{1}{s+5} \end{bmatrix} \quad \begin{bmatrix} \frac{1.3}{s+1.1} & 0 \\ 0 & \frac{1}{s+5.2} \end{bmatrix} \quad \begin{bmatrix} \frac{1.11}{s+1.2} & 0 \\ 0 & \frac{1}{s+5.4} \end{bmatrix} \quad \begin{bmatrix} \frac{1.15}{s+1.3} & 0 \\ 0 & \frac{1}{s+5.6} \end{bmatrix} \quad \begin{bmatrix} \frac{1.09}{s+1.4} & 0 \\ 0 & \frac{1}{s+5.8} \end{bmatrix}$$

Figure 4-1: Five LTI Models to be Collected in an LTI Array

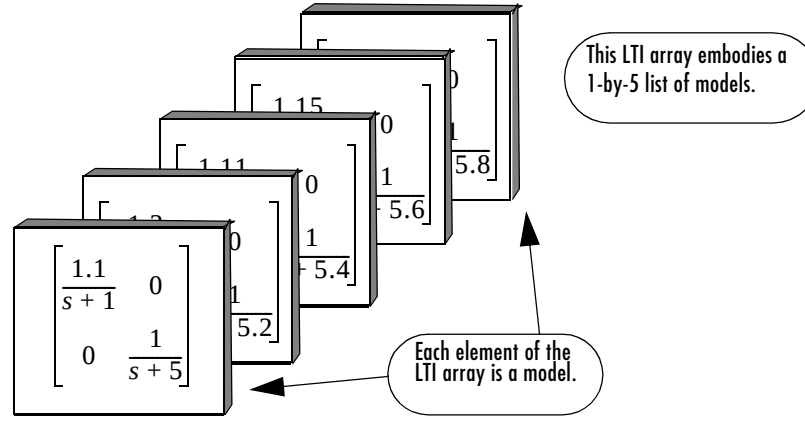
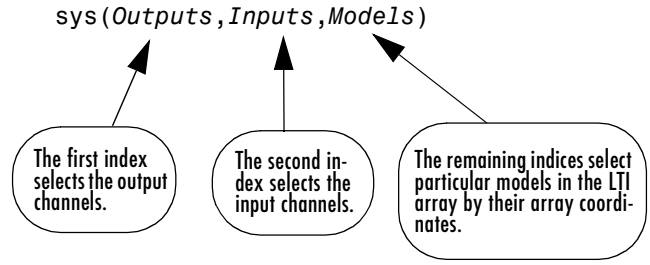


Figure 4-2: An LTI Array Containing These Five Models

Just as you might collect a set of two-by-two matrices in a multidimensional array, you can collect this set of five transfer function models as a list in an LTI array under one variable name, say, sys. Each element of the LTI array is an LTI model.

Individual models in the LTI array sys are accessed via indexing. The general form for the syntax you use to access data in an LTI array is



For example, you can access the third model in sys with sys(:,:,3). The following illustrates how you can use indexing to select models or their components from sys.

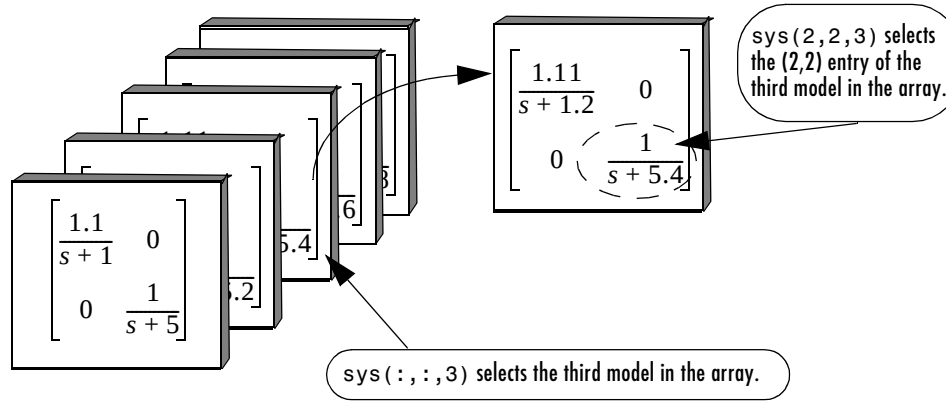


Figure 4-3: Using Indices to Select Models and Their Components

See “Indexing Into LTI Arrays” on page 4-20 for more information on indexing.

Higher Dimensional Arrays of LTI Models

You can also collect a set of models in a two-dimensional array. The following diagram illustrates a 2-by-3 array of six, two-output, one-input models called m2d.

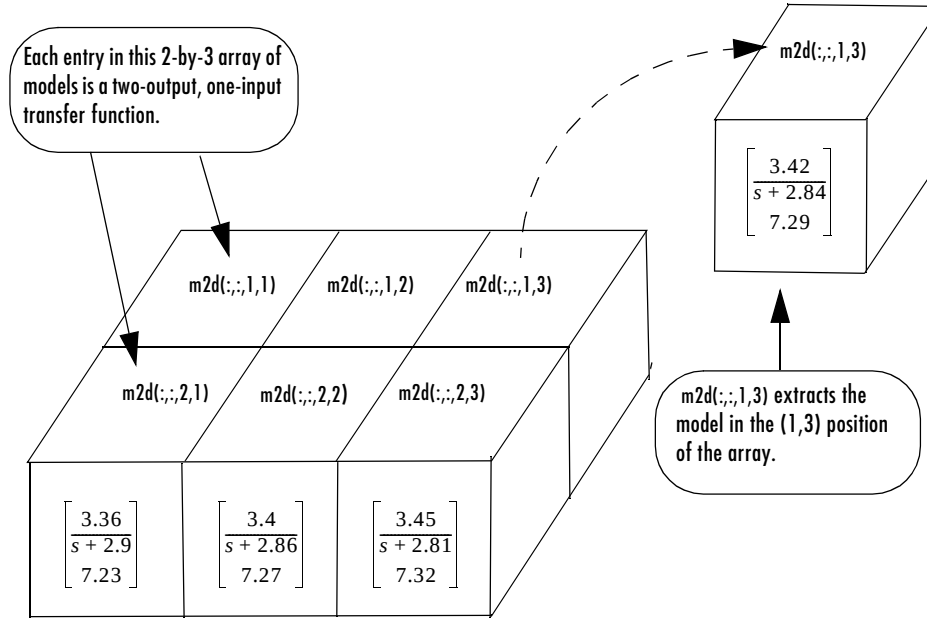


Figure 4-4: m2d: A 2-by-3 Array of Two-Output, One-Input Models

More generally, you can organize models into a 3-D or higher-dimensional array, in much the same way you arrange numerical data into multidimensional arrays (see Chapter 12, "Multidimensional Arrays" in *Using MATLAB).*

Dimensions, Size, and Shape of an LTI Array

The dimensions and size of a single LTI model are determined by the output and input channels. An array of LTI models has additional quantities that determine its dimensions, size, and shape.

There are two sets of dimensions associated with LTI arrays:

- The *I/O dimensions*—the output dimension and input dimension common to all models in the LTI array
- The *array dimensions*—the dimensions of the array of models itself

The size of the LTI array is determined by:

- The lengths of the I/O dimensions—the number of outputs (or inputs) common to all models in the LTI array
- The length of each array dimension—the number of models along that array dimension

The next figure illustrates the concepts of dimension and size for the LTI array m2d, a 2-by-3 array of one-input, two-output transfer function models.

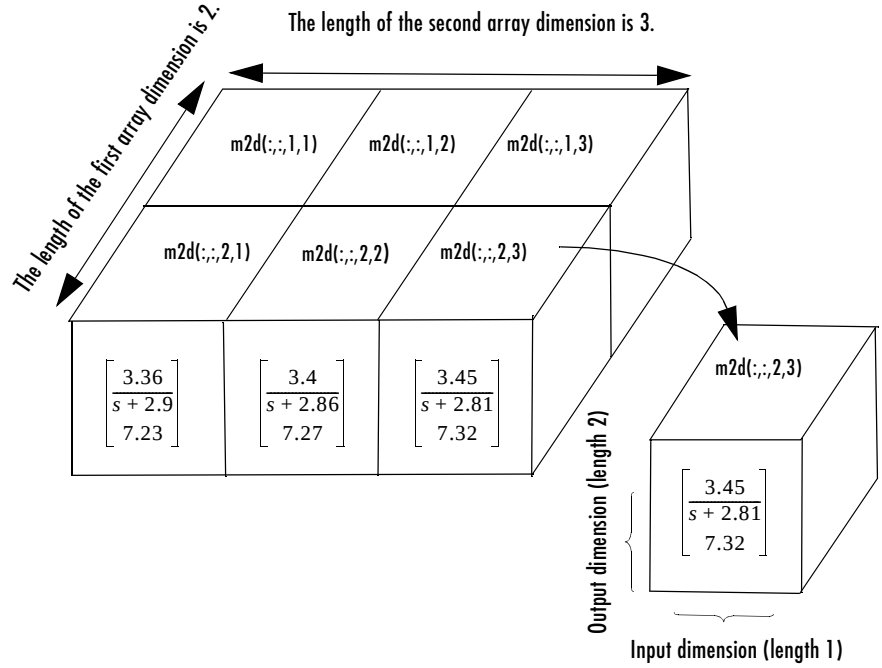


Figure 4-5: Dimensions and Size of m2d, an LTI Array

You can load this sample LTI array into your workspace by typing

```
load LTIexamples
size(m2d)

2x3 array of continuous-time transfer functions
Each transfer function has 2 outputs and 1 input.
```

According to the matrix analogy in "Viewing LTI Systems As Matrices" on page 2-5, the I/O dimensions correspond to the row and column dimensions of the transfer matrix. The two I/O dimensions are both of length 1 for SISO models. For MIMO models the lengths of these dimensions are given by the number of outputs and inputs of the model.

Five related quantities are pertinent to understanding the array dimensions:

- N, the number of models in the LTI array
- K, the number of array dimensions
- $S_1 S_2 \ldots S_K$, the list of lengths of the array dimensions
  - S_i is the number of models along the i^{th} dimension.
- $S_1 – \text{by} – S_2 – \text{by} – \ldots – \text{by} – S_K$, the configuration of the models in the array
  - The configuration determines the shape of the array.
  - The product of these integers $S_1 \times S_2 \times \ldots \times S_K$ is N.

In the example model `m2d`,:

- The length of the output dimension, the first I/O dimension, is 2, since there are two output channels in each model.
- The length of the input dimension, the second I/O dimension, is 1, since there is only one input channel in each model.
- N, the number of models in the LTI array, is 6.
- K, the number of array dimensions, is 2.
- The array dimension lengths are [2 3].
- The array configuration is 2-by-3.

size and ndims

You can access the dimensions and shape of an LTI array using:

- `size` to determine the lengths of each of the dimensions associated with an LTI array
- `ndims` to determine the total number of dimensions in an LTI array

When applied to an LTI array, `size` returns

```
[Ny Nu S1 S2 ... Sk]
```

where

- `Ny` is the number of outputs common to all models in the LTI array.
- `Nu` is the number of inputs common to all models in the LTI array.
- `S1 S2 ... Sk` are the lengths of the array dimensions of a *k*-dimensional array of models. `Si` is the number of models along the *i*th array dimension.

Note:

- By convention, a single LTI model is treated as a 1-by-1 array of models.

  For single LTI models, size returns only the I/O dimensions `[Ny Nu]`.
- For LTI arrays, size always returns at least two array dimensions.

  For example, the size of a 2-by-1 LTI array in `[Ny Nu 2 1]`
- `size` ignores trailing singleton dimensions beyond the second array dimension.

  For example, `size` returns `[Ny Nu 2 3]` for a 2-by-3-by-1-by-1 LTI array of models with `Ny` outputs and `Nu` inputs.
- You can use the syntax `size(sys,'order')` to determine the number of states in an LTI array, `sys`. A multidimensional array is returned if `sys` is an SS model such that the numbers of states in each model in `sys` are not the same.

The function `ndims` returns the total number of dimensions in an LTI array:

- 2, for single LTI models
- 2 + `p`, for LTI arrays, where `p` (greater than 2) is the number of array dimensions

Note that

```
ndims (sys) = length(size(sys))
```

To see how these work on the sample 2-by-3 LTI array `m2d` of two-output, one-input models, type

```
load LTIexamples
s = size(m2d)

s =

     2     1     2     3
```

Notice that `size` returns a vector whose entries correspond to the length of each of the four dimensions of `m2d`: two outputs and one input in a 2-by-3 array of models. Type

```
ndims(m2d)

ans =
     4
```

to see that there are indeed four dimensions attributed to this LTI array.

reshape

Use `reshape` to reorganize the arrangement (array configuration) of the models of an existing LTI array.

For example, to arrange the models in an LTI Array `sys` as a $w_1 \times \ldots \times w_p$ array, type

```
reshape(sys,w1,...,wp)
```

where `w1,...,wp` are any set of integers whose product is N, the number of models in `sys`.

You can reshape the LTI array `m2d` into a 3-by-2, a 6-by-1, or a 1-by-6 array using `reshape`. For example, type

```
load LTIexamples
sys = reshape(m2d,6,1);
size(sys)

6x1 array of continuous-time transfer functions
Each transfer function has 2 outputs and 1 inputs.

s = size(sys)

s =
     2     1     6     1
```

Building LTI Arrays

There are several ways to build LTI arrays:

- Using a `for` loop to assign each model in the array
- Using `stack` to concatenate LTI models into an LTI array
- Using `tf`, `zpk`, `ss`, or `frd`

In addition, you can use the command `rss` to generate LTI arrays of random state-space models.

Generating LTI Arrays Using rss

A convenient way to generate arrays of state-space models with the same number of states in each model is to use `rss`. The syntax is

```
rss(N,P,M,sdim1,...,sdimk)
```

where

- `N` is the number of states of each model in the LTI array.
- `P` is the number of outputs of each model in the LTI array.
- `M` is the number of inputs of each model in the LTI array.
- `sdim1,...,sdimk` are the lengths of the array dimensions.

For example, to create a 4-by-2 array of random state-space models with three states, one output, and one input, type

```
sys = rss(3,2,1,4,2);
size(sys)

4x2 array of continuous-time state-space models
Each model has 2 outputs, 1 input, and 3 states.
```

Building LTI Arrays Using for Loops

Consider the following second-order SISO transfer function that depends on two parameters, ζ and ω

$$.H(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

Suppose, based on measured input and output data, you estimate confidence intervals $[\omega_1,\omega_2]$, and $[\zeta_1,\zeta_2]$ for each of the parameters, ω and ζ. All of the possible combinations of the confidence limits for these model parameter values give rise to a set of four SISO models.

	ω_1	ω_2
ζ_1	$H_{11}(s)=\dfrac{\omega_1^2}{s^2+2\zeta_1\omega_1 s+\omega_1^2}$	$H_{12}(s)=\dfrac{\omega_1^2}{s^2+2\zeta_2\omega_1 s+\omega_1^2}$
ζ_2	$H_{21}(s)=\dfrac{\omega_2^2}{s^2+2\zeta_1\omega_2 s+\omega_2^2}$	$H_{22}(s)=\dfrac{\omega_2^2}{s^2+2\zeta_2\omega_2 s+\omega_2^2}$

Figure 4-6: Four LTI Models Depending on Two Parameters

You can arrange these four models in a 2-by-2 array of SISO transfer functions called `H`.

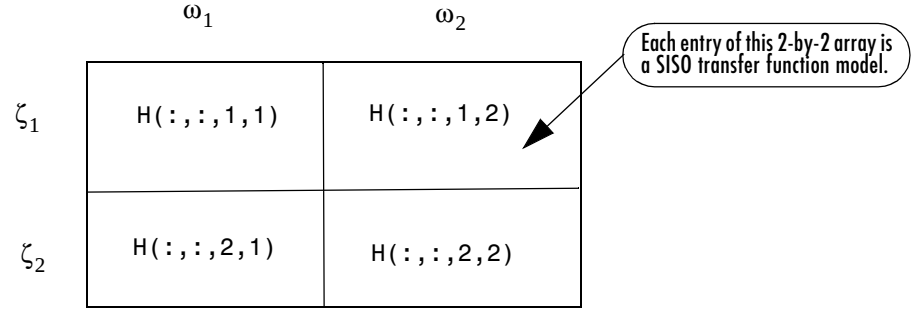


Figure 4-7: The LTI Array H

Here, for $i,j \in \{1, 2\}$, `H(:,:,i,j)` represents the transfer function

$$\frac{\omega_j^2}{s^2+2\zeta_i\omega_j s+\omega_j^2}$$

corresponding to the parameter values $\zeta = \zeta_i$ and $\omega = \omega_j$.

The first two colon indices (:) select all I/O channels from the I/O dimensions of H. The third index of H refers to the first array dimension (ζ), while the fourth index is for the second array dimension (ω).

Suppose the limits of the ranges of values for ζ and ω are [0.66,0.76] and [1.2,1.5], respectively. Enter these at the command line.

```
zeta = [0.66,0.75];
w = [1.2,1.5];
```

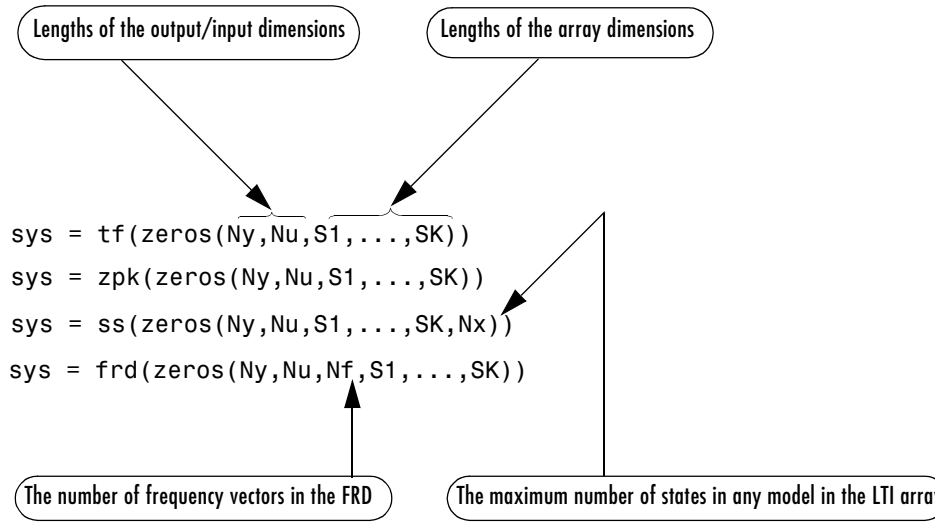
Since the four models have the same parametric structure, it's convenient to use two nested `for` loops to construct the LTI array.

```
for i = 1:2
  for j = 1:2
    H(:,:,i,j) = tf(w(j)^2,[1 2*zeta(i)*w(j) w(j)^2]);
  end
end
```

H now contains the four models in a 2-by-2 array. For example, to display the transfer function in the (1,2) position of the array, type

```
H(:,:,1,2)

Transfer function:
       2.25
-------------------
s^2 + 1.98 s + 2.25
```

For the purposes of efficient computation, you can initialize an LTI array to zero, and then reassign the entire array to the values you want to specify. The general syntax for zero assignment of LTI arrays is



To initialize `H` in the above example to zero, type

```
H = tf(zeros(1,1,2,2));
```

before you implement the nested `for` loops.

Building LTI Arrays Using the stack Function

Another way to build LTI arrays is using the function `stack`. This function operates on single LTI models as well as LTI arrays. It concatenates a list of LTI arrays or single LTI models only along the array dimension. The general syntax for `stack` is

```
stack(Arraydim,sys1,sys2...)
```

where

- `Arraydim` is the array dimension along which to concatenate the LTI models or arrays.
- `sys1`, `sys2`, ... are the LTI models or LTI arrays to be concatenated.

When you concatenate several models or LTI arrays along the *j*th array dimension, such as in

```
stack(j,sys1,sys2,...,sysn)
```

- The lengths of the I/O dimensions of `sys1,...,sysn` must all match.
- The lengths of all but the *j*th array dimension of `sys1,...,sysn` must match.

For example, if two TF models `sys1` and `sys2` have the same number of inputs and outputs,

```
sys = stack(1,sys1,sys2)
```

concatenates them into a 2-by-1 array of models.

Note:

- `stack` only concatenates along an array dimension, not an I/O dimension.
- To concatenate LTI models or LTI arrays along an input or output dimension, use the bracket notation (`[,] [;]`). See "Model Interconnection Functions" on page 3-16 for more information on the use of bracket notation to concatenate models. See also "Special Cases for Operations on LTI Arrays" on page 4-27 for some examples of this type of concatenation of LTI arrays.

Here's an example of how to build the LTI array `H` using the function `stack`.

```
% Set up the parameter vectors.

zeta = [0.66,0.75];
w = [1.2,1.5];

% Specify the four individual models with those parameters.
%
H11 = tf(w(1)^2,[1 2*zeta(1)*w(1) w(1)^2]);
H12 = tf(w(2)^2,[1 2*zeta(1)*w(2) w(2)^2]);
H21 = tf(w(1)^2,[1 2*zeta(2)*w(1) w(1)^2]);
H22 = tf(w(2)^2,[1 2*zeta(2)*w(2) w(2)^2]);
```

```
% Set up the LTI array using stack.

COL1 = stack(1,H11,H21); % The first column of the 2-by-2 array
COL2 = stack(1,H12,H22); % The second column of the 2-by-2 array
H = stack(2, COL1, COL2); % Concatenate the two columns of models.
```

Notice that this result is very different from the single MIMO LTI model returned by

```
H = [H11,H12;H21,H22];
```

Building LTI Arrays Using tf, zpk, ss, and frd

You can also build LTI arrays using the `tf`, `zpk`, `ss`, and `frd` constructors. You do this by using multidimensional arrays in the input arguments for these functions.

Specifying Arrays of TF models tf

For TF models, use

```
sys = tf(num,den)
```

where

- Both `num` and `den` are multidimensional cell arrays the same size as `sys` (see "size and ndims" on page 4-9).
- `sys(i,j,n1,...,nK)` is the (i, j) entry of the transfer matrix for the model located in the $(n_1, \ldots, n_K)$ position of the array.
- `num(i,j,n1,...,nK)` is a row vector representing the numerator polynomial of `sys(i,j,n1,...,nK)`.
- `den(i,j,n1,...,nK)` is a row vector representing denominator polynomial of `sys(i,j,n1,...,nK)`.

See "MIMO Transfer Function Models" on page 2-10 for related information on the specification of single TF models.

Specifying Arrays of ZPK Models Using zpk

For ZPK models, use

```
sys = zpk(zeros,poles,gains)
```

where

- Both `zeros` and `poles` are multidimensional cell arrays whose cell entries contain the vectors of zeros and poles for each I/O pair of each model in the LTI array.
- `gains` is a multidimensional array containing the scalar gains for each I/O pair of each model in the array.
- The dimensions (and their lengths) of `zeros`, `poles`, and `gains`, determine those of the LTI array, `sys`.

Specifying Arrays of SS Models Using ss

To specify arrays of SS models, use

```
sys = ss(a,b,c,d)
```

where `a`, `b`, `c`, and `d` are real-valued multidimensional arrays of appropriate dimensions. All models in the resulting array of SS models have the same number of states, outputs, and inputs.

Note: You cannot use the `ss` constructor to build an array of state-space models with different numbers of states. Use `stack` to build such LTI arrays.

The Size of LTI Array Data for SS Models

The size of the model data for arrays of state-space models is summarized in the following table.

Data	Size (Data)
a	$\left[N_s N_s S_1 S_2 \ldots S_K\right]$
b	$\left[N_s N_u S_1 S_2 \ldots S_K\right]$
c	$\left[N_y N_s S_1 S_2 \ldots S_K\right]$
d	$\left[N_y N_u S_1 S_2 \ldots S_K\right]$

where

- N_s is the maximum of the number of states in each model in the array.
- N_u is the number of inputs in each model.
- N_y is the number of outputs in each model.
- $S_1, S_2, \ldots, S_K$ are the lengths of the array dimensions.

Specifying Arrays of FRD Models Using frd

To specify a *K*-dimensional array of *p*-output, *m*-input FRD models for which $S_1, S_2, \ldots, S_K$ are the lengths of the array dimensions, use

```
sys = frd(response,frequency,units)
```

where

- `frequency` is a real vector of *n* frequency data points common to all FRD models in the LTI array.
- `response` is a *p*-by-*m*-by-*n*-*by*-S_1-by-…-by-S_K complex-valued multidimensional array.
- `units` is the optional string specifying `'rad/s'` or `'Hz'`.

Note that for specifying an LTI array of SISO FRD models, `response` can also be a multidimensional array of 1-by-*n* matrices whose remaining dimensions determine the array dimensions of the FRD.

Indexing Into LTI Arrays

You can index into LTI arrays in much the same way as you would for multidimensional arrays to:

- Access models
- Extract subsystems
- Reassign parts of an LTI array
- Delete parts of an LTI array

When you index into an LTI array `sys`, the indices should be organized according to the following format

`sys(`*`Outputs`*`,` *`Inputs`*`,` $n_1, \ldots, n_K$ `)`

where

- *`Outputs`* are indices that select output channels.
- *`Inputs`* are indices that select input channels.
- $n_1, \ldots, n_K$ are indices into the array dimensions that select one model or a subset of models in the LTI array.

Note on Indexing into LTI Arrays of FRD models: For FRD models, the array indices can be followed by the keyword `'frequency'` and some expression selecting a subset of the frequency points as in

```
sys (outputs, inputs, n1,...,nk, 'frequency', SelectedFreqs)
```

See “Referencing FRD Models Through Frequencies” on page 3-7 for details on frequency point selection in FRD models.

Accessing Particular Models in an LTI Array

To access any given model in an LTI array:

- Use colon arguments `(:,:)` for the first two indices to select all I/O channels.
- The remaining indices specify the model coordinates within the array.

For example, if sys is a 5-by-2 array of state-space models defined by

```
sys = rss(4,3,2,5,2);
```

you can access (and display) the model located in the (3,2) position of the array sys by typing

```
sys(:,:,3,2)
```

If sys is a 5-by-2 array of 3-output, 2-input FRD models, with frequency vector [1,2,3,4,5], then you can access the response data corresponding to the middle frequency (3 rad/s), of the model in the (3,1) position by typing

```
sys(:,:,3,1,'frequency',3.0)
```

To access all frequencies of this model in the array, you can simply type

```
sys(:,:,3,1)
```

Single Index Referencing of Array Dimensions

You can also access models using single index referencing of the array dimensions.

For example, in the 5-by-2 LTI array sys above, you can also access the model located in the (3,2) position by typing

```
sys(:,:,8)
```

since this model is in the eighth position if you were to list the 10 models in the array by successively scanning through its entries along each of its columns.

For more information on single index referencing, see the "Advanced Indexing" section of Chapter 10, "M-File Programming" in *Using MATLAB*.

Extracting LTI Arrays of Subsystems

To select a particular subset of I/O channels from all the models in an LTI array, use the syntax described in "Extracting and Modifying Subsystems" on page 3-5. For example,

```
sys = rss(4,3,2,5,2);
A = sys(1, [1 2])
```

or equivalently,

```
A = sys(1,[1 2],:,:)
```

selects the first two input channels, and the first output channel in each model of the LTI array `A`, and returns the resulting 5-by-2 array of one-output, two-input subsystems.

You can also combine model selection with I/O selection within an LTI array. For example, to access both:

- The state-space model in the `(3,2)` array position
- Only the portion of that model relating the second input to the first output

type

```
sys(1,2,3,2)
```

To access the subsystem from all inputs to the first two output channels of this same array entry, type

```
sys(1:2,:,3,2)
```

Reassigning Parts of an LTI Array

You can reassign entire models or portions of models in an LTI array. For example,

```
sys = rss(4,3,2,5,2); % 5X2 array of state-space models
H = rss(4,1,1,5,2);   % 5X2 array of SISO models
sys(1,2) = H
```

reassigns the subsystem from input two to output one, for all models in the LTI array `sys`. This SISO subsystem of each model in the LTI array is replaced with the LTI array `H` of SISO models. This one-line assignment command is equivalent to the following 10-step nested `for` loop.

```
for k = 1:5
  for j = 1:2
    sys(1,2,k,j) = H(:,:,k,j);
  end
end
```

Notice that you don't have to use the array dimensions with this assignment. This is because I/O selection applies to all models in the array when the array indices are omitted.

Similarly, the commands

```
sys(:,:,3,2) = sys(:,:,4,1);
sys(1,2,3,2) = 0;
```

reassign the entire model in the (3,2) position of the LTI array sys and the (1,2) subsystem of this model, respectively.

LTI Arrays of SS Models with Differing Numbers of States

You must use an entire LTI model for reassignment if you have an LTI array sys of state-space models for which:

- The numbers of states in each model in sys is not constant.
- You want to change the dimensions of the a, b, and c matrices in one model as you reassign its a, b, and c properties.

For example, if

```
sys = ss(stack(1,tf(1,[1 2 1]),tf(1,[1 1])));
```

then the model sys(:,:,1) is of order 2, while sys(:,:,2) is of order 1. To reassign the a, b, and c properties of sys(:,:,1) so that the state dimension of the new model is not 2, you must make the entire model assignment at once. You can do this as follows.

```
sys(:,:,1) = sys2
```

where sys2 represents an LTI model for which size(sys2,'order') is not 2.

Deleting Parts of an LTI Array

You can use indexing to delete any part of an LTI array by reassigning it to be empty ([]). For instance,

```
sys = rss(4,3,2,5,2);
sys(1,:) = [];
size(sys)

5x2 array of continuous-time state-space models
Each model has 2 outputs, 2 inputs, and 4 states.
```

deletes the first output channel from every model of this LTI array.

Similarly,

```
sys(:,:,[3 4],:) = []
```

deletes the third and fourth rows of this two-dimensional array of models.

Operations on LTI Arrays

Using LTI arrays, you can apply almost all of the basic model operations that work on single LTI models to entire sets of models at once. These basic operations, discussed in Chapter 3, "Operations on LTI Models," include:

- The arithmetic operations: +, –, *, /, \, ', .'
- The model interconnection functions: concatenation along I/O dimensions ([,], [;]), `feedback`, `append`, `series`, `parallel`, and `lft`

When you apply any of these operations to two (or more) LTI arrays (for example, `sys1` and `sys2`), the operation is implemented on a model-by-model basis. Therefore, the *k*th model of the resulting LTI array is derived from the application of the given operation to the *k*th model of `sys1` and the *k*th model of `sys2`.

For example, if `sys1` and `sys2` are two LTI arrays and

```
sys = op(sys1,sys2)
```

then the *k*th model in the resulting LTI array `sys` is obtained by adding the *k*th models in `sys1` to the *k*th model in `sys2`

```
sys(:,:,k) = sys1(:,:,k) + sys2(:,:,k)
```

You can also apply any of the response plotting functions such as `step`, `bode`, and `nyquist` described in Chapter 5, "Model Analysis Tools," to LTI arrays. These plotting functions are also applied on a model by model basis. See "The Right-Click Menu for LTI Arrays" on page 6-28 for information on using the model selector for LTI arrays on response plots.

Example: Addition of Two LTI Arrays

The following diagram illustrates the addition of two 3-by-1 LTI arrays `sys1+sys2`.

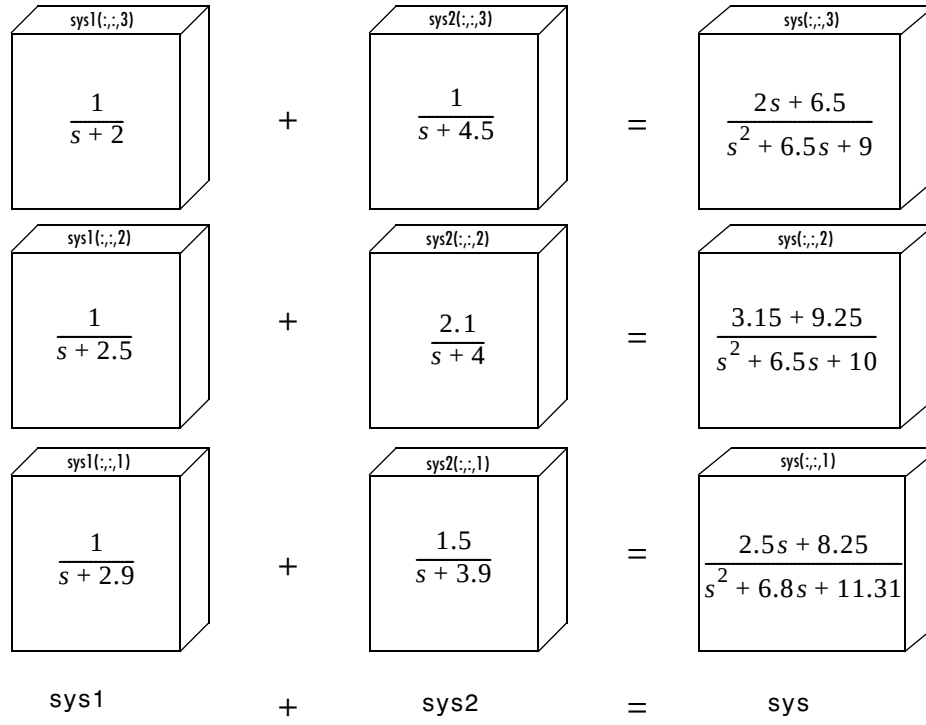


Figure 4-8: The Addition of Two LTI Arrays

The summation of these LTI arrays

```
sys = sys1+sys2
```

is equivalent to the following model-by-model summation.

```
for k = 1:3
  sys(:,:,k)=sys1(:,:,k) + sys2(:,:,k)
end
```

Note that:

- Each model in sys1 and sys2 must have the same number of inputs and outputs. This is required for the addition of two LTI arrays.
- The lengths of the array dimensions of sys1 and sys2 must match.

Dimension Requirements

In general, when you apply any of these basic operations to two or more LTI arrays:

- The I/O dimensions of each of the LTI arrays must be compatible with the requirements of the operation.
- The lengths of array dimensions must match.

The I/O dimensions of each model in the resulting LTI array are determined by the operation being performed. See Chapter 3, "Operations on LTI Models," for requirements on the I/O dimensions for the various operations.

For example, if sys1 and sys2 are both 1-by-3 arrays of LTI models with two inputs and two outputs, and sys3 is a 1-by-3 array of LTI models with two outputs and 1 input, then

```
sys1 + sys2
```

is an LTI array with the same dimensions as sys1 and sys2.

```
sys1 * sys3
```

is a 1-by-3 array of LTI models with two outputs and one input, and

```
[sys1,sys3]
```

is a 1-by-3 array of LTI models with two outputs and three inputs.

Special Cases for Operations on LTI Arrays

There are some special cases in applying operations to LTI arrays.

Consider applying any binary operation called op (such as +, –, or *) to an LTI array sys1

```
sys = op(sys1,sys2)
```

where sys, the result of the operation, is an LTI array with the same array dimensions as sys1. You can use shortcuts for coding `sys = op(sys1,sys2)` in the following cases:

- For operations that apply to LTI arrays, sys2 does not have to be an array. It can be a single LTI model (or a gain matrix) whose I/O dimensions satisfy the compatibility requirements for op (with those of each of the models in sys1). In this case, op applies sys2 to each model in sys1, and the *k*th model in sys satisfies

  ```
  sys(:,:,k) = op(sys1(:,:,k),sys2)
  ```

- For arithmetic operations, such as +, *, /, and \, sys2 can be either a single SISO model, or an LTI array of SISO models, even when sys1 is an LTI array of MIMO models. This special case relies on MATLAB's scalar expansion capabilities for arithmetic operations.
  - When sys2 is a single SISO LTI model (or a scalar gain), op applies sys2 to sys1 on an entry-by-entry basis. The *ij*th entry in the *k*th model in sys satisfies

    ```
    sys(i,j,k) = op(sys1(i,j,k),sys2)
    ```

  - When sys2 is an LTI array of SISO models (or a multidimensional array of scalar gains), op applies sys2 to sys1 on an entry-by-entry basis for each model in sys.

    ```
    sys(i,j,k) = op(sys1(i,j,k),sys2(:,:,k))
    ```

Examples of Operations on LTI Arrays with Single LTI Models

Suppose you want to create an LTI array containing three models, where, for τ in the set $\{1.1, 1.2, 1.3\}$, each model $H_\tau(s)$ has the form

$$H_\tau(s) = \begin{bmatrix} \frac{1}{s+\tau} & 0 \\ -1 & \frac{1}{s} \end{bmatrix}$$

You can do this efficiently by first setting up an LTI array `h` containing the SISO models $1/(s+\tau)$ and then using concatenation to form the LTI array `H` of MIMO LTI models $H_\tau(s)$, $\tau \in \{1.1, 1.2, 1.3\}$. To do this, type

```
tau = [1.1 1.2 1.3];
for i=1:3                    % Form LTI array h of SISO models.
  h(:,:,i)=tf(1,[1 tau]);
end
H = [h 0; -1 tf(1,[1 0])]; %Concatenation: array h & single models
size(H)

3x1 array of continuous-time transfer functions
Each transfer function has 2 output(s) and 2 input(s).
```

Similarly, you can use `append` to perform the diagonal appending of each model in the SISO LTI array `h` with a fixed single (SISO or MIMO) LTI model.

```
S = append(h,tf(1,[1 3])); % Append a single model to h.
```

specifies an LTI array `S` in which each model has the form

$$S_\tau(s) = \begin{bmatrix} \frac{1}{s+\tau} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

You can also combine an LTI array of MIMO models and a single MIMO LTI model using arithmetic operations. For example, if `h` is the LTI array of three SISO models defined above,

```
[h,h] + [tf(1,[1 0]);tf(1,[1 5])]
```

adds the single one-output, two-input LTI model `[1/s 1/(s + 5)]` to every model in the 3-by-1 LTI array of one-output, two-input models `[h,h]`. The result is a new 3-by-2 array of models.

Examples: Arithmetic Operations on LTI Arrays and SISO Models

Using the LTI array of one-output, two-input state-space models `[h,h]`, defined in the previous example,

```
tf(1,[1 3]) + [h,h]
```

adds a single SISO transfer function model to each entry in each model of the LTI array of MIMO models `[h,h]`.

Finally,

```
G = rand(1,1,3,1);
sys = G + [h,h]
```

adds the array of scalars to each entry of each MIMO model in the LTI array `[h,h]` on a model-by-model basis. This last command is equivalent to the following `for` loop.

```
hh = [h,h];
for k = 1:3
  sys(:,:,k) = G(1,1,k) + hh(:,:,k);
end
```

Other Operations on LTI Arrays

You can also apply the analysis functions, such as `bode`, `nyquist`, and `step`, to LTI arrays. See Chapter 5, “Model Analysis Tools,” for more information on these functions.

5

Model Analysis Tools

General Model Characteristics

General model characteristics include the model type, I/O dimensions, and continuous or discrete nature. Related commands are listed in the table below. These commands operate on continuous- or discrete-time LTI models or arrays of LTI models of any type.

General Model Characteristics Commands	
`class`	Display model type (`'tf'`, `'zpk'`, `'ss'`, or `'frd'`).
`hasdelay`	Test true if LTI model has any type of delay.
`isa`	Test true if LTI model is of specified class.
`isct`	Test true for continuous-time models.
`isdt`	Test true for discrete-time models.
`isempty`	Test true for empty LTI models.
`isproper`	Test true for proper LTI models.
`issiso`	Test true for SISO models.
`ndims`	Display the number of model/array dimensions.
`reshape`	Change the shape of an LTI array.
`size`	Output/input/array dimensions. Used with special syntax, `size` also returns the number of state dimensions for state-space models, and the number of frequencies in an FRD model.

This example illustrates the use of some of these commands. See the related reference pages for more details.

```
H = tf({1 [1 -1]},{[1 0.1] [1 2 10]})

Transfer function from input 1 to output:
   1
-------
s + 0.1

Transfer function from input 2 to output:
    s - 1
--------------
s^2 + 2 s + 10


class(H)

ans =
tf

size(H)
Transfer function with 2 input(s) and 1 output(s).

[ny,nu] = size(H)% Note: ny = number of outputs

ny =
     1

nu =
     2

isct(H)% Is this system continuous?

ans =
     1

isdt(H)% Is this system discrete?

ans =
     0
```

Model Dynamics

The Control System Toolbox offers commands to determine the system poles, zeros, DC gain, norms, etc. You can apply these commands to single LTI models or LTI arrays. The following table gives an overview of these commands.

Model Dynamics	
`covar`	Covariance of response to white noise.
`damp`	Natural frequency and damping of system poles.
`dcgain`	Low-frequency (DC) gain.
`dsort`	Sort discrete-time poles by magnitude.
`esort`	Sort continuous-time poles by real part.
`norm`	Norms of LTI systems (H_2 and L_∞).
`pole, eig`	System poles.
`pzmap`	Pole/zero map.
`zero`	System transmission zeros.

With the exception of L_∞ `norm`, these commands are not supported for FRD models.

Here is an example of model analysis using some of these commands.

```
h = tf([4 8.4 30.8 60],[1 4.12 17.4 30.8 60])

Transfer function:
     4 s^3 + 8.4 s^2 + 30.8 s + 60
-------------------------------------
s^4 + 4.12 s^3 + 17.4 s^2 + 30.8 s + 60

pole(h)

ans =
  -1.7971 + 2.2137i
  -1.7971 - 2.2137i
  -0.2629 + 2.7039i
  -0.2629 - 2.7039i

zero(h)
ans =
   -0.0500 + 2.7382i
   -0.0500 - 2.7382i
   -2.0000

dcgain(h)

ans =
     1

[ninf,fpeak] = norm(h,inf)% peak gain of freq. response

ninf =
1.3402     % peak gain

fpeak =
1.8537     % frequency where gain peaks
```

These functions also operate on LTI arrays and return arrays. For example, the poles of a three dimensional LTI array sysarray are obtained as follows.

```
sysarray = tf(rss(2,1,1,3))

Model sysarray(:,:,1,1)
=======================
  Transfer function:
    -0.6201 s - 1.905
  ---------------------
  s^2 + 5.672 s + 7.405

Model sysarray(:,:,2,1)
=======================
  Transfer function:
  0.4282 s^2 + 0.3706 s + 0.04264
  -------------------------------
      s^2 + 1.056 s + 0.1719

Model sysarray(:,:,3,1)
=======================
  Transfer function:
    0.621 s + 0.7567
  ---------------------
  s^2 + 2.942 s + 2.113

3x1 array of continuous-time transfer functions.

pole(sysarray)
ans(:,:,1) =
   -3.6337
   -2.0379
ans(:,:,2) =
   -0.8549
   -0.2011
ans(:,:,3) =
   -1.6968
   -1.2452
```

State-Space Realizations

The following functions are useful to analyze, perform state coordinate transformations on, and derive canonical state-space realizations for single state-space LTI models or LTI arrays of state-space models.

State-Space Realizations	
`canon`	Canonical state-space realizations.
`ctrb`	Controllability matrix.
`ctrbf`	Controllability staircase form.
`gram`	Controllability and observability gramians.
`obsv`	Observability matrix.
`obsvf`	Observability staircase form.
`ss2ss`	State coordinate transformation.
`ssbal`	Diagonal balancing of state-space realizations.

The function `ssbal` uses a simple *diagonal* similarity transformation

$$(A, B, C) \to (T^{-1}AT, T^{-1}B, CT)$$

to balance the state-space data (A, B, C). This is accomplished by reducing the norm of the matrix.

$$\begin{bmatrix} T^{-1}AT & T^{-1}B \\ CT & 0 \end{bmatrix}$$

Such balancing usually improves the numerical conditioning of subsequent state-space computations. Note that conversions to state-space using `ss` produce balanced realizations of transfer functions and zero-pole-gain models.

By contrast, the canonical realizations produced by `canon`, `ctrbf`, or `obsvf` are often badly scaled, sensitive to perturbations of the data, and poorly suited for

state-space computations. Consequently, it is wise to use them only for analysis purposes and not in control design algorithms.

Time and Frequency Response

The Control System Toolbox contains a set of commands that provide the basic time and frequency domain analysis tools required for control system engineering. These commands apply to any kind of LTI model (TF, ZPK, or SS, continuous or discrete, SISO or MIMO). You can only apply the frequency domain analysis tools FRDs. The LTI Viewer provides an integrated graphical user interface (GUI) to analyze and compare LTI models (see Chapter 6, "The LTI Viewer" for details).

Time Responses

Time responses investigate the time-domain transient behavior of LTI models for particular classes of inputs and disturbances. You can determine such system characteristics as rise time, settling time, overshoot, and steady-state error from the time response. The Control System Toolbox provides functions for step response, impulse response, initial condition response, and general linear simulations. You can apply these functions to single TF, SS, or ZPK models or arrays of these types of models. Note that you can simulate the response to white noise inputs using `lsim` and the function `rand` (see *Using MATLAB* to generate random input vectors.

Time Response	
`impulse`	Impulse response.
`initial`	Initial condition response.
`gensig`	Input signal generator.
`lsim`	Simulation of response to arbitrary inputs.
`step`	Step response.

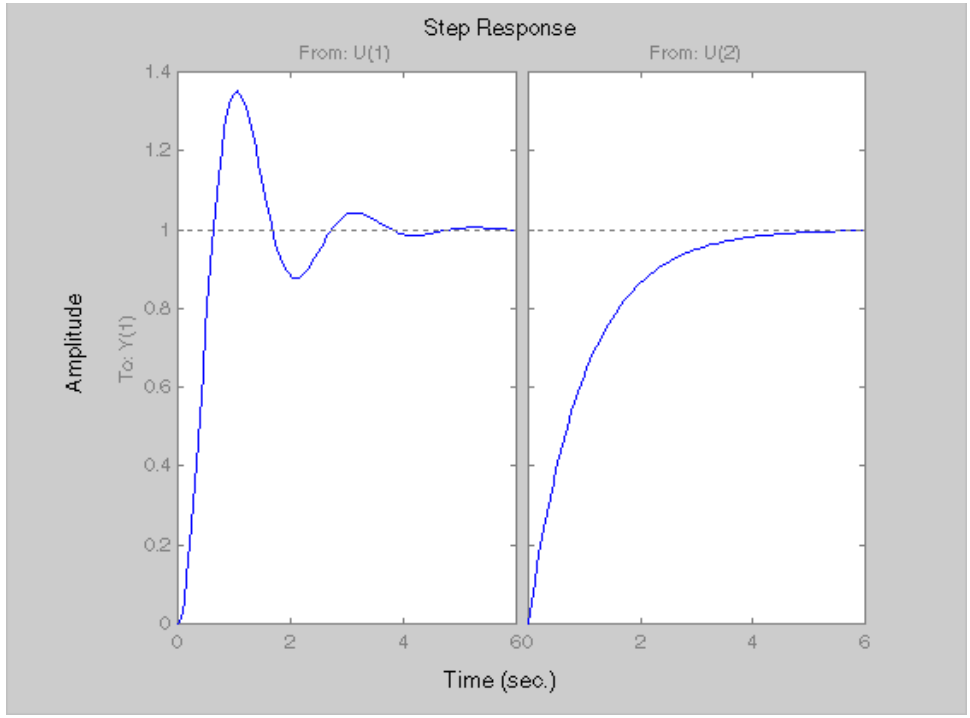
The functions `step`, `impulse`, and `initial` automatically generate an appropriate simulation horizon for the time response plots. Their syntax is

```
step(sys)
impulse(sys)
initial(sys,x0)    % x0 = initial state vector
```

where sys is any continuous or discrete LTI model or LTI array. For MIMO models, these commands produce an array of plots with one plot per I/O channel (or per output for `initial` and `lsim`). For example,

```
h = [tf(10,[1 2 10]) , tf(1,[1 1])]
step(h)
```

produces the following plot.



The simulation horizon is automatically determined based on the model dynamics. You can override this automatic mode by specifying a final time

```
step(sys,10)  % simulates from 0 to 10 seconds
```

or a vector of evenly spaced time samples.

```
t = 0:0.01:10  % time samples spaced every 0.01 second
step(sys,t)
```

Note: When specifying a time vector `t = [0:dt:tf]`, remember the following constraints on the spacing `dt` between time samples:

- For discrete systems, `dt` should match the system sample time.
- Continuous systems are first discretized using zero-order hold and `dt` as sampling period, and `step` simulates the resulting discrete system. As a result, you should pick `dt` small enough to capture the main features of the continuous transient response.

The syntax `step(sys)` automatically takes these issues into account.

Finally, the function `lsim` simulates the response to more general classes of inputs. For example,

```
t = 0:0.01:10
u = sin(t)
lsim(sys,u,t)
```

simulates the zero-initial condition response of the LTI system `sys` to a sine wave for a duration of 10 seconds.

Note: You can also implement several plotting options by using the right-click menus accessible from the (white) plot region of all time and frequency plots. These options are listed on the menu. To learn more about the right-click menus on plots, see "The Right-Click Menus" on page 6-18

Frequency Response

The Control System Toolbox provides response-plotting functions for the following frequency domain analysis tools:

- Bode plots
- Nichols charts
- Nyquist plots
- Singular value plots

In addition, the function `margin` determines the gain and phase margins for a given SISO open-loop model. These functions can be applied to single LTI models or LTI arrays.

Table 5-1: Frequency Response

Function Name	Description
`bode`	Computes the Bode plot.
`evalfr`	Computes the frequency response at a single complex frequency (not for FRD models).
`freqresp`	Computes the frequency response for a set of frequencies.
`margin`	Computes gain and phase margins.
`ngrid`	Applies grid lines to a Nichols plot.
`nichols`	Computes the Nichols plot.
`nyquist`	Computes the Nyquist plot.
`sigma`	Computes the singular value plot.

As for time response functions, the commands

```
bode(sys)
nichols(sys)
nyquist(sys)
sigma(sys)
```

handle both continuous and discrete models. These functions produce a frequency response plot for SISO LTI models, and an array of plots in the MIMO case. The frequency grid used to evaluate the response is automatically selected based on the system poles and zeros.

The Bode plot produced by `bode` plots the magnitude of the frequency response in decibels (dB), as `20*log10(abs(response))`. Phase is plotted in degrees.

To set the frequency range explicitly to some interval `[wmin,wmax]`, use the syntax

```
bode(sys,{wmin , wmax})    % Note the curly braces
```

For example,

```
bode(sys,{0.1 , 100})
```

draws the Bode plot between 0.1 and 100 radians/second. You can also specify a particular vector of frequency points as in

```
w = logspace(-1,2,100)
bode(sys,w)
```

The `logspace` command generates a vector `w` of logarithmically spaced frequencies starting at $10^{-1} = 0.1$ rad/s and ending at $10^{2} = 100$ rad/s. See the reference page for `linspace` for linearly spaced frequency vectors.

Note: In discrete time, the frequency response is evaluated on the unit circle and the notion of "frequency" should be understood as follows. The upper half of the unit circle is parametrized by

$$z = e^{j\omega T_s}, \qquad 0 \le \omega \le \omega_N = \frac{\pi}{T_s}$$

where T_s is the system sample time and ω_N is called the *Nyquist frequency*. The variable ω plays the role of continuous-time frequency. We use this "equivalent frequency" as an x-axis variable in all discrete-time frequency response plots. In addition, the frequency response is plotted only up to the Nyquist frequency ω_N because it is periodic with period $2\omega_N$ (a phenomenon known as *aliasing*).

Note: An easy way to implement these response-plotting functions is through the LTI Viewer. See Chapter 6, "The LTI Viewer" for more information.

Plotting and Comparing Multiple Systems

The LTI Viewer provides one method of plotting various responses for multiple models. See Chapter 6, "The LTI Viewer" to see how to accomplish this. You can also use the command line response-plotting functions to plot the response of

several LTI models on a single plot. To do so, invoke the corresponding command line function using the list `sys1,..., sysN` of models as the inputs.

```
step(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN)
...
bode(sys1,sys2,...,sysN)
nichols(sys1,sys2,...,sysN)
...
```

All models in the argument lists of any of the response plotting functions (except for `sigma`) must have the same number of inputs and outputs. To differentiate the plots easily, you can also specify a distinctive color/linestyle/marker for each system just as you would with the `plot` command. For example,

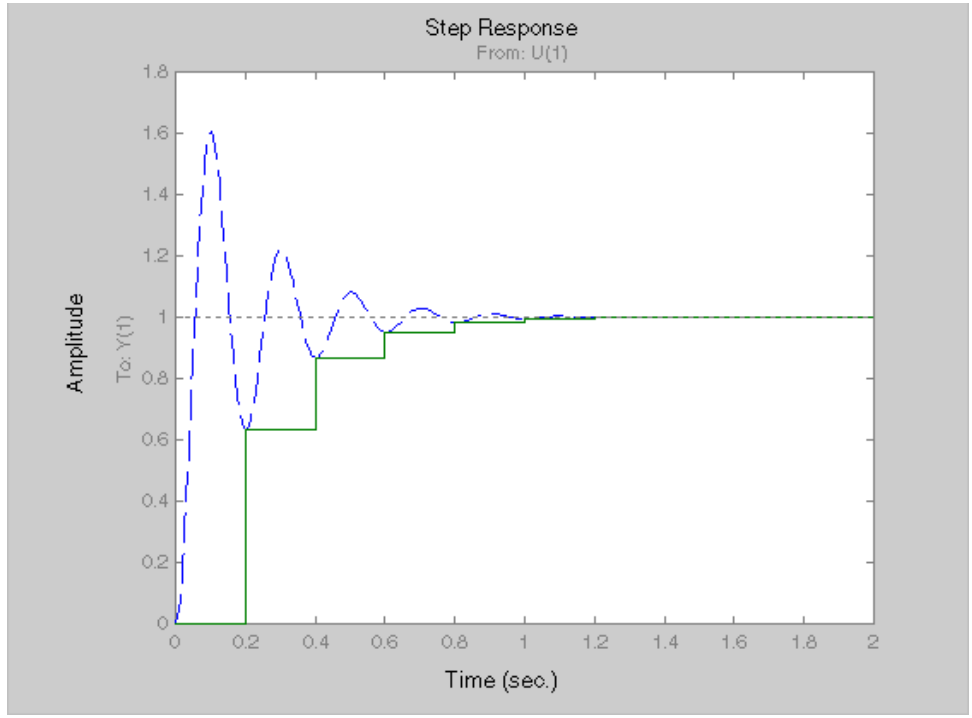
```
bode(sys1,'r',sys2,'y--',sys3,'gx')
```

plots `sys1` with solid red lines, `sys2` with yellow dashed lines, and `sys3` with green `x` markers.

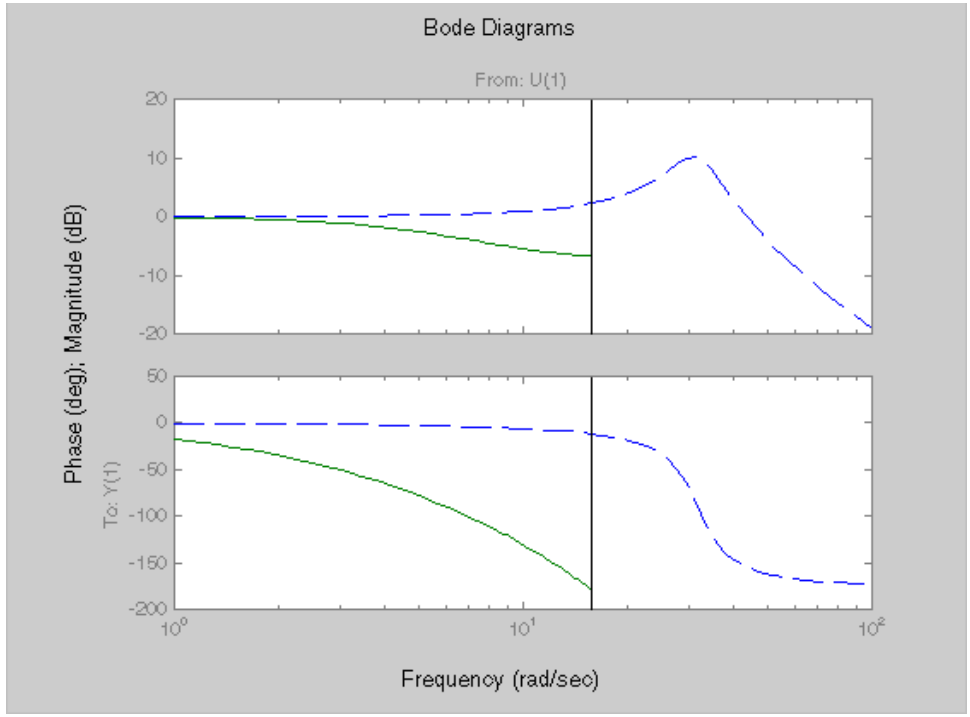
You can plot responses of multiple models on the same plot. These models need not be all continuous-time or all discrete-time.

The following example compares a continuous model with its zero-order-hold discretization.

```
sysc = tf(1000,[1 10 1000])
sysd = c2d(sysc,0.2)       % ZOH sampled at 0.2 second

step(sysc,'--',sysd,'-')   % compare step responses
```



```
bode(sysc,'--',sysd,'-')    % compare Bode responses
```



A comparison of the continuous and discretized responses reveals a drastic undersampling of the continuous system. Specifically, there are hidden oscillations in the discretized time response and aliasing conceals the continuous-time resonance near 300 rad/sec.

Customizing the Plot Display

You can plot data generated by several response analysis functions applied to one or several LTI models, as well as your own data. There are several ways you can customize how you display plots:

- Store the time or frequency response data in MATLAB arrays by invoking response analysis functions such as `step`, and `bode` with output arguments

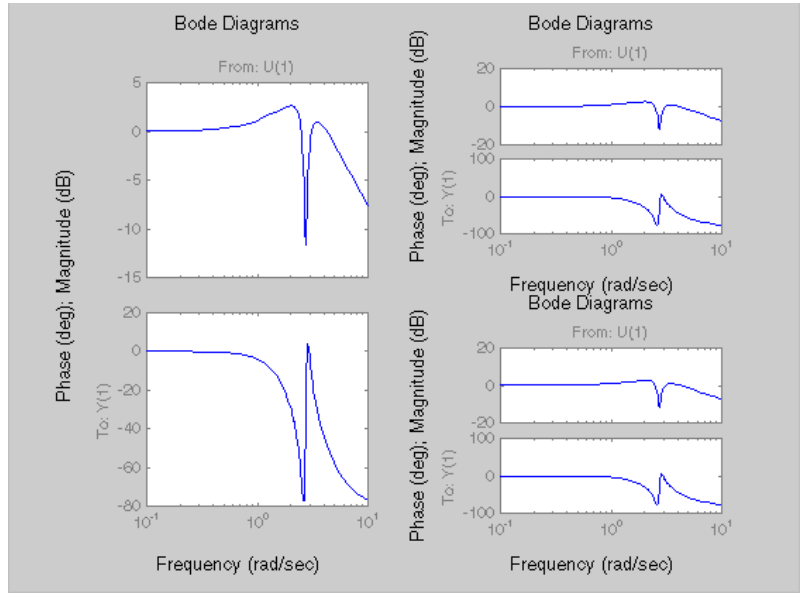
  ```
  [y,t] = step(sys)
  [mag,phase,w] = bode(sys)
  [re,im,w] = nyquist(sys)
  ```

  and use the `plot` command to display the result.

- Use the `subplot` and `hold` commands to plot several sets of data in a single figure window.
- Use the plot configuration menu in the LTI Viewer for time and frequency responses of LTI models. For more information, see “Viewer Configuration Window” on page 6-39.

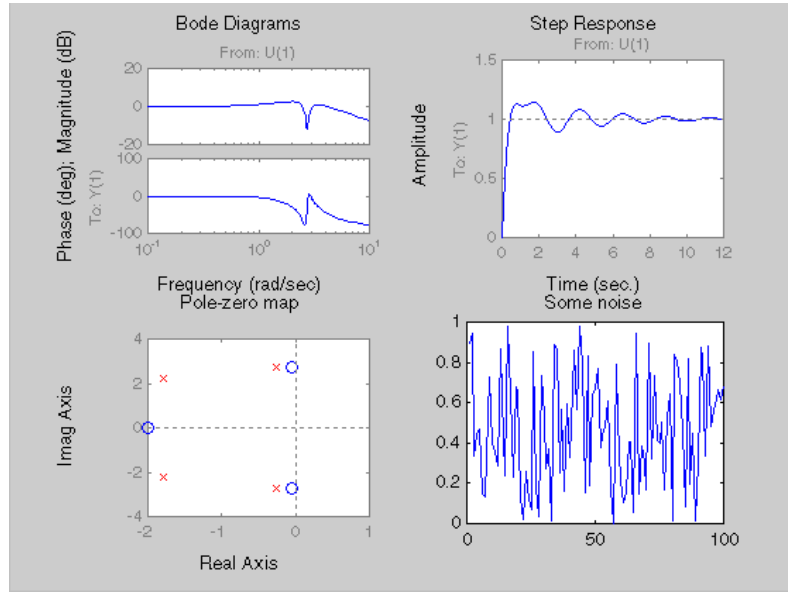
For example, the following sequence of commands displays the Bode plot, step response, pole/zero map, and some additional data in a single figure window.

```
h = tf([4 8.4 30.8 60],[1 4.12 17.4 30.8 60]);
subplot (121)
bode(h)

subplot(222)
bode(h)

subplot(224)
bode(h)
```



Another example is

```
subplot(221)
bode(h)
subplot(222)
step(h)
subplot(223)
pzmap(h)
subplot(224)
plot(rand(1, 100))  % any data can go here
title('Some noise')
```



Note: Each of the plots generated by response analysis functions in these figures (here, `bode`, `step`, and `pzmap`) has its own right-click menu (similar to those in the LTI viewer. For more information, see "The Right-Click Menus" on page 6-18.

Model Order Reduction

You can derive reduced-order models with the following commands.

Model Order Reduction	
`balreal`	Input/output balancing.
`minreal`	Minimal realization or pole/zero cancellation.
`modred`	State deletion in I/O balanced realization.
`sminreal`	Structurally minimal realization

Use `minreal` to delete uncontrollable or unobservable state dynamics in state-space models, or cancel pole/zero pairs in transfer functions or zero-pole-gain models. Use `sminreal` to remove any states from a model that don't affect the I/O response. For already minimal models, you can further reduce the model order using a combination of `balreal` and `modred`. See the corresponding Reference pages for details.

6

The LTI Viewer

Introduction

The LTI Viewer is a graphical user interface for viewing and manipulating the response plots of LTI models.

You can display the following plot types for LTI models using the LTI Viewer:

- Step response (only for TF, SS, or ZPK models)
- Impulse response (only for TF, SS, or ZPK models)
- Bode plot
- Nyquist plot
- Nichols chart
- Singular values of the frequency response
- Poles and zeros (only for TF, SS, or ZPK models)
- LTI model response to a general input (only for TF, SS, or ZPK models)
- Initial state LTI response (only for SS models)

The LTI Viewer displays up to six of these different response analysis plot types simultaneously. You can also analyze the response plots of several LTI models at once. However, in order to analyze models with different numbers of inputs and outputs, you must display them in separate LTI Viewers.

A special version of the LTI Viewer can also be used to analyze Simulink models. The operation of the Simulink LTI Viewer is discussed at the end of this chapter in “Simulink LTI Viewer” on page 6-48.

Functionality of the LTI Viewer

The basic function of the LTI Viewer is to display the plots of LTI model responses. Several menus are included for operations such as importing models into the LTI Viewer or printing response plots. In addition to these data operations, you can manipulate the LTI Viewer response plots in several ways, including:

- Change the type of plot being displayed in each plot region of the LTI Viewer
- Toggle on and off the response plots of individual LTI models loaded in the LTI Viewer
- Display response plot characteristics for a given plot type, such as settling time for a step response plot

- Zoom in on or out from the individual displayed plots
- Toggle the grid on or off on a plot
- Select which I/O channels the LTI Viewer displays for MIMO models in each plot
- For a given plot type, select how the LTI Viewer displays the I/O channels for MIMO models
- Select which models of an LTI array you want displayed in the LTI Viewer by indexing into dimensions or model characteristics
- Control plot characteristics such as the ranges for time and frequency used in various types of plots
- Control linestyle preferences such as the color and marker for each model response plotted
- Initialize the LTI Viewer from the command line to display multiple plot types (e.g., Bode plot and step response) simultaneously
- Control of the number of response plot regions that appear (one to six regions) in the LTI Viewer

Many of these features are accessed and controlled through plot-specific right-click menus.

Getting Started Using the LTI Viewer: An Example

This section contains a brief introduction to the LTI Viewer through an example that leads you through the following steps:

1. Load two LTI models into the LTI Viewer, initialized with the step responses and Bode plots of both models.
2. Use the right-click menu to display markers for
   - The settling time on the step responses
   - The peak magnitude response on the Bode plots
3. Use the mouse to display the values of these response characteristics on the plots.
4. Import a third LTI model to the LTI Viewer for comparison.
5. Use the right-click menu to zoom in on a plot.

Suppose you have a set of compensators you've designed to control a system, and you want to compare the closed-loop step responses and Bode plots. You can do this with the LTI Viewer.

A sample set of closed-loop transfer function models are included (along with some other models) in the MAT-file `LTIexamples.mat`. Type

```
load LTIexamples
```

In this example, you analyze the response plots of these three transfer function models.

```
Gcl1, Gcl2, Gcl3

Transfer function:
     4 s^3 + 8.4 s^2 + 30.8 s + 60
-------------------------------------
s^4 + 4.12 s^3 + 17.4 s^2 + 30.8 s + 60
```

```
Transfer function:
     2 s^3 + 1.2 s^2 + 15.1 s + 7.5
-------------------------------------
s^4 + 2.12 s^3 + 10.2 s^2 + 15.1 s + 7.5

Transfer function:
    1.2 s^3 + 1.12 s^2 + 9.1 s + 7.5
-------------------------------------
s^4 + 1.32 s^3 + 10.12 s^2 + 9.1 s + 7.5
```

Initializing the LTI Viewer with Multiple Plots

For a given LTI model, you can use the LTI Viewer to simultaneously display multiple response plot types, such as the Bode plot and the step response. You can also initialize the LTI Viewer to display the plots of several different models at once. The general syntax for initializing the LTI Viewer to plot up to six plot types is

```
ltiview({'type1';'type2';...;'typek'},sys1,...,sysn)
```

where:

- `{'type1';'type2';...;'typek'}` is a cell array listing up to six strings for the names of the plot types ($k \leq 6$).
- `sys1`, ..., `sysn` is a list of the MATLAB workspace variable names for the systems whose responses you want to initially display in the LTI Viewer.

The plot type names can be any of the following.

Plot Type	Description
`bode`	Bode plot
`impulse`	Impulse response
`initial`	Initial state response for SS models
`lsim`	LTI model response to general input
`nichols`	Nichols chart
`nyquist`	Nyquist plot

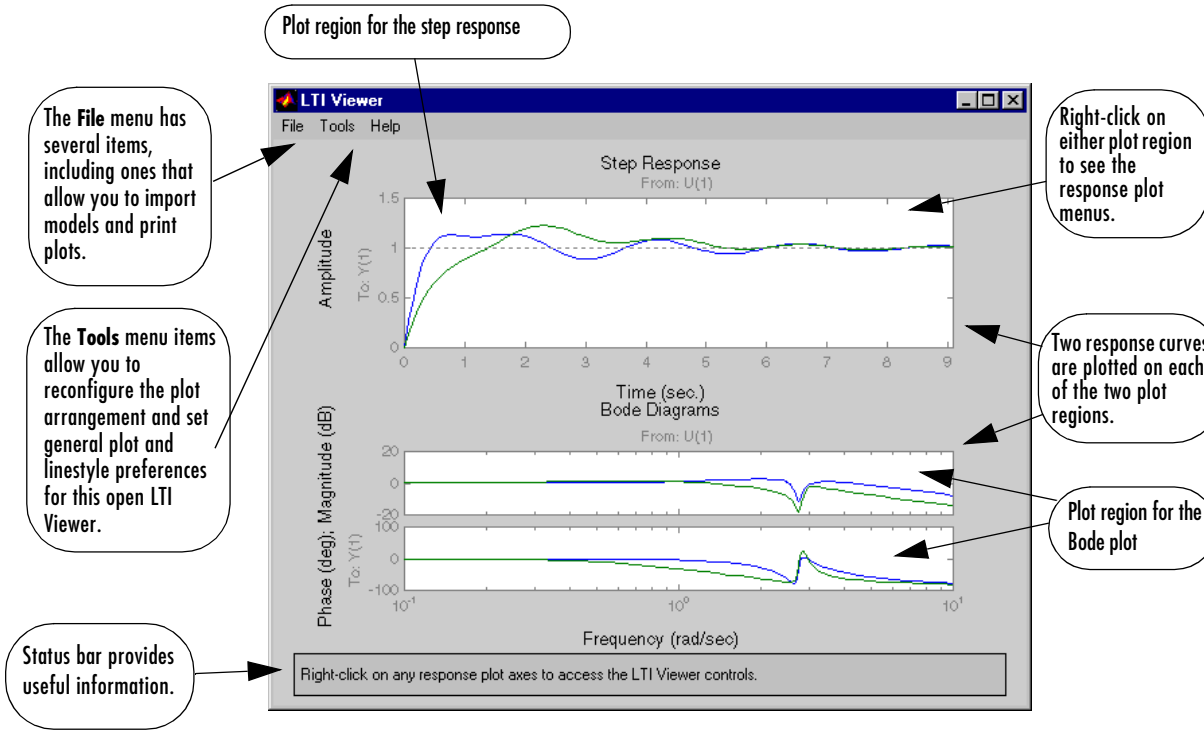
Plot Type	Description
`pzmap`	Plot of poles and zeros
`sigma`	Singular values of the frequency response
`step`	Step response

Note: When you initialize the LTI Viewer with `lsim` or `initial`, these plot types require some extra arguments. For more information on the syntax for calling `ltiview`, see `ltiview` on page 11-133.

To load the two models `Gcl1`, and `Gcl2` into the LTI Viewer so that it displays the step responses and Bode plots of both models, type

```
ltiview({'step';'bode'},Gcl1,Gcl2)
```

This opens the following LTI Viewer.



Right-Click Menus: Setting Response Characteristics

To access the individual response plot controls, use the right-click menus available from any of the plot regions displayed. These right-click menus vary, depending on if the LTI Viewer is displaying the response plots of SISO models, MIMO models, or LTI arrays. Some of the menu items are also plot type dependent.

For example, suppose you want to mark the settling time on the step responses, and the peak magnitude response on the Bode plots. To do this:

1 Right-click anywhere in the plot region of the step response plots. This opens the following menu list in the plot region.

Figure 6-1: The Right-Click Menu for SISO Models

2 Place your mouse pointer on the **Characteristics** menu item, and select **Settling Time** with your left mouse button.

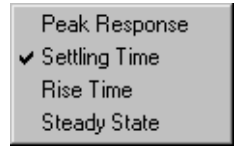


Figure 6-2: The Step Response Characteristics Submenu

3 Right-click anywhere in the plot region of the Bode plots to open a right-click menu.

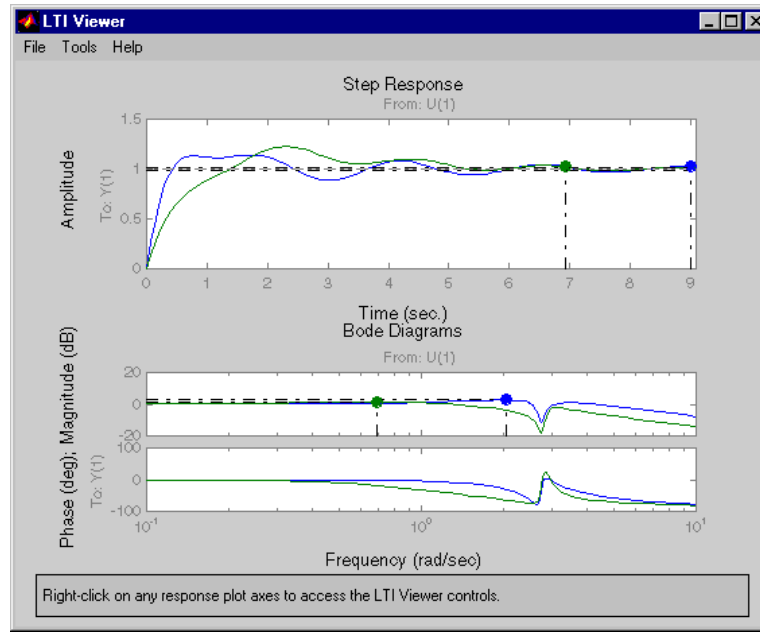
4 Place your mouse pointer on the **Characteristics** menu item.

The submenu items of the **Characteristics** menu for the Bode plot are different than those of the **Characteristics** menu for the step response right-click menu.

Figure 6-3: Bode Plot Characteristics Submenu

5 Select **Peak Response** with your left mouse button.

Your LTI Viewer should now look like this.



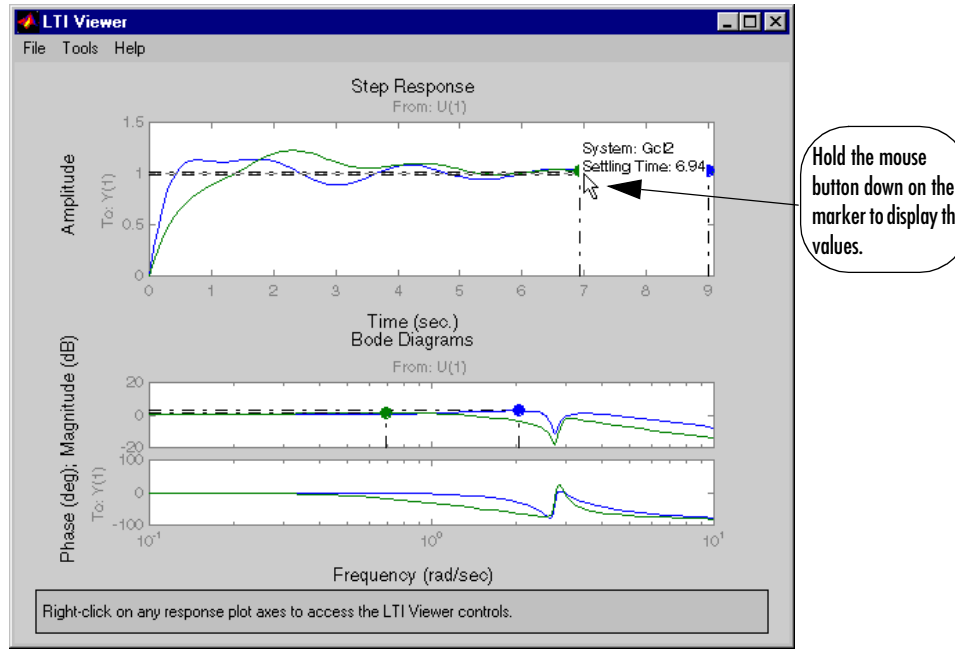
Notice that there is one settling time or peak magnitude marker for each LTI model displayed in the LTI Viewer.

Displaying Response Characteristics on a Plot

To display the values of any plot characteristic marked on a plot:

1 Click on the marker

2 Hold the left or right mouse button down to read the values off the plot.



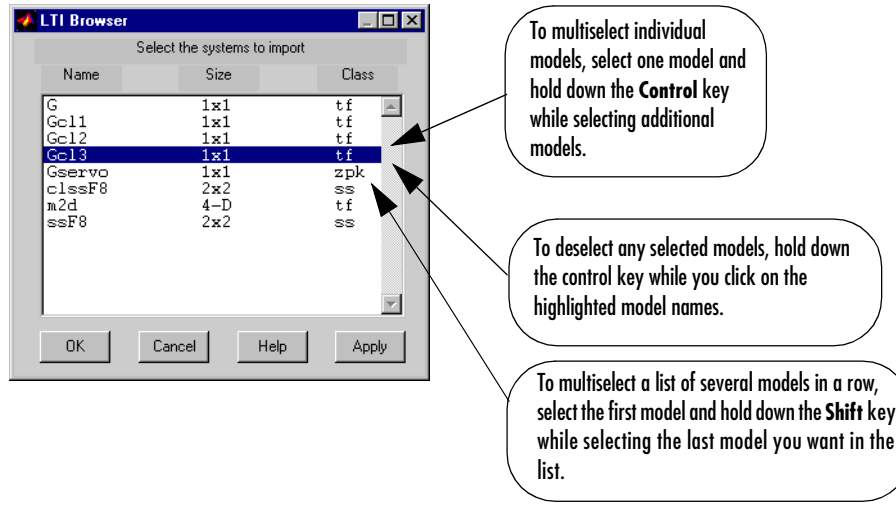
Note that you can:

- Use either the right or the left mouse button when you select a marker on a plot.
- Left-click anywhere on a particular plot line to see the response values of that plot at that point.
- Right-click anywhere on a plot line to see I/O and model information.

Importing Models

If the closed-loop models `Gcl1` and `Gcl2` do not meet your specifications, you may want to design another compensator at the command line, and import the resulting closed-loop model `Gcl3` for comparison:

1. Select **Import** from the **File** menu. This opens a browser listing all of the LTI models currently available in the MATLAB workspace.



   This browser allows you to copy LTI models from the MATLAB workspace into the *LTI Viewer workspace*. LTI model variable names have to be in the LTI Viewer workspace before you can analyze the response plots of these models using the LTI Viewer.

2. Select `Gcl3` from the list of models in the workspace browser.

3. Select **OK**.

The LTI Viewer now shows the step response of `Gcl3` in addition to those of `Gcl1` and `Gcl2`.

Note: A given LTI Viewer can only be used to analyze models with the same number of inputs and outputs. If you want to analyze models with different numbers of inputs or outputs, you must import these into separate LTI Viewers. See "Opening a New LTI Viewer" on page 6-16 for more information.

Zooming

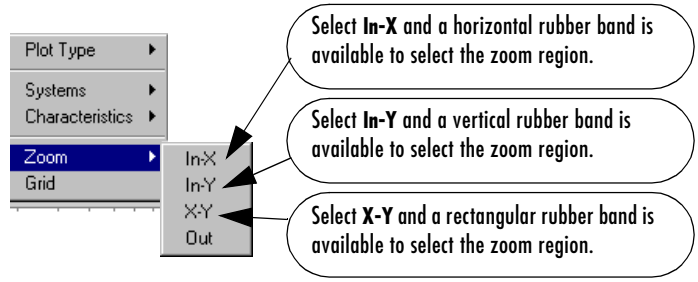
With three models loaded into the LTI Viewer, you may want to zoom in on one region of a given plot, in order to inspect the response behavior in that region more closely. For example, let's zoom in on the step responses of these three models in the vicinity of 4.5 seconds on the time axis.

To zoom in on a region on any of the plots, use the **Zoom** menu item available from the right-click menu:

1. Right-click on a plot region (in this case, the step response) to open the right-click menu.

2. Move your mouse pointer over the **Zoom** menu.

   Your menu looks like this.



3. Select **X-Y** to zoom in both the horizontal and vertical directions.

4. Use your mouse to create the rectangular rubberband that indicates the zoom region:

a Point your mouse to any corner of the rectangle of the region you want to zoom in on.

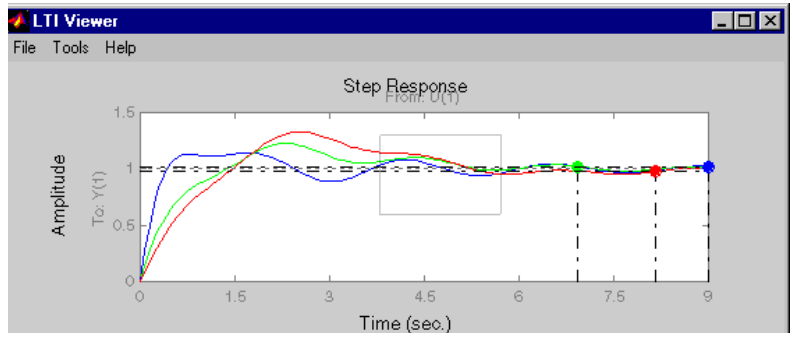
b Left-Click there, and hold the mouse button down.

c Drag the mouse pointer until the rectangle covers the region you want to zoom in on.

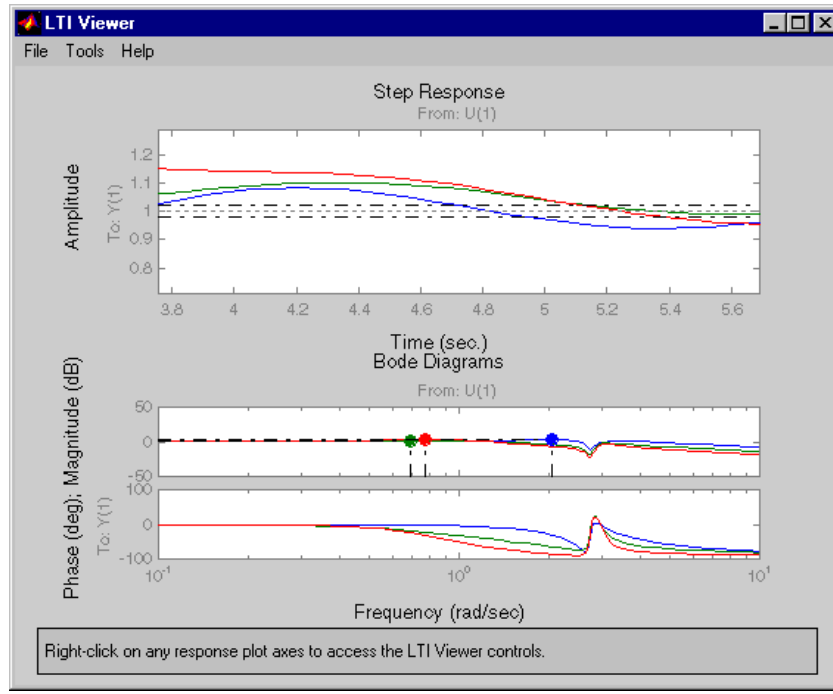
d Release your mouse.

For this example, zoom in around the region near 4.5 seconds on the step response plot.

Your step response plot looks like this as you select the zoom region.



After releasing the mouse on the zoom region, the LTI Viewer looks like this.



Notice that you've only zoomed on the step response plot; the Bode plot remains unchanged.

Note: To *zoom out*, i.e., to revert back to the original coordinate limits that were in place before you zoomed, follow the steps for zooming again, only this time select **Out** from the **Zoom** menu.

The LTI Viewer Menus

The LTI Viewer has three main menus:

- **File**
- **Tools**
- **Help**

The **File** menu provides features pertinent to bringing data in and out of the LTI Viewer. The **Help** provides help on the LTI Viewer features. The **File** and **Help** menus are covered in this section. The **Tools** menu allows you to control certain features common to all of the plots. You can read about the **Tools** menu items in "The LTI Viewer Tools Menu" on page 6-39.

The File Menu

The **File** menu gives you th following options:

- **New Viewer**—Open a new LTI Viewer.
- **Import**—Bring in new models into the LTI Viewer workspace.
- **Export**—Export models to the MATLAB workspace or to a disk.
- **Delete Systems**—Delete some or all of the LTI models in the LTI Viewer workspace.
- **Refresh Systems**—Update the LTI Viewer with any changes you made at the MATLAB command line to models in the LTI Viewer workspace.
- **Print**—Generate a hardcopy of the LTI model response.
- **Print to Figure**—Send the LTI Viewer plots to a MATLAB figure window.
- **Close Viewer**—Close an open LTI Viewer.

Details on some of these menu items are described below.

Importing a New Model into the LTI Viewer Workspace

LTI models in the MATLAB workspace can only be viewed by the LTI Viewer if they are in the LTI Viewer workspace. There are two ways of loading models into the LTI Viewer workspace:

- Load them into the LTI Viewer workspace when you open the LTI Viewer.
- Import them into an open LTI Viewer using the browser that is opened when you select the **Import** menu item under the **File** menu.

For directions for loading LTI models into the LTI Viewer workspace when you open it, see “Initializing the LTI Viewer with Multiple Plots” on page 6-5. For directions for importing models into the workspace of an open LTI Viewer, see “Importing Models” on page 6-11.

Opening a New LTI Viewer

The **New Viewer** option in the **File** menu enables you to initialize a new LTI Viewer. This is the same as typing `ltiview` at the MATLAB prompt. You can use this feature to compare response plots of LTI models that don't have the same numbers of inputs and outputs.

Refreshing Systems in the LTI Viewer Workspace

If you modify the characteristics of an LTI model in the MATLAB workspace, select **Refresh Systems** in the **File** menu to update the models in the LTI Viewer workspace with the changes you made.

Printing Response Plots

To print your response plots, go to the **File** menu:

- Choose the **Print** option to obtain a hardcopy.
- Choose the **Print to Figure** option to send the plots to a MATLAB figure window.

The **Print** option allows you to print the plots, exactly as you see them displayed in the LTI Viewer. The **Print to Figure** option allows you to use the **Plot Tools** feature of MATLAB figure windows to edit the plots before printing them.

Getting Help

You can obtain instructions on how to use the LTI Viewer directly from the **Help** menu.

Static Help

The **Help** menu contains three submenus:

- **Overview**
- **Response Preferences**
- **Linestyle Preferences**

The first submenu, **Overview**, opens the help text describing how to use the LTI Viewer menus and right-click menus that control the LTI Viewer. The remaining help menu items pertain to LTI Viewer controls you access from the **Tools** menu: the **Response Preferences** and **Linestyle Preferences** windows. These windows provide additional tools for manipulating the system responses. See the “Response Preferences” on page 6-40” and the “Linestyle Preferences” on page 6-44” sections for more information on the **Response Preferences** and **Linestyle Preferences** windows.

Interactive Help

The status bar at the bottom of the LTI Viewer provides you with instructions, hints, and error messages as you proceed through your analysis. In general, you can consult the status bar to learn:

- If you have tried to perform an unsupported function
- If a function has successfully been completed
- If additional information on the use of an LTI Viewer control is available

The Right-Click Menus

You can access most of the controls for the individual response plots displayed by the LTI Viewer through the right-click menus located in any plot region. There is one right-click menu per plot region displayed on the LTI Viewer. These menus vary slightly, depending on the model dimensions and plot type:

- The menu items that appear on the plot regions of the responses for SISO models are the basic right-click menu items.
- Additional menu items appear when you are displaying the plots of MIMO LTI models or LTI arrays.

You can't access the right-click menu for the LTI Viewer plots until at least one response plot is displayed in the LTI Viewer.

The Right-Click Menu for SISO Models

If you have not already done so, load the three SISO LTI models into the LTI Viewer workspace. You can do this by typing

```
load LTIexamples
ltiview({'step';'bode'},Gcl1,Gcl2,Gcl3)
```

Once you have loaded models into the LTI Viewer, right-click on the plot region of the step response plot. The following menu appears in the plot region.

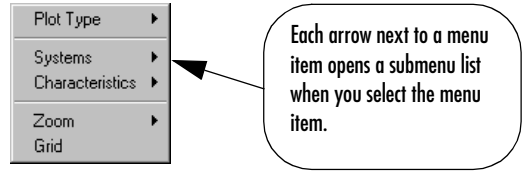


Figure 6-4: The Right-Click Menu for SISO Models

This is the right-click menu for SISO models. These menu items control the LTI Viewer plots for all models:

- **Plot Type**—You can choose which plot type you want displayed from this list of nine plot types.

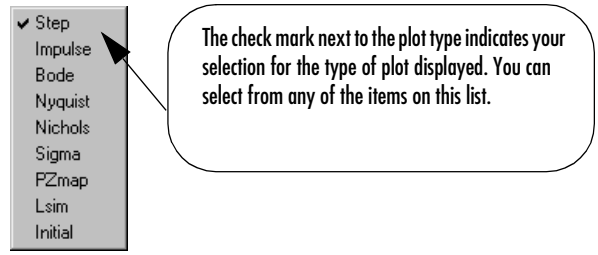


Figure 6-5: Plot Type Submenu

- **Systems**—The **Systems** submenu lists the models in the LTI Viewer workspace. You can choose to display or hide the plots of LTI models in the LTI Viewer workspace from this menu item:

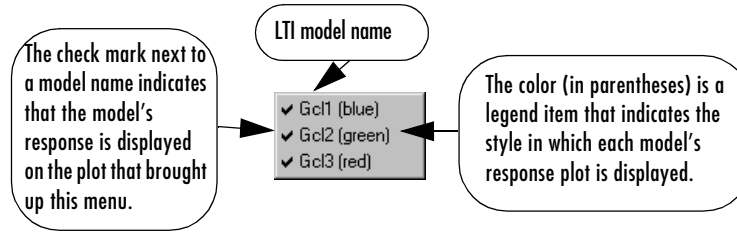


Figure 6-6: The Systems Submenu for Three Models

  - The legend for the individual response curves (described as linestyle, marker types, or color preferences) is indicated next to the name of each model. For information on how to modify these preferences, see "Linestyle Preferences" on page 6-44.
  - Check marks appear next to the names of models whose plots are displayed. Names of models that do not have a check mark next to them

are in the LTI Viewer workspace, but their responses are not displayed on the plot associated with the open menu.

- You can select any model in the list with your mouse to toggle on (or off) the visibility of its response curve in the selected plot region.

- **Characteristics**—You can toggle on and off the option to display a marker for various response characteristics for each plot type. For more information, see "Displaying Response Characteristics on a Plot" on page 6-9.

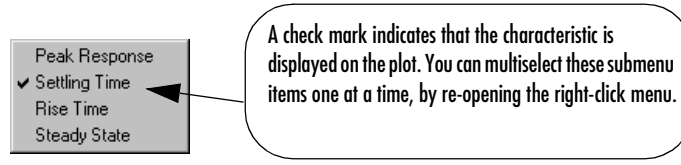


Figure 6-7: The Characteristics Submenu for the Step Response

- **Zoom**—You can zoom in or out of a given plot using the four submenu items in the **Zoom** menu. For more information, see "Zooming" on page 6-12.
- **Grid**—You can toggle a grid on and off by selecting this menu item.

Selecting a Menu Item

To select any menu item on the right-click menu:

1. Move your mouse over the menu item until it becomes highlighted, and its submenu (if it has one) is displayed.

2. Click on any (sub)menu item you want to select:

   a. For options menus such as **Characteristics** and **Systems**: If the menu item does not have a check next to it, selecting the menu item will produce a check next to that item. Selecting a checked item unchecks that menu selection (deselects that menu option).

   b. For exclusive menus such as **Plot Type**, one menu item must be checked, and selecting a plot type other than the one checked changes the plot type according to your selection.

Note: To multiselect submenu items (such as in the **Characteristics** or the **Systems** menus), re-open the right-click menu for each submenu item selection.

The Right-Click Menu for MIMO Models

When you load a MIMO model into the LTI Viewer, the right-click menu has a few more options than the right-click menu for SISO models does. These additional menu items are:

- **Axes Grouping** for grouping I/O channels
- **Select I/Os** for hiding the plots from some I/O channels

To see these menu items, you much have a MIMO model loaded in the LTI Viewer workspace.

The model `ssF8` in the file `LTIexamples.mat` is a MIMO state-space model for an F-8 aircraft. This model has two inputs and two outputs. The `InputNames` have been assigned as `Elevator` and `Flaperon`, and the `OutputNames` have been assigned as `Acceleration` and `FlightPath`.

If you haven't already done so, load this model into the MATLAB workspace by typing

```
load LTIexamples
```

Now that `ssF8` is in the MATLAB workspace, you can load it into an LTI Viewer workspace. To open a new LTI Viewer that displays the four nyquist plots for each of the I/O channels of this model, type

```
ltiview('nyquist',ssF8)
```

Your LTI Viewer looks like this.

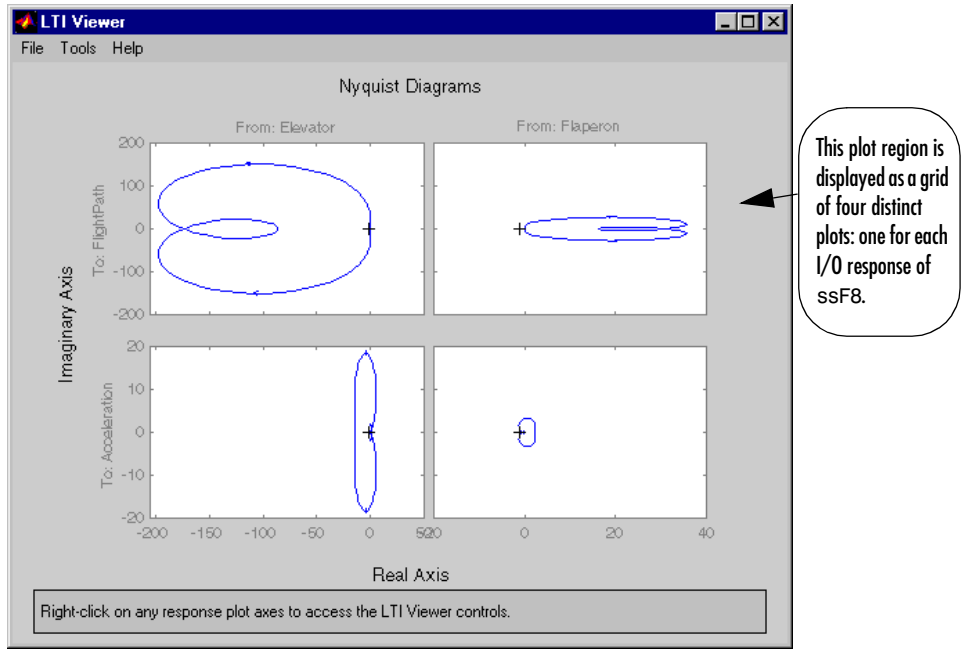


Figure 6-8: Nyquist Plots of the Four I/O Responses in ssF8

Notice that the I/O names for this model appear on the Nyquist plot. Each of the four plots displayed represents the I/O response from a single input to a single output.

Right-click on any part of the plot region (anywhere on the grid of plots). This opens the following menu.

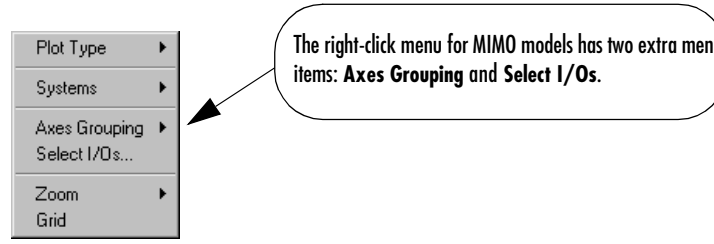
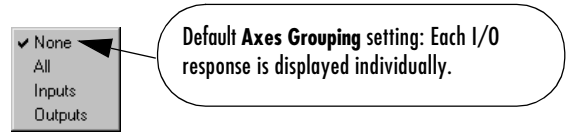


Figure 6-9: The Right-Click Menu for MIMO Models

The Axes Grouping Submenu

The **Axes Grouping** submenu is as follows.



When you first load a MIMO model into the LTI Viewer, it displays each I/O response curve in a separate portion of the plot region. For example, if you have two inputs and three outputs, the LTI Viewer initially displays a three-by-two grid of six separate plots. In this example, a two-by-two grid of four separate plots is displayed in the plot region. This initial (default) **Axes Grouping** setting is indicated by (checked) submenu item, **None**.

You can use the **Axes Grouping** submenu to reconfigure the grouping of these I/O response curves with the following submenu items.

- **Inputs**: The response curves from all of the inputs to a given output are plotted in the same portion of the plot region. There are as many separate portions of the plot region displayed as there are outputs.

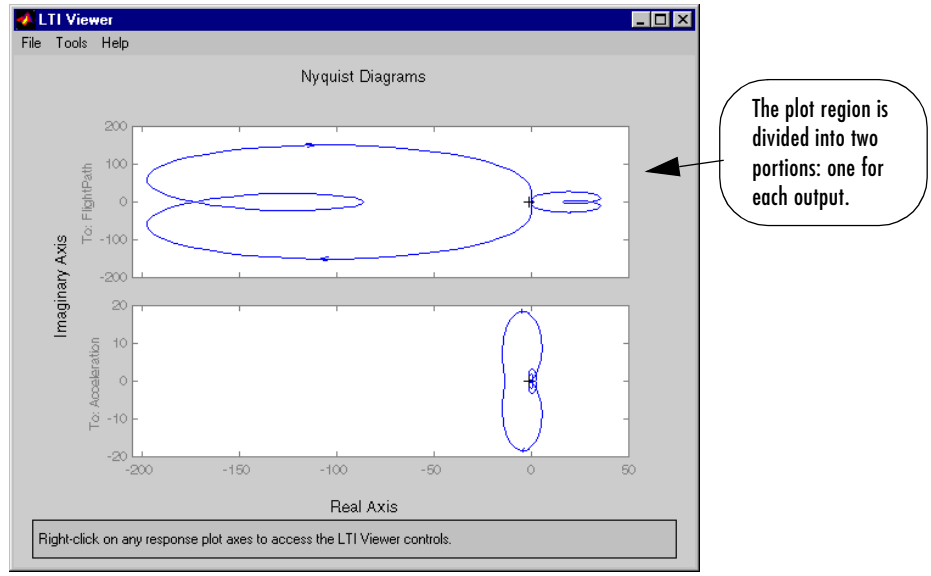


Figure 6-10: Axes Grouping: Inputs

- **Outputs**: The response curves from a given input to all of the outputs are plotted in the same portion of the plot region. There are as many separate portions of the plot region displayed as there are inputs.

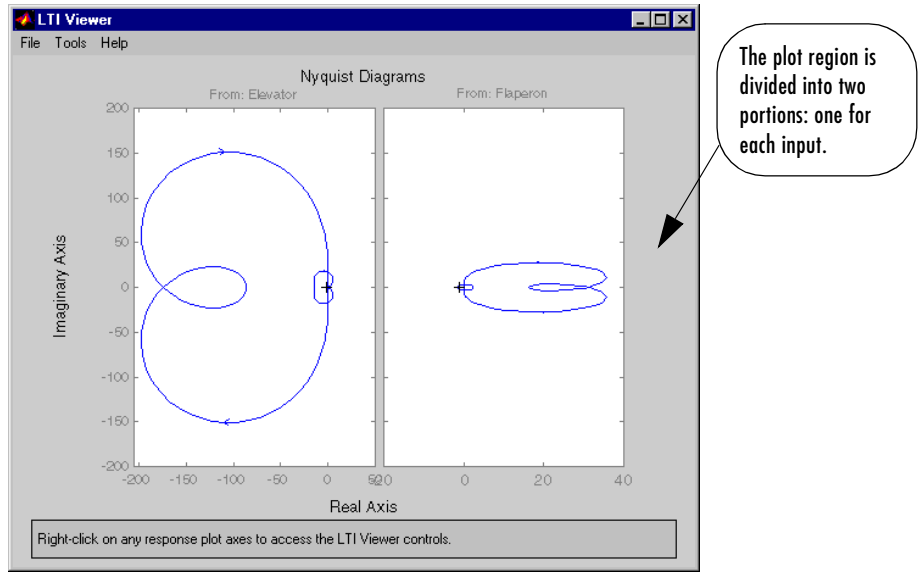


Figure 6-11: Axes Grouping: Outputs

- **All**: All of the I/O response curves are displayed (grouped) in a single plot region.

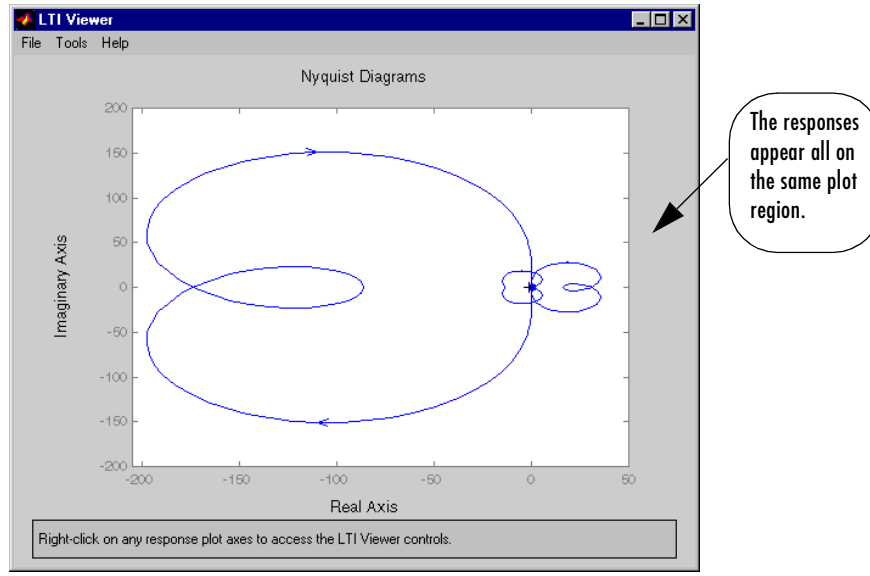
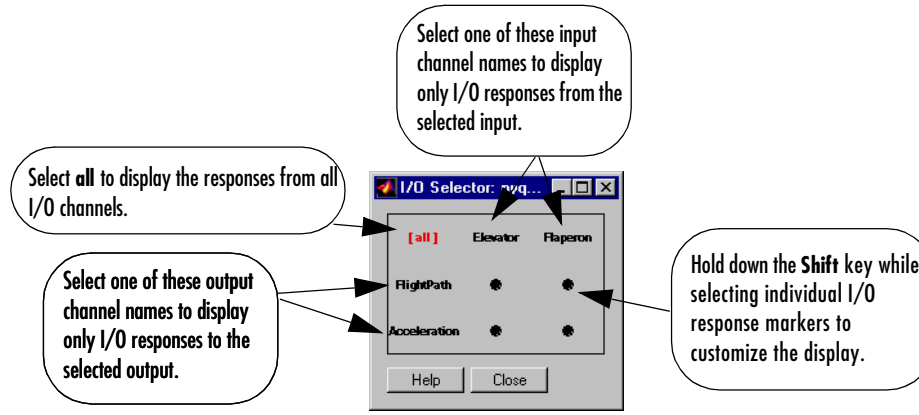


Figure 6-12: Axes Grouping: All

The Select I/Os Menu Item

The LTI Viewer initially displays all of the I/O response curves from each input channel to each output channel. You can select the **Select I/Os** menu item to customize the display with respect to the input and output channels.

When you select **Select I/Os** from the right-click menu, the following window opens.

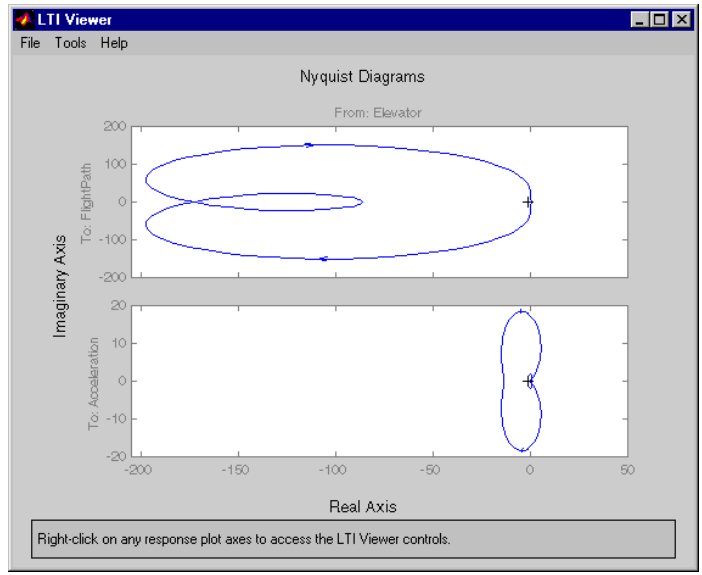


This **I/O Selector** window allows you to customize the I/O channel display for the plots of MIMO models. You can:

- Select any input channel to display the responses from only that input.
- Select any output channel to display the responses only to that output.
- Select **All** to display the I/O responses from all inputs to all outputs.
- Select individual I/O response channels. You can multiselect channels by holding down the **Shift** key while selecting the channels, or by rubberbanding a box around a selected set of I/O response markers on the grid.

For example, select **Elevator** to display the responses from only this input. Notice that the name of this input is now highlighted in red in the **I/O Selector** window.

With the **Axes Grouping** set to **None**, the display looks like this.



Note: To reset the **Axes Grouping** to **None**, open the right-click menu on the plot region, and select **None**.

The Right-Click Menu for LTI Arrays

When you load an LTI array into the LTI Viewer, all models in the LTI array are initially displayed. Using the **Select from LTI Array** menu item available to you through the right-click menu, you can choose to display the plots from a subset of the models in any LTI array in the LTI Viewer workspace, while hiding the plots of the other models.

The **Select from LTI Array** menu item opens the **Model Selector for LTI Arrays** window.

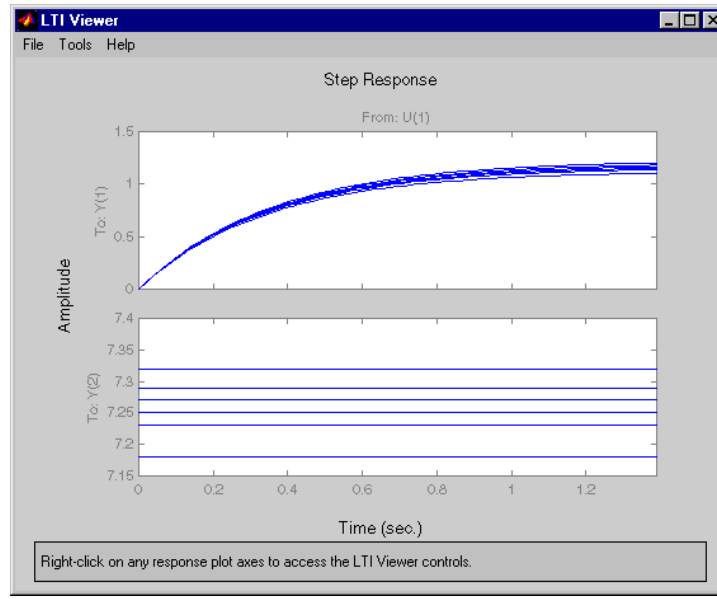
For a given LTI array in the LTI Viewer workspace, you can use this interface to display the plots of a subset of models in the LTI array, using either or both of the following options:

- Indexing into the array dimensions
- Indexing into the array through design specification criteria

In order to have access to right-click menu item for LTI arrays, you must have at least one LTI array loaded in the LTI Viewer workspace. For example, type

```
load LTIexamples
ltiview('step',m2d)
```

`m2d` is a 2-by-3 array of two-output, one-input models. Your LTI Viewer looks like this.



Notice that for each I/O map in `m2d`, the step responses of all of the models are plotted in the same plot region.

To display the responses of only some of the models in the LTI array, you must first complete the following two steps:

1 Right-click anywhere in the plot region to open the following right-click menu.

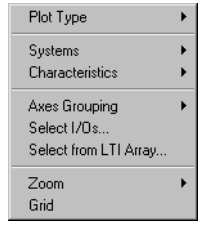


Figure 6-13: Right-Click Menu for LTI Arrays

2 Select the **Select from LTI Array** menu item.

This opens the **Model Selector for LTI Arrays** window in the (default) **Index into Dimensions** setup.

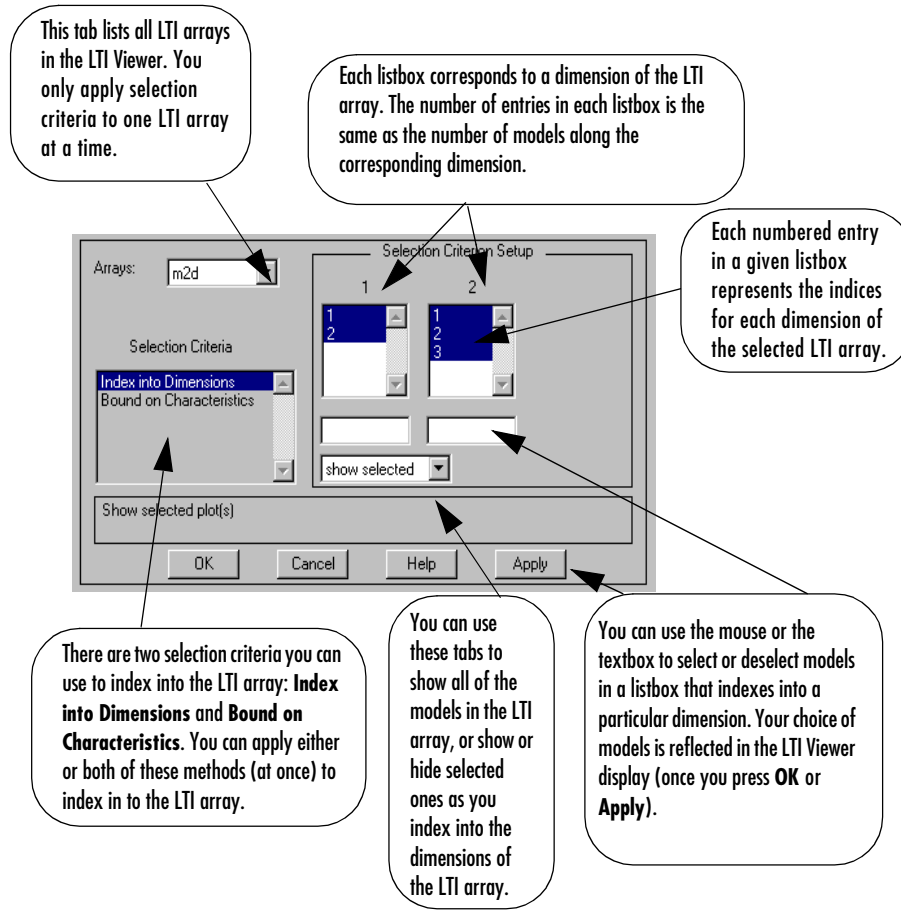


Figure 6-14: LTI Array Model Selector for a 2-by-3 Array of Models

The Model Selector for LTI Arrays

For any of the LTI arrays loaded into the LTI Viewer workspace, you can use the **Model Selector for LTI Arrays** window to display the responses of only a subset of the models in the LTI array. To do this, you must first select the LTI array name from the **Arrays** pull-down tab.

Once you have selected the name of an LTI array in the **Model Selector for LTI Arrays** window, you can select models in the LTI array whose response plots you want displayed using either or both of the following:

- Index into the array dimensions of the LTI array (**Index into Dimensions**)
- Index into LTI array using design specification criteria (**Bound on Characteristics**)

Indexing into the Array Dimensions of an LTI Array

To index into the array dimensions of an LTI Array:

1. Select **Index into Dimensions** in the **Selection Criteria** listbox. This item is initially selected for you by default.
2. Leave the **show selected** tab as is (or change it to **hide selected**). This allows you to use your mouse or the textboxes to index into each dimension of the LTI array.
3. Select indices of models whose plots you want displayed (or hidden) using the listboxes corresponding to the dimensions of the LTI array by either:
   - Using your mouse (using the **Control** key for multiselection) to select model indices from each array dimension of the LTI array (from each listbox)
   - Typing a vector of indices or any MATLAB expression that specifies a vector of indices in the textbox below the listbox
4. Select **Apply** to implement your model selection choice without closing the window, or **OK** to implement your model selection choice and close the **Model Selector for LTI Arrays** window.

Note: Choosing the **show all** tab while in the **Indexing into Dimensions** selection criterion is equivalent to selecting all of the indices in the listboxes. However, any previous selections you made using the **Indexing into Dimensions** selection criterion are not lost. They can be reinstalled by applying **show selected**.

For example to display only the first row of models in the 2-by-3 LTI array `m2d`, either:

- Select the first index in the first listbox (corresponding to the first dimension of the LTI array) with your mouse.
- Type the vector `[1]` in the textbox below the first listbox.

The following figure depict the **Model Selector for LTI Arrays** window for selecting to display the responses of the first row of models in the LTI array `m2d`.

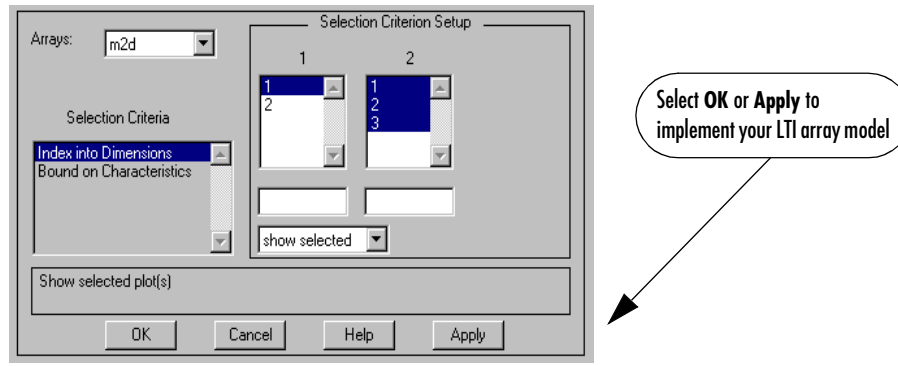


Figure 6-15: LTI Array Model Selector to Select the First Row of m2d

The next figure shows the LTI Viewer display that results from selecting **Apply** or **OK**.

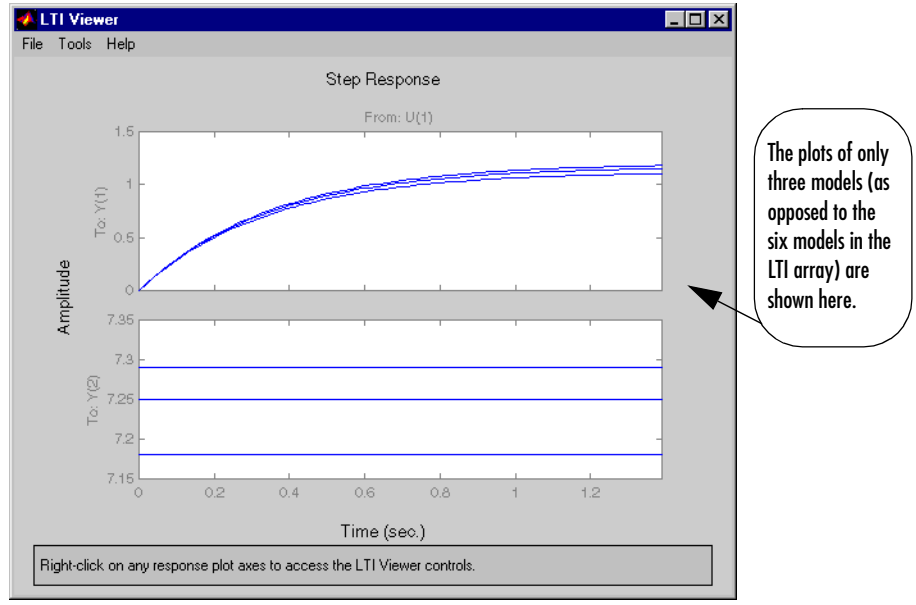


Figure 6-16: Step Response of the First Row of Models in m2d

There are a variety of ways you can index into the dimensions of an LTI array using the textboxes located below each listbox. You can type both logical expressions, or ones that define indices directly.

For example, suppose you have a variable p defined in the MATLAB workspace, representing a vector of parameters associated with the second dimension of `m2d`.

```
p = [1.1 5.3 10]
```

The variable p might, for example, represent three different operating conditions for which you created the LTI array. To select indices specified by these parameters in the second array dimension of `m2d`, you can, for example, type the following under the second listbox.

```
p>5   % Choose the 2nd and 3rd indices in the 2nd array dim. of m2d
```

or

```
find(p<2)  % Choose the 1st index in the 2nd array dim. of m2d
```

Indexing into the LTI Array Using Design Specification Criteria

You can also use several plot-specific design criteria to select those models in the LTI array whose responses you want displayed. You index into the LTI array through these design criteria (response plot characteristics) using Boolean expressions. To plot selected models by indexing into the LTI array in this manner:

1. Select **show all** in the **Index into Dimensions** set-up, and select **Apply**. This step is not required, but if you don't do this, then any indexing into design specifications you perform applies only to the models whose plots are selected to be displayed using **Index into Dimensions**.

2. Select **Bound on Characteristics** in the **Selection Criteria** listbox. The right side of the interface now reflects the plot-specific design specification characteristics available for you to select models from.

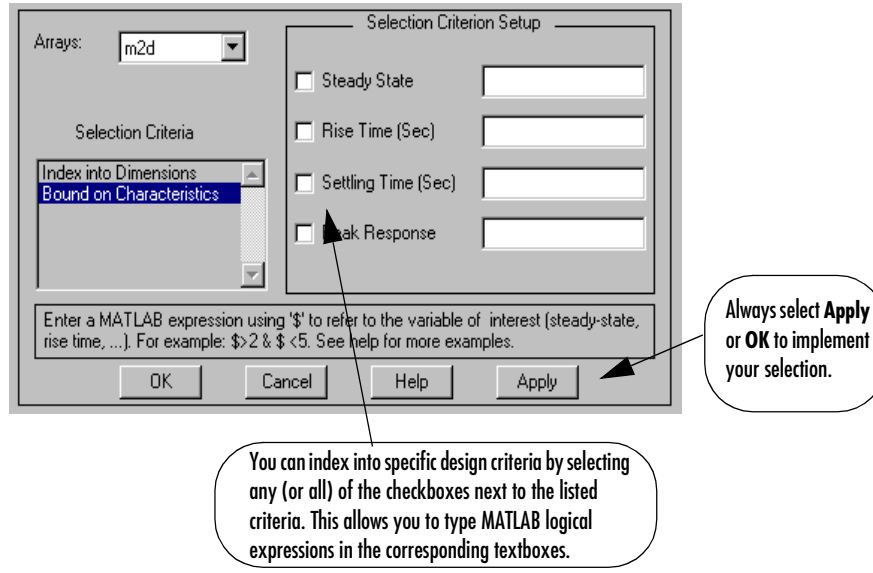


Figure 6-17: Selector for LTI Arrays Using Design Specification Criteria

3. Select the checkbox next to a design specification characteristic you want to index through.

4 Position your mouse pointer in the textbox next to the design specification characteristic.

5 Type a MATLAB relational expression in the textbox, using $ as a variable name in the expression. Note that for arrays of MIMO models, you can consider $ to be an Ny-by-Nu matrix, if each model in the LTI array has Ny outputs and Nu inputs. This allows you to specify different requirements on different I/O channels.

6 Press **Apply** to implement your indexing selection.

For example, to display only the plot of the model with the maximum rise time in the step response from the input to the first output, type

```
$(1,1) == max($(1,1))
```

in the textbox next to **Rise Time**.

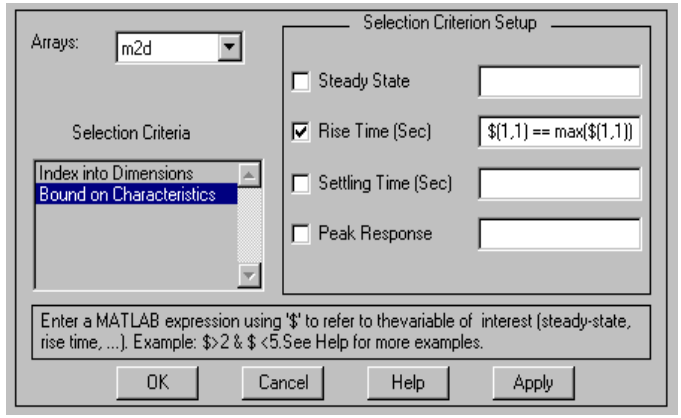


Figure 6-18: Specifying Design Criteria to Select Models in the LTI Array

After selecting the **Apply** button, your LTI Viewer displays only the step response of the model with the maximum rise time in the step response from the input to the first output. The result of your action is displayed on the status bar.

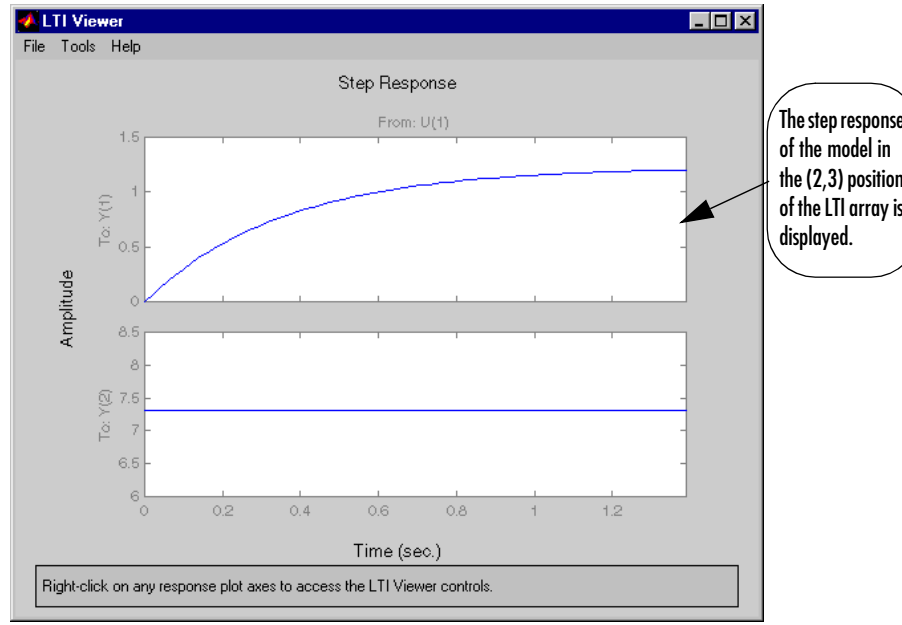


Figure 6-19: Step Response of the Model with the Maximum Rise Time

You can also use any logical expression in variables defined in the MATLAB workspace to index into a specific design criterion. For example, typing

```
$(2,1) < 7.25 & $(1,1) > 1.2
```

next to **Steady State** (after unchecking **Rise Time**), displays the responses of any models for which the steady-state response has a value less than 7.25 for in the second I/O channel, and greater than 1.2 in the first.

Suppose you have a design specification requirement for each I/O map of each model of the LTI array, and store this requirement in a matrix `q` in the MATLAB workspace. For example, if `q` is an `Ny-by-Nu` matrix (2-by-1, in this case), and `N` is the number of models in the LTI array (6, in this case), then you can type

```
N = 6;
Q = repmat(q,[1,1,N]);
```

at the MATLAB command line, and

```
$ > any(any(Q))
```

in the **Model Selector for LTI Arrays** window. This displays only the plots of those models for which the required bound is not satisfied on *any* of the I/O channels. You must use the `any` command twice, once for each I/O dimension. Typing `$>Q` would only display the plots of those models for which is bound is not satisfied on *all* of the I/O channels.

The LTI Viewer Tools Menu

Three preferences windows provide additional options for customizing the LTI Viewer display. You can access these windows from the **Tools** menu. The preference windows you can access from the **Tools** menu are:

- **Viewer Configuration**—To change the number and type of plots displayed by the LTI Viewer
- **Response Preferences**—To set various parameters such as the ranges of values for the response plot time and frequency scales
- **Linestyle Preferences**—To set options for changing linestyles, colors, and markers for all of the response curves

Viewer Configuration Window

Select the **Viewer Configuration** menu item under the **Tools** menu. The following window opens the **Available LTI Viewer Configurations** window.

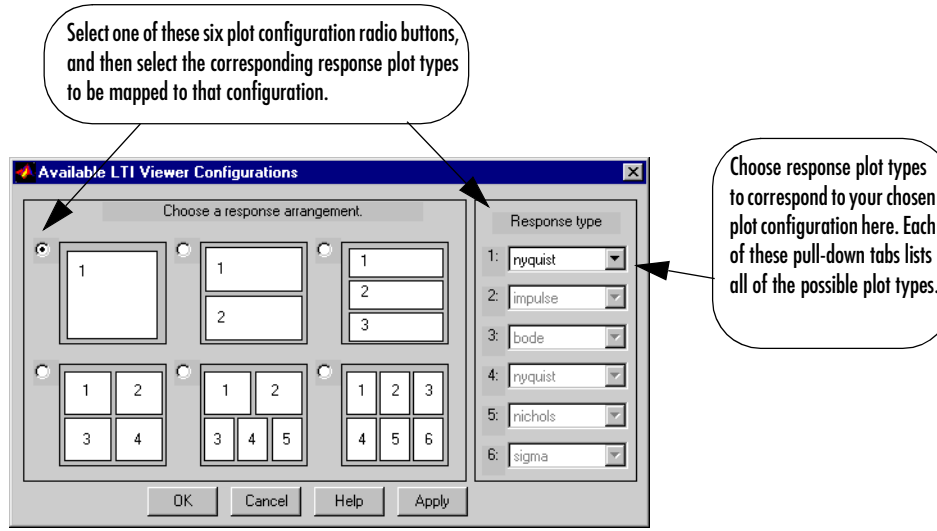


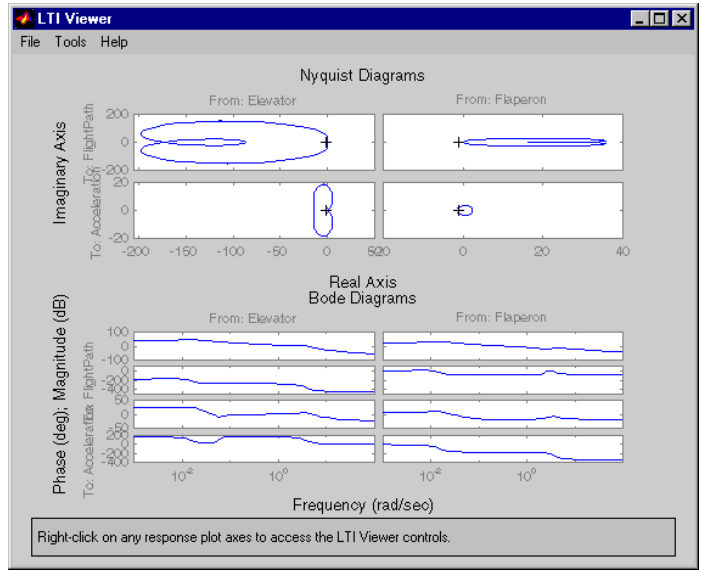
Figure 6-20: The Available LTI Viewer Configurations Window

For example, load the model `ssF8` into the LTI Viewer workspace (see "The Right-Click Menu for MIMO Models" on page 6-21 for instructions on how to

load this model). With the **Available LTI Viewer Configurations** window open:

1. Select the radio button for the two-plot configuration.
2. Use the pull-down tab next to **1.** to set the first plot to **nyquist**.
3. Use the pull-down tab next to **2.** to set the second plot to **bode**.
4. Select **OK**.

With the `ssF8` model loaded, the LTI Viewer now displays two plot types on separate plots:



Response Preferences

When you select **Response Preferences** from the **Tools** menu, the **Response Preferences** window shown below opens.

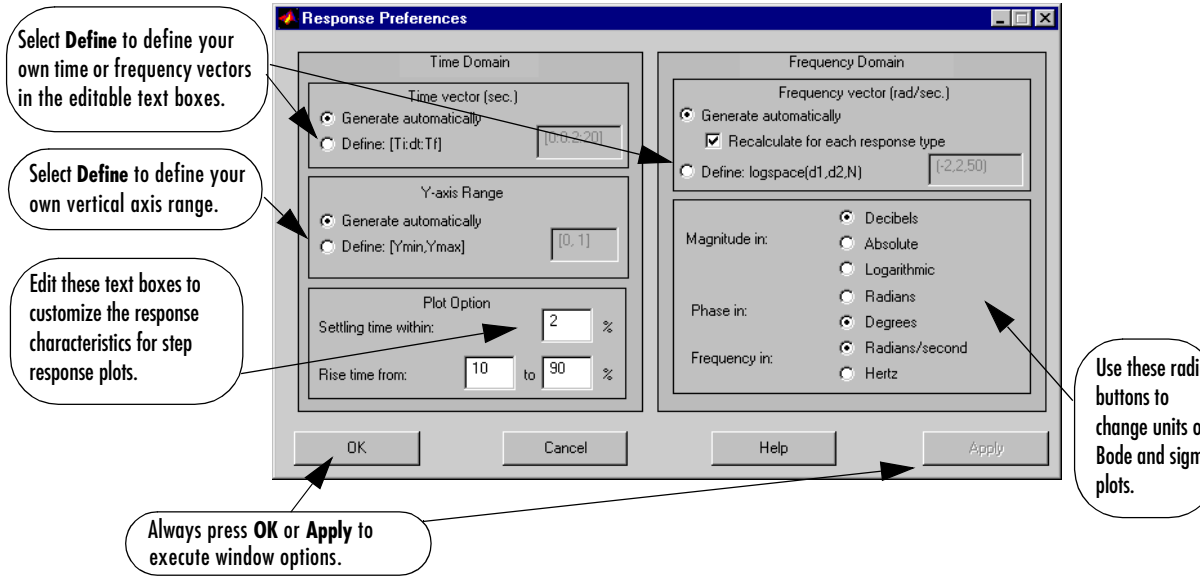


Figure 6-21: Response Preferences Window

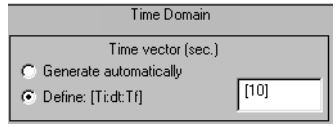
You can use the **Response Preferences** window to:

- Specify the time duration for time domain response plots and the frequency range for frequency domain response plots.
- Choose units for plotting the magnitude, phase, and frequency of Bode plots. This choice of units also assigns the magnitude and frequency units for singular value (`sigma`) plots.
- Specify vertical axis limits for time domain responses.
- Set target percentages for settling time and rise time calculations on step response plots.

Setting Response Time Durations and Frequency Ranges

To get a smoother time domain response curve over a specified time duration, you can override the default time range and reset the time vector to a desired

value. You can use the **Time vector (sec.)** portion of the **Time Domain** field shown below to do this.



The **Time vector (sec.)** field accepts one, two, or three arguments, separated by colons and surrounded by square brackets:

- `[Tf]` specifies only the final time.
- `[Ti:Tf]` specifies the initial and final time.
- `[Ti:dt:Tf]` specifies the initial and final time and provides the incremental step `dt` to use when generating the time vector.

The **Generate automatically** radio button is selected when you initially open this window. With this option selected, the LTI Viewer automatically determines the time vector to use for plotting the response. To override the default setting:

1. Select the **Define** radio button.
2. Enter the desired final time `Tf` or the new time vector as `[Ti:dt:Tf]` as described previously.
3. Select either:
   - **Apply** to keep the **Response Preferences** window open when you apply these changes
   - **OK** to apply the changes and close the **Response Preferences** window

You can also use this window for setting:

- The **Y-axis Range** section of the **Time Domain** field to override the vertical axis default settings: Enter the desired vertical axis limit as a row vector of the lower and upper axis limit.
- The **Frequency vector (rad/sec.)** section of the **Frequency Domain** field: Enter the desired frequencies as if they were the input arguments of the `logspace` function.

The **Frequency vector (rad/sec.)** field also provides you with the option to recalculate a new frequency vector for each frequency response type. When this checkbox is selected along with **Generate automatically**, a new frequency vector and response is calculated each time you toggle between different frequency responses, e.g.from Bode to Nyquist. If you deselect the **Recalculate for each response type** checkbox, the frequency vector used to calculate the previous frequency response is used and the frequency response data is simply converted to the new response type.

Note: Whenever you override any of the default settings in the **Response Preferences** window, the values you enter are used on each plot, and during every applicable response calculation.

Customizing Step Response Specifications

You can also use the **Response Preferences** window to customize the percentage values used in the step response settling time or rise time calculation.

The *settling time* percentage value determines the time after which the envelope of the step response remains within that percentage of the steady state value of the step response.

The *rise time* percentages are marked by two values. These determine the time it takes for the step response to increase from the first percentage value to the second percentage value of the steady state step value response.

The default values for these step response characteristics are:

- 2% of the steady state value for settling time
- 10% to 90% of the steady state value for rise time

You can use the editable text box in the **Plot Option** section of the **Time Domain** field shown below to change the percentages for the settling time or

rise time. For example, you can change the value for the settling time to 5% as shown below.

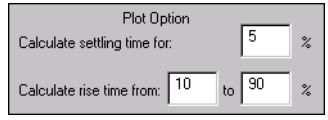
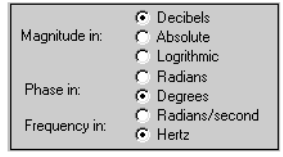


Figure 6-22: Changing the Settling Time Percentage Value to 5%

Changing the Frequency Domain Plot Units

In addition to providing options for specifying the frequencies used in the frequency responses, the **Frequency Domain** field allows you to choose the units used for Bode plots and singular value (`sigma`) plots. You can use the radio buttons in the **Frequency Domain** field shown below to override the default units for these plots.



By default, the LTI Viewer plots:

- Magnitude in decibels
- Phase in degrees
- Frequency in radians per second

Linestyle Preferences

You can use the controls in the **Linestyle Preferences** window to modify the response curve plot styles.

To do this:

1. Open the **Tools** menu.
2. Select **Linestyle Preferences**.

After selecting **Linestyle Preferences** from the **Tools** menu, the following **Linestyle Preferences** window opens.

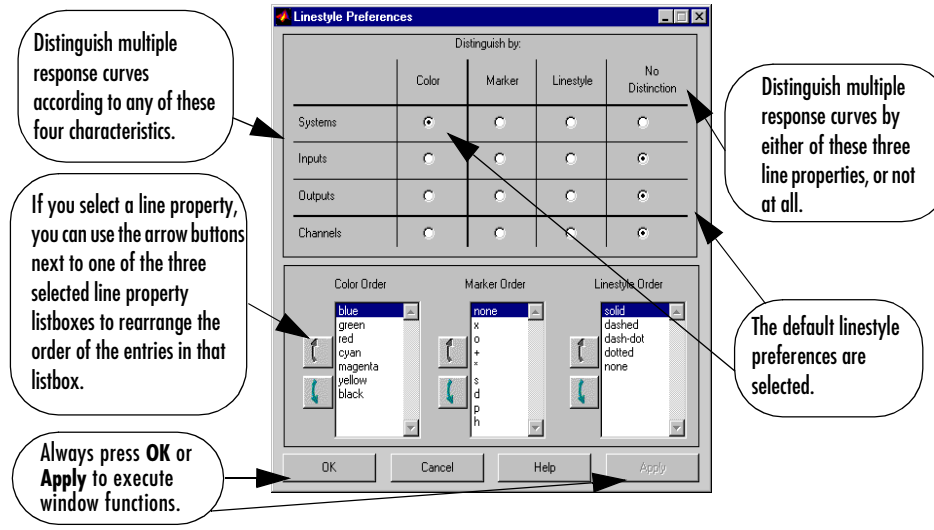


Figure 6-23: The Linestyle Preferences Window

You can use the **Linestyle Preferences** window to:

- Select the property used to distinguish the response curves for different LTI models, inputs, outputs, or I/O channels.
- Change the order in which the line properties are applied.

The settings you select in the **Linestyle Preferences** window override any plot styles you may have entered in the original `ltiview` command.

To distinguish multiple response curves in each of the plot regions, you can distinguish the plotted response curves using combinations of any of the following preferences:

- Colors (red, blue, green, etc.)
- Markers (circles, crosses, etc.)
- Linestyle, the type of curve drawn (solid, dashed, etc.)

You can designate that the chosen preference (color, marker, or linestyle) distinguish the plotted response curves by any (or all) of the following.

- **Systems**: The line properties vary with the models.
- **Inputs**: The line properties vary with the input channels.
- **Outputs**: The line properties vary with the output channels.
- **Channels**: The line properties vary with the I/O channels.

Changing the Response Curve Linestyle Properties

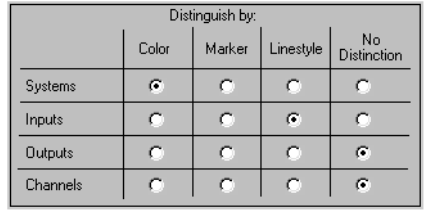
You can use the radio buttons in the **Distinguish by** field of the **Linestyle Preferences** window to vary a line property by model, input, output, or I/O channel.

When you open the **Linestyle Preferences** window, the radio button in the **Systems** row and **Color** column of the **Distinguish by** field is selected and the radio buttons in the **No Distinction** column are selected in the remaining rows. This is the default setting for the plot styles used for all of the response plots.

For example, to distinguish the responses from different inputs of a MIMO model using different linestyles (in addition to distinguishing multiple models by color):

1 Select the radio button in the **Inputs** row and **Linestyle** column.

As soon as you select this radio button, the previously selected radio button in the **Inputs** row is turned off, as shown in the figure below.



The radio buttons are mutually exclusive along each row and each column with the exception of the **No Distinction** column. In other words, you can use only one line property to distinguish the different systems, inputs, outputs, or channels, and that same property cannot be applied to any other row of the **Linestyle Preferences** window.

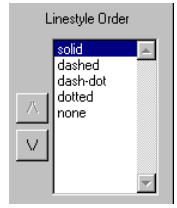
2 Select either:

- **Apply** to keep the **Linestyle Preferences** window open when you apply these changes
- **OK** to apply the changes and close the **Linestyle Preferences** window

The Order in which Line Properties are Assigned

You can determine the order in which the line properties are applied to models (or inputs, outputs, or I/O channels) by referring to the order of the line properties in the listboxes. The three listboxes tell you the default order in which each of the line properties will be applied. For example, look at the **Linestyle Order** listbox and notice that the responses from the first input will be plotted with a solid line, and the second input with a dashed line.

If, for example, (while **Linestyle** is assigned to distinguish models by inputs), you want to plot the response from the first input with dashed lines and those from the second input with dotted lines, you can use the up and down arrows to the left of each listbox to reorder the entries in the listboxes. The figure below shows the up and down arrows and the **Linestyle Order** listbox.



To change any listbox order:

1 Select the line property you want to move in the list.

2 Press the up and down arrows to the left of that listbox to move the highlighted property in the desired direction within the listbox.

Simulink LTI Viewer

If you have Simulink, you can use the *Simulink LTI Viewer*, a version of the LTI Viewer that performs linear analysis on any portion of a Simulink model.

The Simulink LTI Viewer features:

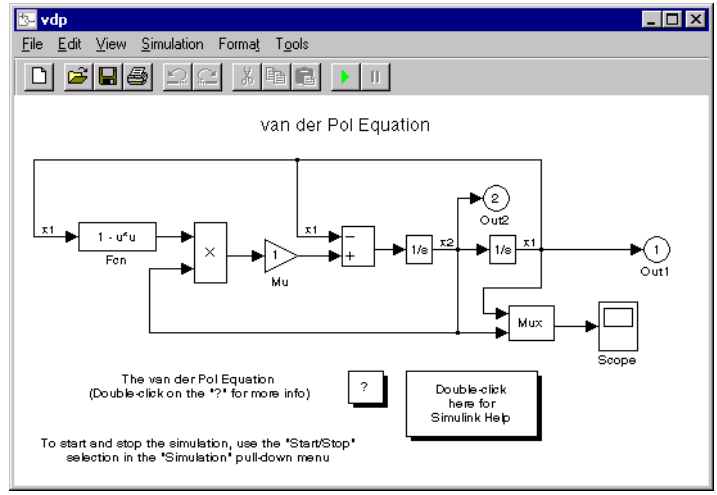
- Drag-and-drop blocks that identify the location for the inputs and outputs of the portion of a Simulink model you want to analyze.
- The ability to specify the operating conditions about which the Simulink model is linearized for analysis in the LTI Viewer.
- Access to all time and frequency response tools featured in the LTI Viewer.
- The ability to compare a set of (linearized) models obtained by varying either the operating conditions or some model parameter values.

Using the Simulink LTI Viewer

To learn about the Simulink LTI Viewer, we will perform some analysis on a Simulink model for a van der Pol oscillator. To open this model, type

```
vdp
```

at the MATLAB prompt. This brings up the following diagram:



Notice that the title of this Simulink model is **vdp**, and that it contains static nonlinearities.

A Sample Analysis Task

Suppose you want to:

- Analyze the Bode plot of the linear response between the input `x2` to the Product block and the output `x1` of the second Integrator block.
- Determine the effect of changing the value of the Gain block labeled `Mu` on this response.

The basic procedure for carrying out this type of analysis is outlined below:

1. Open the Simulink LTI Viewer.
2. Specify your *analysis model*:
   a. Specify the portion of the Simulink model you want to analyze. This involves using special Simulink blocks to locate the inputs and outputs of this analysis model on your Simulink diagram.
   b. Set the operating conditions for linear analysis (optional). If your Simulink model includes nonlinear components, the Simulink LTI Viewer linearizes the model around the specified operation point. The default operating conditions have all state and input values set to zero.
   c. Modify any Simulink model block parameters (optional).
3. Perform linear analysis with the Simulink LTI Viewer:
   a. Import a linearized analysis model to the Simulink LTI Viewer.
   b. Analyze the Bode plot.
   c. Specify a second analysis model by changing the value of the Gain block, `Mu`.
   d. Import the second linearized analysis model, and compare the Bode plots of the two linearized analysis models.
4. Save the analysis models for future use.

In the remaining sections of this chapter, we explain how to carry out each of these steps on the van der Pol oscillator example.

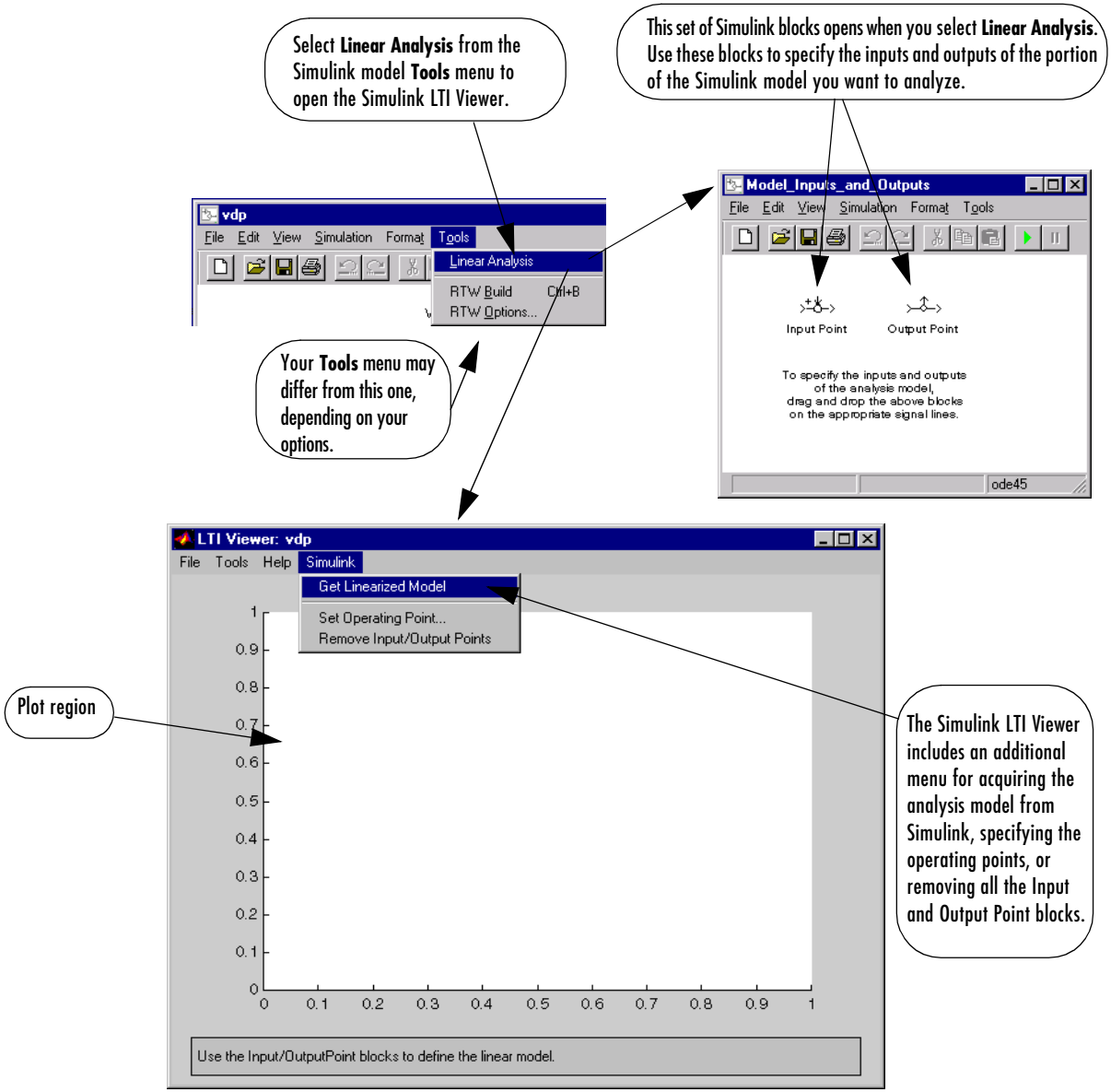
Opening the Simulink LTI Viewer

To open a Simulink LTI Viewer linked to the **vdp** Simulink model:

1. Go to the **Tools** menu on the Simulink model.
2. Select **Linear Analysis**.

When you select **Linear Analysis**, two new windows open: an LTI Viewer window and a Simulink diagram called **Model_Inputs_and_Outputs** containing two blocks: Input Point and Output Point.

The following figure depicts how to open the Simulink LTI Viewer.



The Simulink LTI Viewer differs from the *regular* LTI Viewer, in that:

- The title bar shows the name of the Simulink model to which it is linked.
- The menu bar contains an additional menu called **Simulink** that contains the following items:
  - **Get Linearized Model** linearizes the Simulink model and imports the resulting linearized analysis model to the LTI Viewer. Each time you select this menu item, a new version of the linearized analysis model is added to the Simulink LTI Viewer workspace.
  - **Set Operating Point** allows you to set or reset the operating conditions.
  - **Remove Input/Output Points** clears all Input Point and Output Point blocks from the diagram.

Specifying the Simulink Model Portion for Analysis

To specify the portion of the Simulink model you want to analyze, mark its input and output signals on the Simulink model using the Input Point and Output Point blocks in the **Model_Inputs_and_Outputs** window. This defines an input/output relationship that is linearized and analyzed by the LTI Viewer.

Adding Input Point or Output Point Blocks to the Diagram

To designate the input and output signals of your analysis model, insert Input Point and Output Point blocks on the corresponding signal lines in your Simulink diagram.

For example, to insert an Input Point block on the Simulink model:

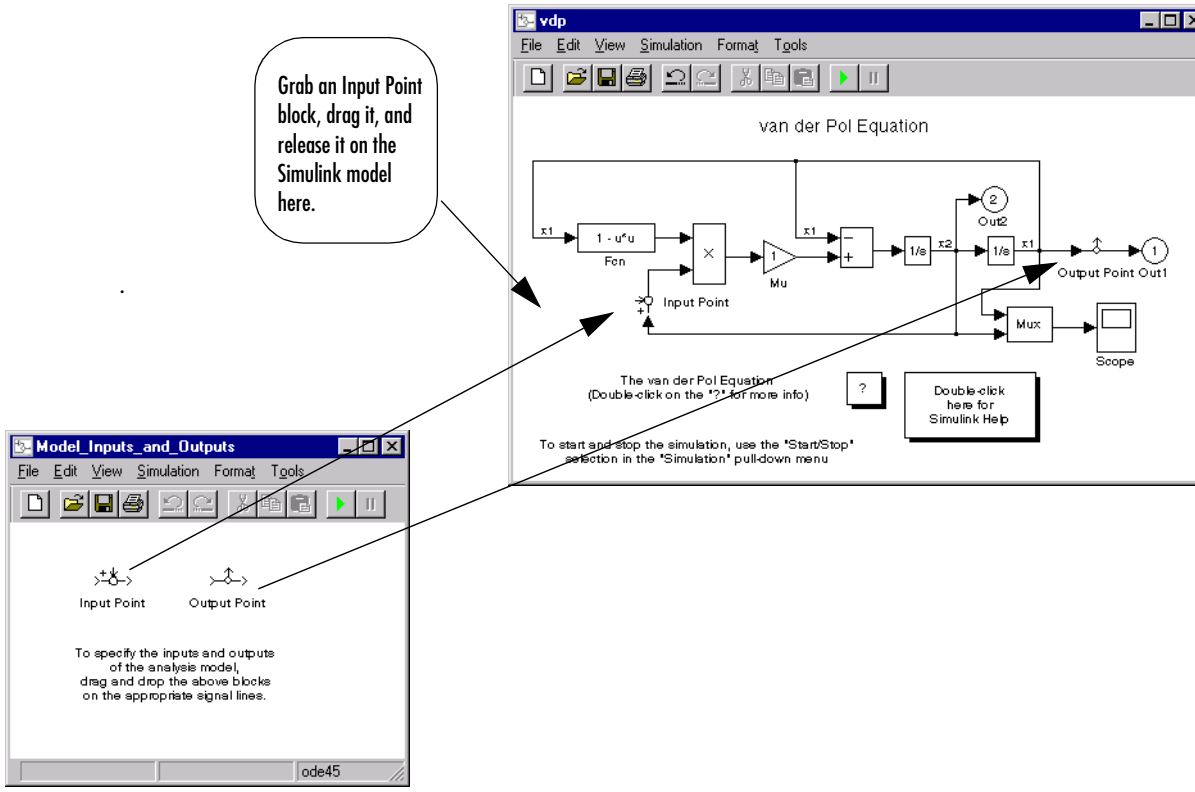
1. Grab the Input Point block in the **Model_Inputs_and_Outputs** window by clicking on the block and holding the mouse button down.
2. Drag the block to your Simulink model and place it over the line associated with the desired signal.
3. Release the mouse button. The block should automatically connect to the line.
4. If the block fails to connect (this may occur, for example, when the line is too short), resize the line and double-click on the block to force the connection.

To set up the analysis model for the **vdp** Simulink model:

1. Insert an Input Point block on the line labeled `x2` going into the Product block.

2. Insert an Output Point block on the line entering the Outport block, `Out1`.

This results in the following diagram.


Grab an Input Point block, drag it, and release it on the Simulink model here.
vdp
File Edit View Simulation Format Tools
van der Pol Equation
x1
1 - u^2*u
Fcn
x1
1
Mu
1/s
x2
1/s
x1
2
Out2
Output Point Out1
Input Point
Mux
Scope
The van der Pol Equation
(Double-click on the "?" for more info)
?
Double-click here for Simulink Help
To start and stop the simulation, use the "Start/Stop" selection in the "Simulation" pull-down menu
Model_Inputs_and_Outputs
File Edit View Simulation Format Tools
Input Point
Output Point
To specify the inputs and outputs of the analysis model, drag and drop the above blocks on the appropriate signal lines.
ode45

Keep the following in mind when using the Input Point and Output Point blocks to specify analysis models:

- You must place at least one Input Point block and one Output Point block somewhere in the diagram in order to specify an analysis model.
- You can place the Input Point and Output Point blocks on any scalar or vector signal line in the Simulink model, with the exception of signal lines connected to any block in the Power System Blockset.
- You can insert Input Point and Output Point blocks at different levels of a Simulink model hierarchy.
- There is no limit on the number of these blocks you can use.

Removing Input Points and Output Points

There are two ways you can remove Input Point or Output Point blocks from the Simulink model:

1. **One by one**: Select the Input Point or Output Point block you want to remove and delete it as you would any other Simulink block.

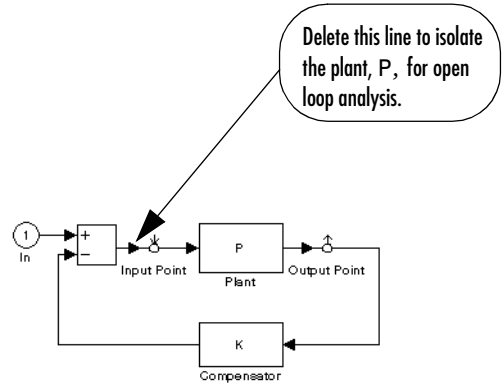
2. **All at once:** To remove all Input Point and Output Point blocks, select **Remove Input/Output Points** from the **Simulink** menu in the LTI Viewer.

When you delete an Input Point or an Output Point block, the signal lines coming into and out of this block are automatically reconnected.

Specifying Open- Versus Closed-Loop Analysis Models

Placing the Input Point and Output Point blocks on your Simulink model does not break any connection or isolate any component. As a result, the Simulink LTI Viewer performs closed-loop analysis whenever your diagram contains feedback loops. This may sometimes lead to counter-intuitive results, as is illustrated by the next example.

Consider the following simple diagram.



Based on the location of the Input Point and Output Point blocks, you might think that the analysis model specified by these blocks is simply the plant model, *P*. However, due to the feedback loop, this analysis model is actually the closed-loop transfer function $P/(1+PK)$.

If you want to analyze the (open-loop) plant *P* instead, you need to open the loop, for example, by deleting the line between the Sum and Input Point blocks.

Setting the Operating Conditions

If you have nonlinear components in your Simulink model, the Simulink LTI Viewer automatically linearizes them when you select **Get Linearized Model**. The Simulink LTI Viewer uses the initial state values you set in the Simulink diagram as default settings for linearization points for the states in the diagram. The default input values for this linearization are zero. You also have the option to linearize about the operating conditions of your choice.

If you want your analysis model to be linearized about zero state, or other state and input operating conditions, follow these steps *before* selecting **Get Linearized Model**:

1 Select **Set Operating Point** in the **Simulink** menu. This opens the **Operating Point** window.

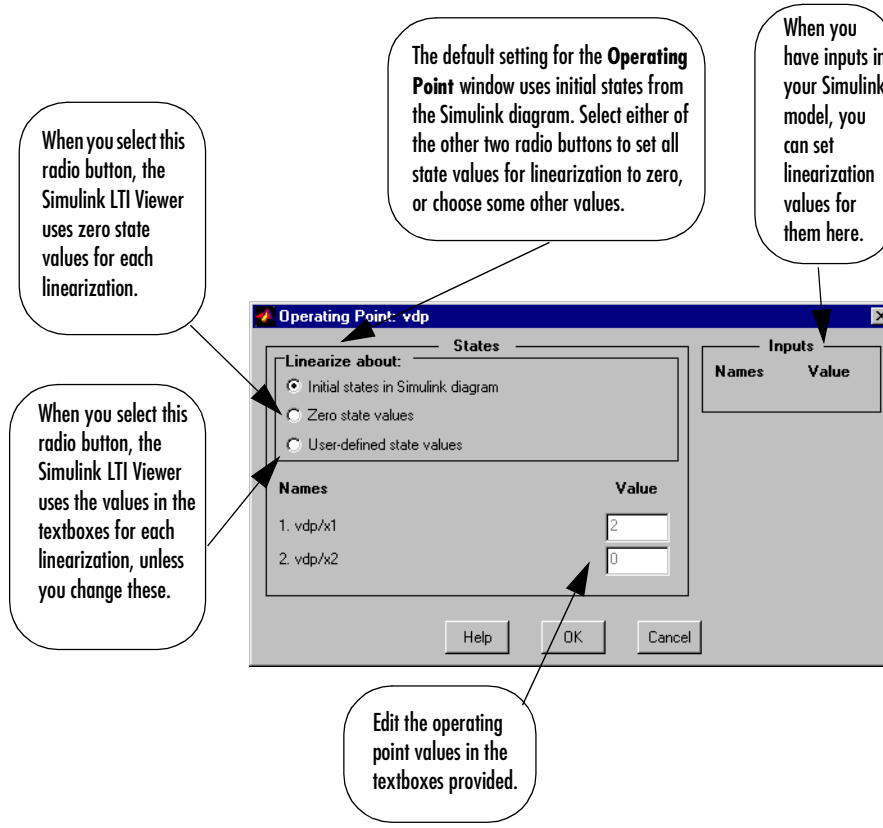


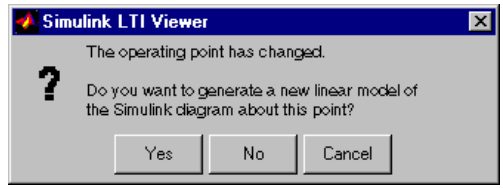
Figure 6-24: The Operating Point Window for Changing Linearization Points

2 Change the radio button selection to either:

- Set all state values for the linearization to zero.
- Define your own state values for the linearization.

3 Use the white textboxes to specify the operating conditions for each input (and state) listed in the **Operating Point** window. You don't have to specify the states if you choose the **Zero state values** radio button.

4 Select **OK**. A dialog box opens.

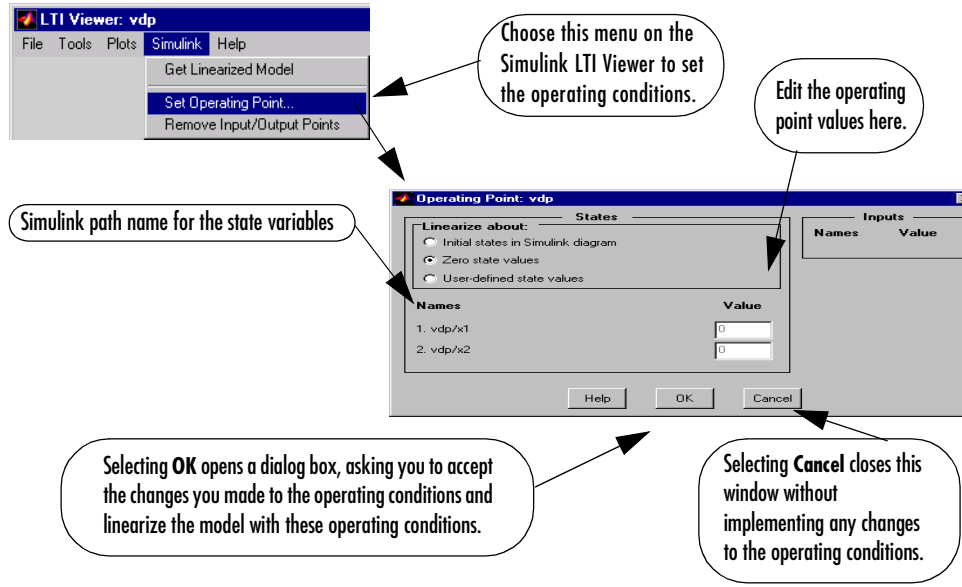


a Selecting **Yes** closes the **Operating Point** window and linearizes the model automatically. The new operating conditions you selected remain in effect for any future linearizations.

b Selecting **No** closes the **Operating Point** window without linearizing the model. The new operating conditions you selected remain in effect for any future linearizations.

c Selecting **Cancel** closes the dialog box and returns you to the **Operating Point** window without linearizing the model or changing operating conditions.

For this example, we use the **Zero initial states** setting, shown in the figure below.



Note the following:

- The inputs listed on the **Operating Point** window correspond to the Inport blocks on the top level of your Simulink model.
- All states and inputs in the Simulink diagram are listed in this window, not just those associated with your analysis model.
- If you want to change the operating conditions, you need only change those values associated with your analysis model.
- While the **Operating Point** window is in the **User-defined initial state values** mode, the values listed in the **Operating Point** window remain in effect throughout your Simulink LTI Viewer session unless you change these.

- While the **Operating Point** window is in the **Initial state in Simulink diagram** mode, the linearization values used by the Simulink LTI Viewer are updated as you change any state values in your Simulink diagram.
- To use the MATLAB command line to change Simulink diagram initial state values
  - Select **Parameters** under the **Simulation** menu on your Simulink diagram.
  - Choose the **Workspace I/O** tab in the **Simulation Parameters** window.
  - Load initial states from the MATLAB workspace using the appropriate textbox.

Modifying the Block Parameters

You have the option of modifying any of your Simulink model block parameters, such as Gain block gain values or LTI block poles and zeros, before you import your analysis model into the Simulink LTI Viewer.

Performing Linear Analysis

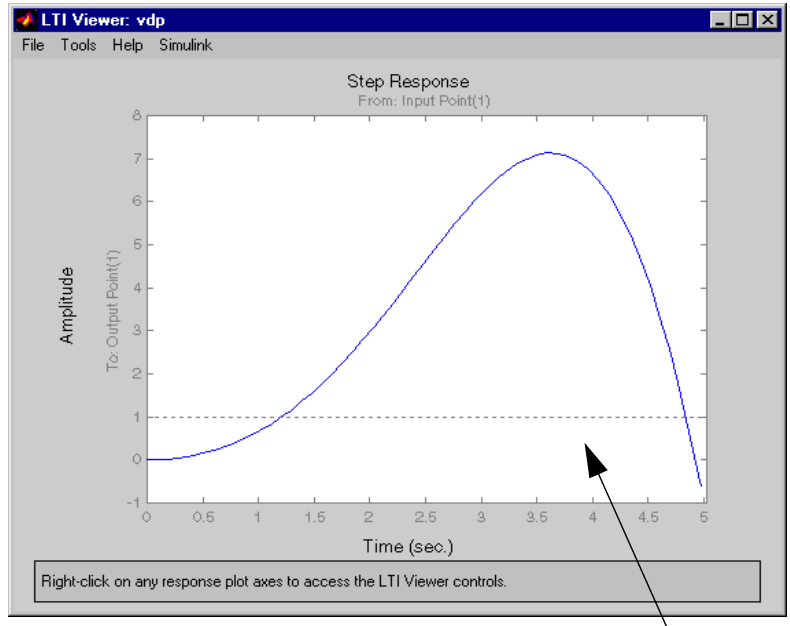
Once you have specified your analysis model, you are ready to analyze it with the Simulink LTI Viewer.

Let's use the Simulink LTI Viewer to compare the Bode plots of two different linearized analysis models. The procedure for carrying out this analysis on the van der Pol example involves:

- Importing a linearized analysis model to the Simulink LTI Viewer.
- Analyzing the Bode plot of the linearized analysis model.
- Specifying another analysis model.
- Importing the second linearized analysis model to compare the Bode plots of both linearized analysis models.

Importing a Linearized Analysis Model to the LTI Viewer

To linearize and load your first analysis model for the van der Pol example into the LTI Viewer, select **Get Linearized Model** on the **Simulink** menu on the LTI Viewer. This produces the following response plot:



The LTI Viewer displays the selected response plot for all of the models in the LTI Viewer workspace. These are listed in the **Systems** menu available by right-clicking in the plot region. The default plot type is step response.

Each time you select **Get Linearized Model** in the LTI Viewer's **Simulink** menu:

- The linearized analysis model is imported into the LTI Viewer workspace.
- The step response of the linearized analysis model is the default plot type displayed.

You can view the linearized models in the LTI Viewer workspace or change the plot type using the right-click menus. See “The Right-Click Menus” on page 6-18 for more information.

Analyzing the Bode Plot of the Linearized Analysis Model

To analyze the Bode plot for this model,

1. Right-click anywhere in the plot region.
2. Select the **Plot Type** menu, and then the **Bode** submenu item.

Specifying Another Analysis Model

Once an analysis model is specified on a Simulink diagram, you can specify other analysis models from the same Simulink diagram in one of the following three ways:

- Modify any of the model parameters.
- Change the operating conditions.
- Change the location of any of the Input Point or Output Point blocks.

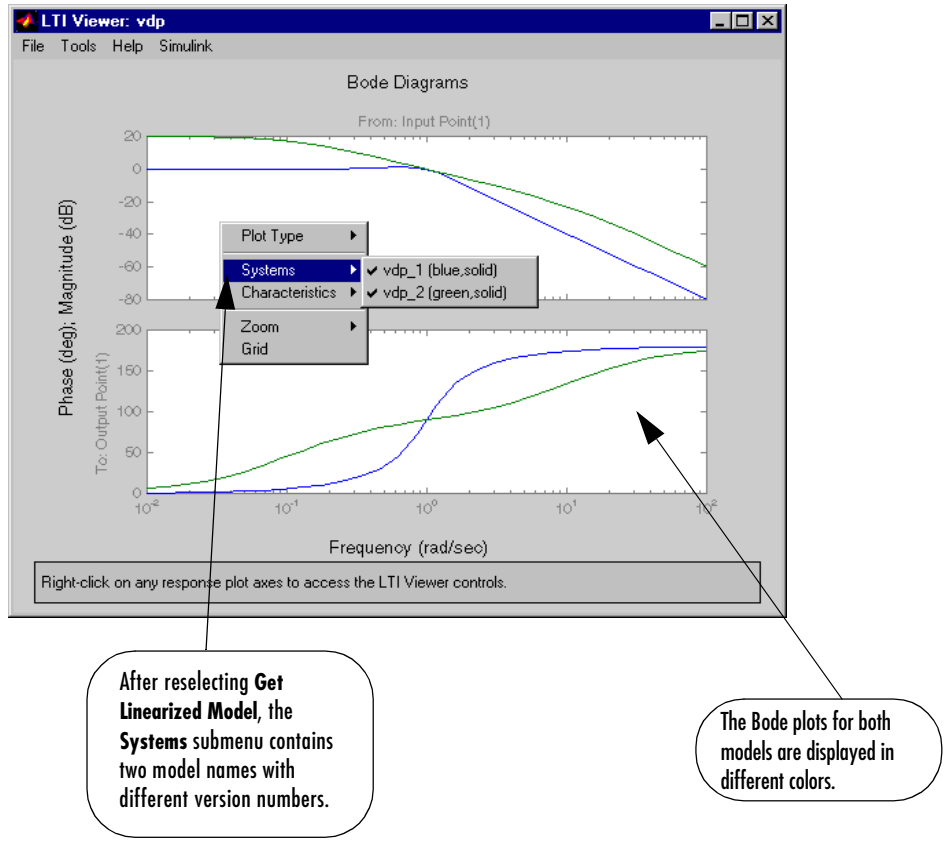
For the van der Pol example, we want to create a second analysis model by modifying a model parameter. Specifically, we modify the Gain block labeled `Mu`. To do this:

- Return to the Simulink model, and open the `Mu` Gain block by double-clicking on it.
- Change the gain to 10; click on **OK**.

Comparing the Bode Plots of the Two Linearized Analysis Models

Having just specified a new analysis model, let's load its linearization into the LTI Viewer and compare the Bode plots of the two models. To do this, reselect the **Get Linearized Model** menu item under **Simulink** on the LTI Viewer.

As is shown below, the linearized model for the new value of `Mu` appears as the last item in the **Systems** submenu, and the Bode plots for both models are displayed.



As you might expect, changing the gain alters the Bode plot.

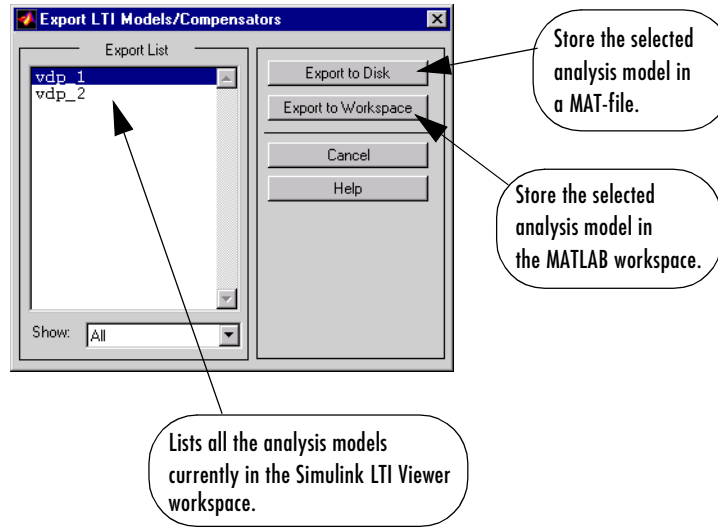
Notice the following about the models listed in the **Systems** list on the right-click menu:

- The names reflect the title of the Simulink model.
- The version number is incremented every time **Get Linearized Model** is selected.
- The LTI Viewer simultaneously displays the response plots of all of the checked models listed.

Saving Analysis Models

The analysis models obtained each time you select **Get Linearized Model** are stored only in the Simulink LTI Viewer workspace. You can save these models into the main MATLAB workspace by selecting **Export** from the Simulink LTI Viewer **File** menu.

Selecting **Export** opens the window shown below.



To export analysis models from the LTI Viewer workspace:

1. Highlight the models you want to save in the **Export List**. (You can multiselect models on the list by holding down the control key while selecting list items).

2. Save the selected models by clicking on the appropriate button on this GUI. You can export the linearized models to either:

   a. A MAT-file

   b. The MATLAB workspace

Note: If you save models to a MAT-file, you are prompted to name the file. The variable names contained in that file are the same as those you selected from the **Export List**. The variable names of each model you save to the MATLAB workspace are also the same as those listed in the **Export List**. It's up to you to modify the names of these variables after you've saved them.

7

Control Design Tools

We use the term control system *design* to refer to the process of selecting feedback gains in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and physical insight.

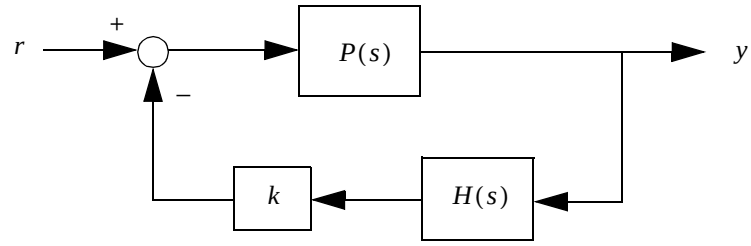
The Control System Toolbox offers functions to:

- Design the control system gains using either classical root locus techniques or modern pole placement and LQG techniques.
- Close the loop for simulation and validation purposes.

Root Locus Design

The root locus method is used to describe the trajectories of the closed-loop poles of a feedback system as one parameter varies over a continuous range of values. Typically, the root locus method is used to tune a feedback gain so as to specify the closed-loop poles of a SISO control system.

Consider, for example, the tracking loop



where $P(s)$ is the plant, $H(s)$ is the sensor dynamics, and k is a scalar gain to be adjusted. The closed-loop poles are the roots of

$$q(s) = 1 + k\,P(s)H(s)$$

The root locus technique consists of plotting the closed-loop pole trajectories in the complex plane as k varies. You can use this plot to identify the gain value associated with a given set of closed-loop poles on the locus.

The command `rltool` opens the Root Locus Design GUI. In addition to plotting the root locus, the Root Locus Design GUI can be used to design a compensator interactively to meet some system design specifications.

The Root Locus Design GUI can be used to:

- Analyze the root locus plot for a SISO LTI feedback loop
- Specify feedback compensator parameters: poles, zeros, and gain
- Examine how the compensator parameters change the root locus, as well as various open and closed-loop system responses (step response, Bode plot, Nyquist plot, or Nichols chart)

Chapter 8 provides more detail on the Root Locus Design GUI.

If you are interested in just the root locus plot, use the command `rlocus`. This command takes one argument: a SISO model of the open loop system, created with `ss`, `tf`, or `zpk`. In the tracking loop depicted on the previous page, this model would represent $P(s)H(s)$. You can also use the function `rlocfind` to select a point on the root locus plot and determine the corresponding gain k.

The following table summarizes the commands for root locus design.

Root Locus Design	
`pzmap`	Pole-zero map.
`rltool`	Root Locus Design GUI.
`rlocfind`	Interactive root locus gain selection.
`rlocus`	Evans root locus plot.
`sgrid`	Continuous ω_n, ζ grid for root locus.
`zgrid`	Discrete ω_n, ζ grid for root locus.

Pole Placement

The closed-loop pole locations have a direct impact on time response characteristics such as rise time, settling time, and transient oscillations. This suggests the following method for tuning the closed-loop behavior:

1. Based on the time response specifications, select desirable locations for the closed-loop poles.
2. Compute feedback gains that achieve these locations.

This design technique is known as *pole placement*.

Pole placement requires a state-space model of the system (use `ss` to convert other LTI models to state space). In continuous time, this model should be of the form

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where u is the vector of control inputs and y is the vector of measurements. Designing a dynamic compensator for this system involves two steps: state-feedback gain selection, and state estimator design.

State-Feedback Gain Selection

Under state feedback $u = -Kx$, the closed-loop dynamics are given by

$$\dot{x} = (A - BK)\, x$$

and the closed-loop poles are the eigenvalues of $A - BK$. Using pole placement algorithms, you can compute a gain matrix K that assigns these poles to any desired locations in the complex plane (provided that (A, B) is controllable).

State Estimator Design

You cannot implement the state-feedback law $u = -Kx$ unless the full state x is measured. However, you can construct a state estimate ξ such that the law $u = -K\xi$ retains the same pole assignment properties. This is achieved by designing a state estimator (or observer) of the form

$$\dot{\xi} = A\xi + Bu + L(y - C\xi - Du)$$

The estimator poles are the eigenvalues of $A - LC$, which can be arbitrarily assigned by proper selection of the estimator gain matrix L. As a rule of thumb, the estimator dynamics should be faster than the controller dynamics (eigenvalues of $A - BK$).

Replacing x by its estimate ξ in $u = -Kx$ yields the dynamic output-feedback compensator

$$\dot{\xi} = \left[A - LC - (B - LD)K\right]\xi + Ly$$
$$u = -K\xi$$

Note that the resulting closed-loop dynamics are

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}, \qquad e = x - \xi$$

Hence, you actually assign all closed-loop poles by independently placing the eigenvalues of $A - BK$ and $A - LC$.

Pole Placement Tools

The Control System Toolbox contains functions to:

- Compute gain matrices K and L that achieve the desired closed-loop pole locations
- Form the state estimator and dynamic compensator using these gains

Pole Placement	
`acker`	SISO pole placement.
`estim`	Form state estimator given estimator gain.
`place`	MIMO pole placement.
`reg`	Form output-feedback compensator given state-feedback and estimator gains.

The function `acker` is limited to SISO systems and should only be used for systems with a small number of states. The function `place` is a more general and numerically robust alternative to `acker`.

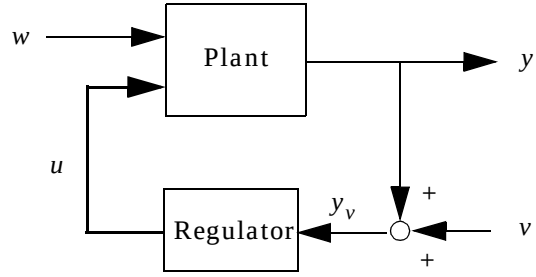
Caution: Pole placement can be badly conditioned if you choose unrealistic pole locations. In particular, you should avoid:

- Placing multiple poles at the same location
- Moving poles that are weakly controllable or observable. This typically requires high gain, which in turn makes the entire closed-loop eigenstructure very sensitive to perturbations.

LQG Design

Linear-Quadratic-Gaussian (LQG) control is a modern state-space technique for designing optimal dynamic regulators. It enables you to trade off regulation performance and control effort, and to take into account process and measurement noise. Like pole placement, LQG design requires a state-space model of the plant (use `ss` to convert other LTI models to state space). This section focuses on the continuous-time case (see related Reference pages for details on discrete-time LQG design).

LQG design addresses the following regulation problem.



The goal is to regulate the output y around zero. The plant is driven by the process noise w and the controls u, and the regulator relies on the noisy measurements $y_v = y + v$ to generate these controls. The plant state and measurement equations are of the form

$$\dot{x} = Ax + Bu + Gw$$
$$y_v = Cx + Du + Hw + v$$

and both w and v are modeled as white noise.

The LQG regulator consists of an optimal state-feedback gain and a Kalman state estimator. You can design these two components independently as shown next.

Optimal State-Feedback Gain

In LQG control, the regulation performance is measured by a quadratic performance criterion of the form

$$J(u) = \int_0^\infty \{x^T Q x + 2x^T N u + u^T R u\}\, dt$$

The weighting matrices Q, N, R are user specified and define the trade-off between regulation performance (how fast $x(t)$ goes to zero) and control effort.

The first design step seeks a state-feedback law $u = -Kx$ that minimizes the cost function $J(u)$. The minimizing gain matrix K is obtained by solving an algebraic Riccati equation. This gain is called the *LQ-optimal* gain.

Kalman State Estimator

As for pole placement, the LQ-optimal state feedback $u = -Kx$ is not implementable without full state measurement. However, we can derive a state estimate $\hat{x}$ such that $u = -K\hat{x}$ remains optimal for the output-feedback problem. This state estimate is generated by the Kalman filter.

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y_v - C\hat{x} - Du)$$

with inputs u (controls) and y_v (measurements). The noise covariance data

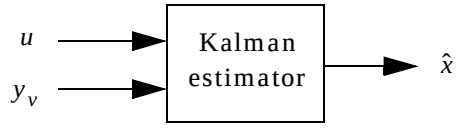
$$E(ww^T) = Q_n, \qquad E(vv^T) = R_n, \qquad E(wv^T) = N_n$$

determines the Kalman gain L through an algebraic Riccati equation.

The Kalman filter is an optimal estimator when dealing with Gaussian white noise. Specifically, it minimizes the asymptotic covariance

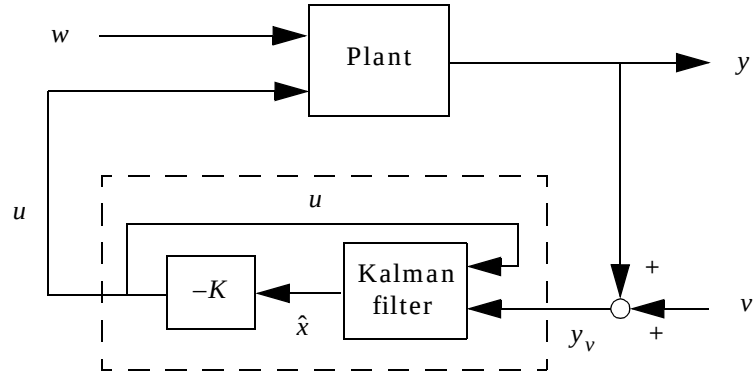
$$\lim_{t \to \infty} E((x - \hat{x})(x - \hat{x})^T)$$

of the estimation error $x - \hat{x}$.



LQG Regulator

To form the LQG regulator, simply connect the Kalman filter and LQ-optimal gain K as shown below:



LQG regulator

This regulator has state-space equations

$$\dot{\hat{x}} = \left[A - LC - (B - LD)K\right]\hat{x} + Ly_v$$

$$u = -K\hat{x}$$

LQG Design Tools

The Control System Toolbox contains functions to perform the three LQG design steps outlined above. These functions cover both continuous and

discrete problems as well as the design of discrete LQG regulators for continuous plants.

LQG Design	
`care`	Solve continuous-time algebraic Riccati equations.
`dare`	Solve discrete-time algebraic Riccati equations.
`dlqr`	LQ-optimal gain for discrete systems.
`kalman`	Kalman estimator.
`kalmd`	Discrete Kalman estimator for continuous plant.
`lqgreg`	Form LQG regulator given LQ gain and Kalman filter.
`lqr`	LQ-optimal gain for continuous systems.
`lqrd`	Discrete LQ gain for continuous plant.
`lqry`	LQ-optimal gain with output weighting.

See the case study on page 9-31 for an example of LQG design. You can also use the functions `kalman` and `kalmd` to perform Kalman filtering; see the case study on page 9-50 for details.

8

The Root Locus Design GUI

Introduction

The root locus method is used to describe the trajectories in the complex plane of the closed-loop poles of a SISO feedback system as one parameter (usually a gain) varies over a continuous range of values.

Along with the MATLAB command line function `rlocus`, the Control System Toolbox provides the Root Locus Design graphical user interface (GUI) for implementing root locus methods on single-input/single-output (SISO) LTI models defined using `zpk`, `tf`, or `ss`.

In addition to plotting root loci, the Root Locus Design GUI is an interactive design tool that can be used to:

- Analyze the root locus plot for a SISO LTI control system
- Specify the parameters of a feedback compensator: poles, zeros, and gain
- Examine how changing the compensator parameters effects changes in the root locus and various closed-loop system responses (step response, Bode plot, Nyquist plot, or Nichols chart)

This chapter explains how to use the Root Locus Design GUI, in part, through an example involving an electrohydraulic servomechanism comprised of an electrohydraulic amplifier, a valve, and a ram. These types of servomechanisms can be made quite small, and are used for position control. Details on the modeling of electrohydraulic position control mechanisms can be found in [1].

After an explanation of the servomechanism control system, the following operations on the Root Locus Design GUI are covered in the section, “Controller Design Using the Root Locus Design GUI” on page 8-6:

- Opening the Root Locus Design GUI
- Importing models into the Root Locus Design GUI
- Changing the gain set point and zooming
- Displaying system responses
- Designing the compensator to meet specifications:
  - Specifying the design region boundaries on the root locus
  - Placing compensator poles and zeros

  - Editing the compensator pole and zero locations
  - Opening the LTI Viewer
- Saving the compensator and models to the workspace or the disk

Other important features listed below and not covered through this example are described in the section, “Additional Root Locus Design GUI Features” on page 8-38:

- Specifying design models: general concepts
- Getting help with the Root Locus Design GUI
- Erasing compensator poles and zeros
- Listing poles and zeros
- Printing the root locus
- Drawing a Simulink diagram of the closed-loop model
- Converting between continuous and discrete models
- Clearing data

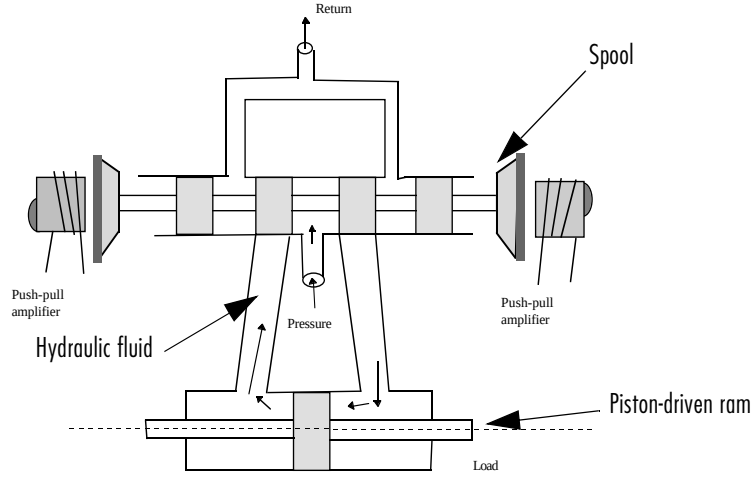
A Servomechanism Example

A simple version of an electrohydraulic servomechanism model consists of:

- A push-pull amplifier (a pair of electromagnets)
- A sliding spool in a vessel of high pressure hydraulic fluid
- Valve openings in the vessel to allow for fluid to flow
- A central chamber with a piston-driven ram to deliver force to a load
- A symmetrical fluid return vessel

The force on the spool is proportional to the current in the electromagnet coil. As the spool moves, the valve opens, allowing the high pressure hydraulic fluid to flow through the chamber. The moving fluid forces the piston to move in the opposite direction of the spool. In [1], linearized models for the electromagnetic amplifier, the valve spool dynamics, and the ram dynamics are derived, and a detailed description of this type of servomechanism is provided.

A schematic of this servomechanism is depicted below.



If you want to use this servomechanism for position control, you can use the input voltage to the electromagnet to control the ram position. When measurements of the ram position are available, you can use feedback for the ram position control.

A closed-loop model for the electrohydraulic valve position control can be set up as follows.

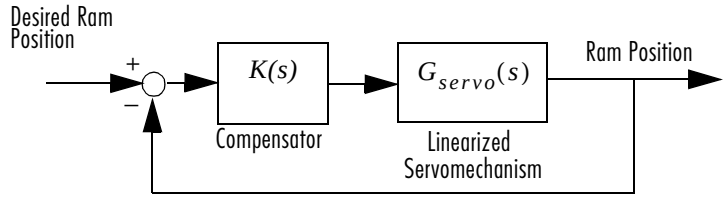


Figure 8-1: Feedback Control for an Electrohydraulic Servomechanism

K(s) represents the compensator for you to design. This compensator can be either a gain or a more general LTI system.

A linearized plant model for the electrohydraulic position control mechanism is given by

$$G_{servo}(s) = \frac{4 \times 10^7}{s(s + 250)(s^2 + 40s + 9 \times 10^4)}$$

For this example, you want to design a controller so that the step response of the closed-loop system meets the following specifications:

- The two-percent settling time is less than 0.05 seconds.
- The maximum overshoot is less than 5 percent.

For details on how these specifications are defined, see [2].

In the remainder of this chapter you learn how to use the Root Locus Design GUI. In the process, you design a controller to meet these specifications.

Controller Design Using the Root Locus Design GUI

In this section, we use the servomechanism example to describe some of the main Root Locus Design GUI features and operations:

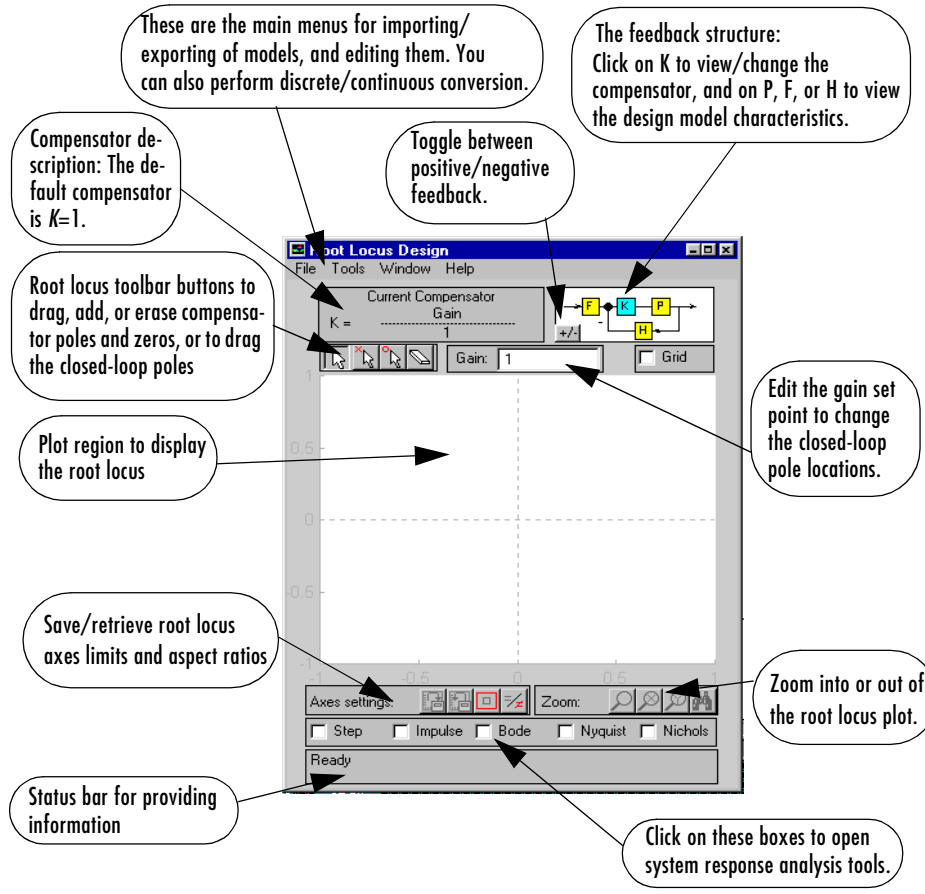
- Opening the Root Locus Design GUI
- Importing models into the Root Locus Design GUI:
  - Opening the **Import LTI Design Model** window
  - Choosing a feedback structure
  - Specifying the design model
- Changing the gain set point and zooming:
  - Dragging closed-loop poles to change the gain set point
  - Zooming
  - Storing and retrieving axes limits
- Displaying system responses
- Designing the compensator to meet specifications:
  - Specifying the design region boundaries on the root locus
  - Placing compensator poles and zeros: general information
  - Placing compensator poles and zeros using the root locus toolbar
  - Editing compensator pole and zero locations
- Saving the compensator and models to the workspace or the disk

Opening the Root Locus Design GUI

To open the Root Locus Design GUI, at the MATLAB prompt type

```
rltool
```

This brings up the following GUI.



Importing Models into the Root Locus Design GUI

The Root Locus Design GUI operates on SISO LTI models constructed using either `tf`, `zpk`, or `ss` (for detail on creating models, see “Creating LTI Models” in Chapter 2).

There are four ways to import SISO LTI models into the Root Locus Design GUI:

- Load a model from the MATLAB workspace.
- Load a model from a MAT-file on your disk.
- Load SISO LTI blocks from an open or saved Simulink diagram.
- Create models using `tf`, `ss`, or `zpk` within the GUI.

You can also use any combination of these methods to import models for root locus analysis and design. However, before you can import a model into the Root Locus Design GUI from the MATLAB workspace, you must have at least one SISO LTI model loaded into your workspace. Similar requirements hold for loading models from the disk or an open Simulink diagram.

For this example, we import our servomechanism model for root locus analysis from the MATLAB workspace. You can find a zero-pole-gain model for $G_{servo}(s)$ in a set of LTI models provided in the file `LTIexamples.MAT`.

To load these LTI models, at the MATLAB prompt type

```
load LTIexamples
```

The model for the position control system is contained in the variable `Gservo`. To view the information on this model, at the MATLAB prompt type

```
Gservo
```

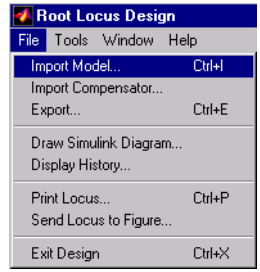
Now that a model for $G_{servo}(s)$ is loaded into the workspace, you can begin your root locus analysis and design for this example.

There are three steps involved in importing a model for our example covered in this section:

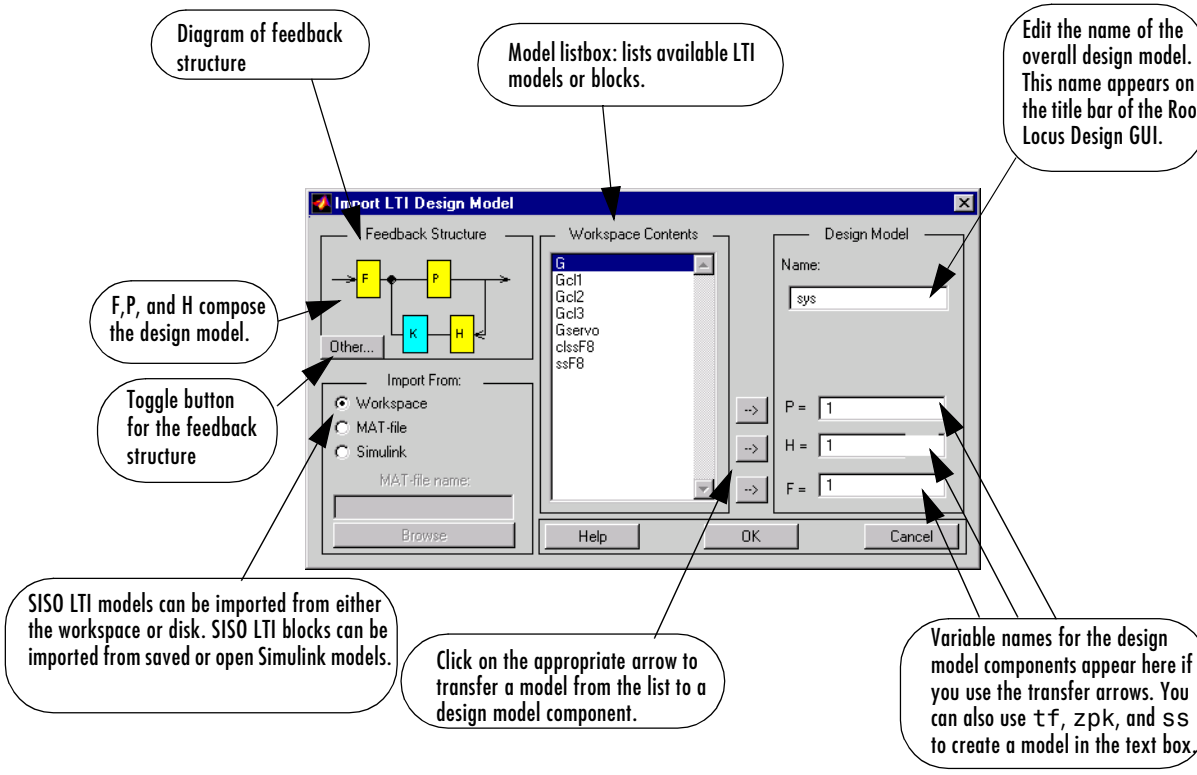
- Opening the **Import LTI Design Model** window
- Choosing a feedback structure
- Specifying the design model

Opening the Import LTI Design Model Window

To import the linearized electrohydraulic servomechanism model into the Root Locus Design GUI, first open the **Import LTI Design Model** window. To do this, select the **Import Model** menu item in the **File** menu.

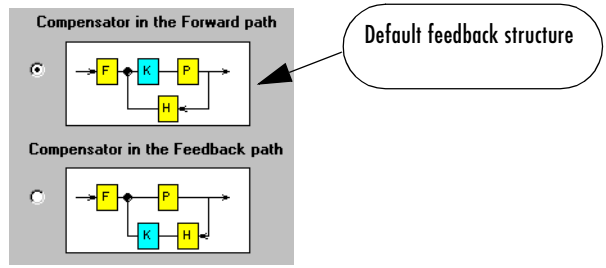


The **Import LTI Design Model** window that appears on your screen is as follows.



Choosing a Feedback Structure

The Root Locus design tool can be applied to SISO LTI systems whose feedback structure is in one of the following two configurations.



The **Feedback Structure** portion of the **Import LTI Design Model** window shows the current selection for the closed-loop structure. The **Other** button toggles the location of the compensator between the two configurations shown above. For this example you want the compensator in the forward path.

Specifying the Design Model

The SISO LTI models in either feedback configuration are coded as follows:

- **F** represents a pre-filter.
- **P** is the plant model.
- **H** is the sensor dynamics.
- **K** is the compensator to be designed.

In terms of the GUI design procedure, once they are set, **F**, **P**, and **H** are *fixed* in the feedback structure. This triple, along with the choice of feedback structure, is referred to throughout this chapter as the *design model*.

The default values for **F**, **P**, **H**, and **K** are all 1.

When you specify your design model, in addition to **F**, **P**, **H**, and the feedback structure, you can specify the design model name. To name the design model, click in the editable text box in the **Import LTI Design Model** window below **Name**, and enter the name you want for the design model. For this example, change the design model name to `Gservo`.

To specify **F**, **P**, and **H** for this example, we use the **Workspace** radio button, which is the default selection. A list of the LTI objects in your workspace appears in the model listbox under **Workspace Contents**.

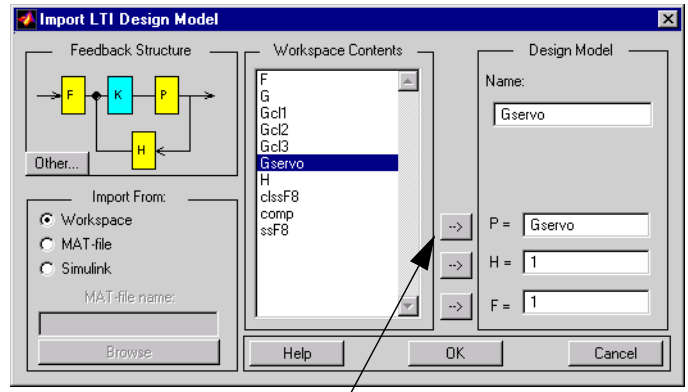
In general, to import design model components from the workspace to the GUI:

1. Select a given model in the **Workspace Contents** listbox to be loaded into either **F**, **P**, or **H**.
2. Click on the arrow buttons next to the design model component you want to specify.

To specify the design model components for this servomechanism example:

1 Load the linearized servomechanism model `Gservo` into the plant **P**, by first selecting it and then selecting the arrow button.

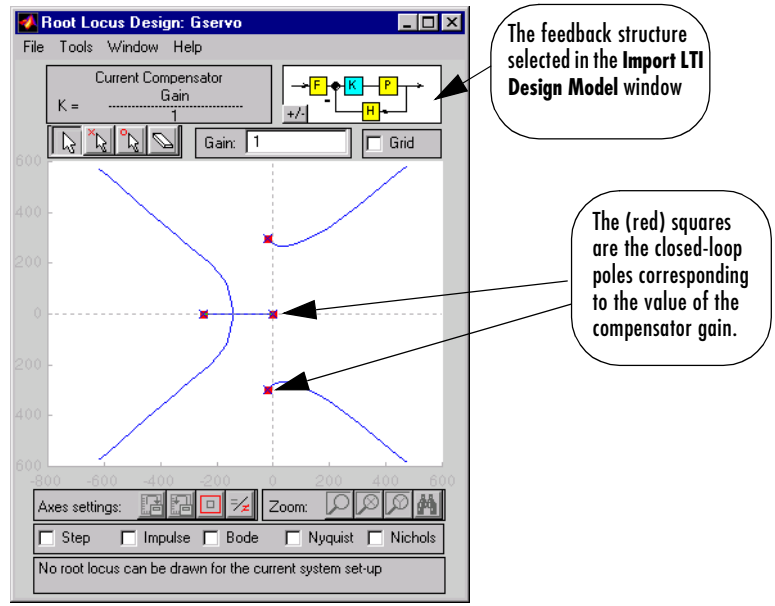
The **Import LTI Design Model** window looks like this after you specify both the plant, **P**, and the model name as `Gservo`.



Use the arrow buttons to transfer selected models from the listbox to the design model components, in this case, to the plant, **P**.

2 Press the **OK** button.

The root locus of the design model is displayed in the plot region of the GUI. Your Root Locus Design GUI looks like this.



Notice that the design model name appears in the title bar.

Changing the Gain Set Point and Zooming

The *gain set point* is the value of the gain you apply to the compensator to determine the closed-loop poles. This value appears in the **Gain** text box on the GUI.

In this section, we use some of the basic functions of the Root Locus Design GUI to analyze a root locus in the plot region. We cover how to change the gain set point, along with the closed-loop pole locations, and how to use the GUI zoom tools.

Let's begin by seeing how much gain you can apply to the compensator and still retain stability of the closed-loop system.

The red squares on each branch of the root locus mark the locations of the closed-loop poles associated with the gain set point. The system becomes

unstable if the gain set point is increased so as to place any of the closed-loop poles in the right-half of the complex plane.

You can test the limits of stability by either:

- Dragging the closed-loop poles around the locus to change the gain set point
- Changing the gain set point manually

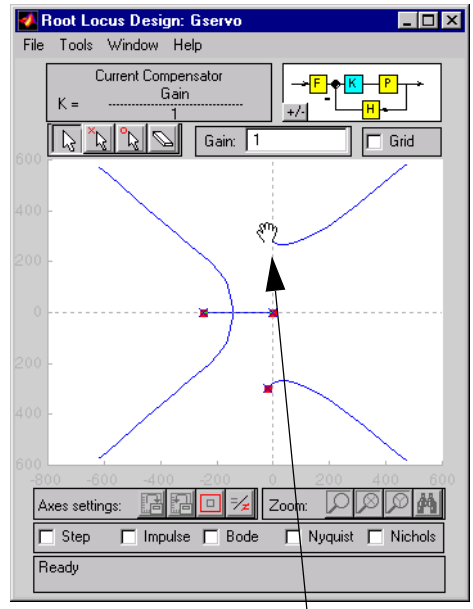
In this example we do the former.

Dragging Closed-loop Poles to Change the Gain Set Point

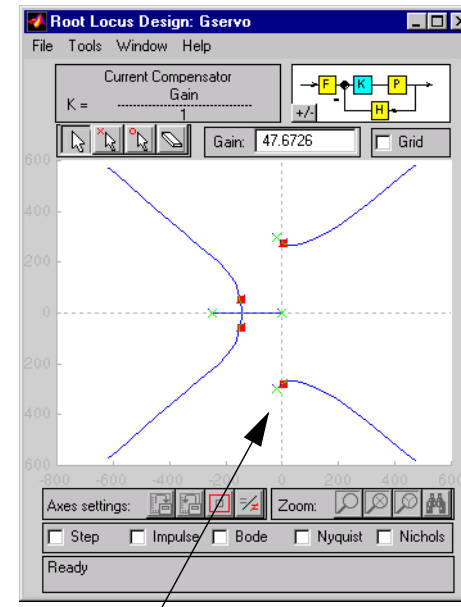
To see by how much the gain can be increased while maintaining the closed-loop poles in the left-half of the complex plane:

1. Move the mouse pointer over a red square marking one of the complex poles nearest the imaginary axis. Notice how the pointer becomes a hand.
2. Grab the closed-loop pole by holding down the left mouse button when the hand appears.
3. Drag the pole close to the imaginary axis.
4. Release the mouse button. The gain changes as the closed-loop set point is recomputed.

The following two figures capture this procedure.



As you get close to a closed-loop pole on the locus, the mouse pointer becomes a hand.



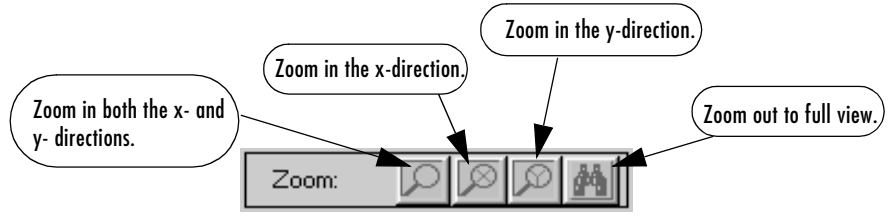
The poles appear to be on the imaginary axis.

The right-most pair of closed-loop poles seem to be on the imaginary axis. Actually, they are only close. Let's use the zoom controls to improve this result.

Zooming

You can use the **Zoom** controls on the lower right of the Root Locus Design GUI to zoom in on a region of the locus, or zoom out to show the entire locus.

The **Zoom** controls are shown below.



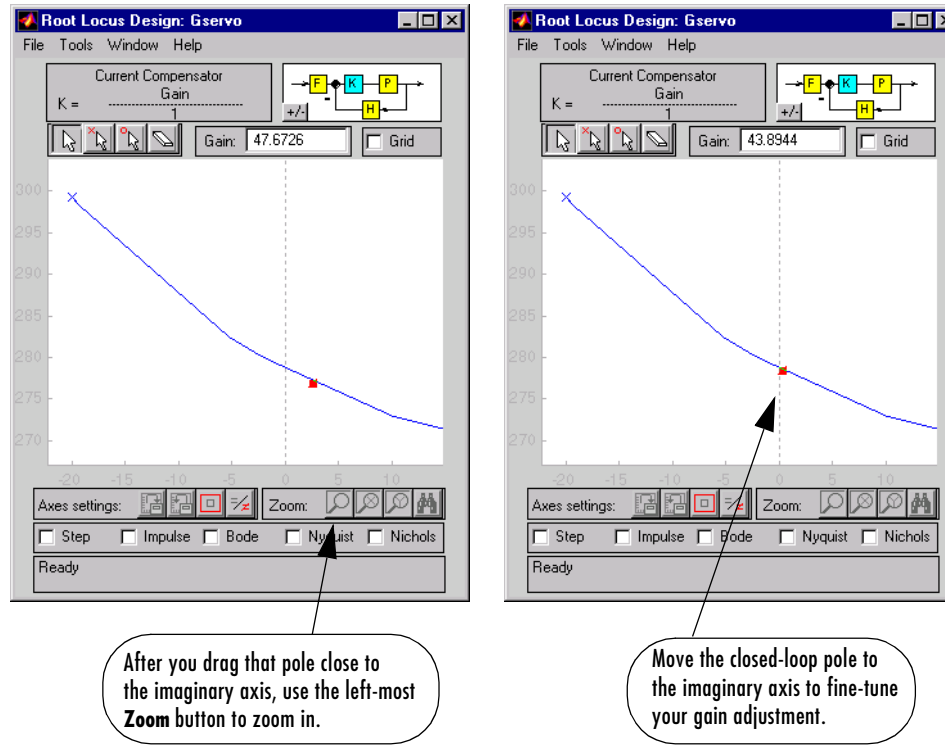
Once it is selected, you can operate any of the first three **Zoom** buttons in one of two ways.

- Use your mouse to *rubberband* around the area on the plot region you want to focus on. Rubberbanding involves clicking and holding the mouse down, while dragging the mouse pointer around the region of interest on the screen.
- Click in the plot region in the vicinity on which you want to focus.

To move the closed-loop poles closer to the imaginary axis on your root locus by zooming with the rubberband method:

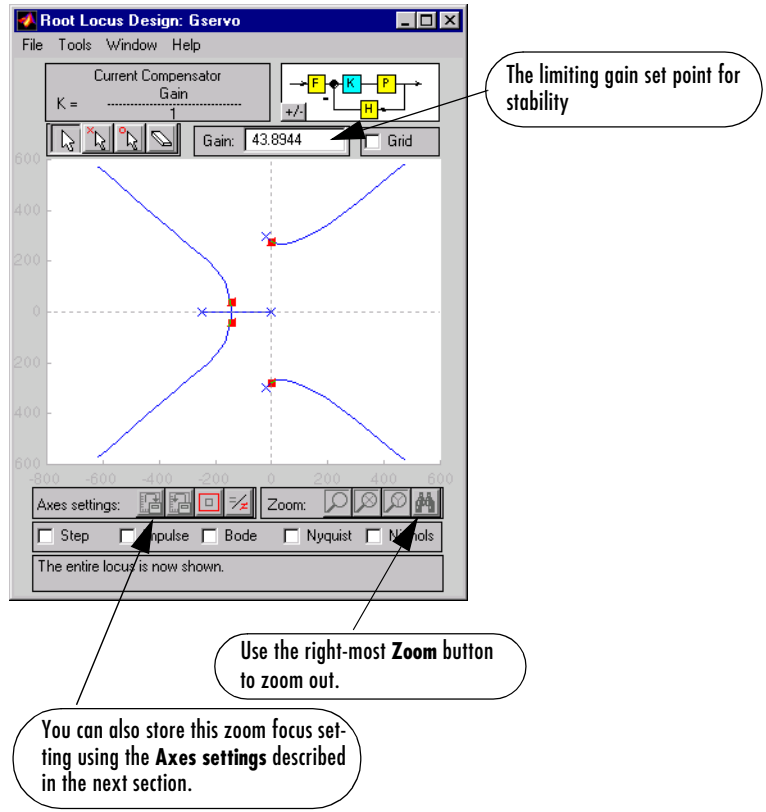
1. Zoom in X-Y by selecting the left-most **Zoom** button.
2. Use your mouse to rubberband the region of the imaginary axis near the closed-loop pole there.
3. Readjust the closed-loop pole position by grabbing it with the mouse and moving it until it rests on the imaginary axis.
4. Zoom out by clicking on the fourth **Zoom** button, (the icon is a pair of binoculars).

The use of zooming to find the limits of stability on the root locus is depicted in the following three figures.



After you drag that pole close to the imaginary axis, use the left-most **Zoom** button to zoom in.

Move the closed-loop pole to the imaginary axis to fine-tune your gain adjustment.



The critical gain value for stability is approximately 43.9.

Note: You can also test how much the gain can be increased while maintaining stability by arbitrarily applying different values of the gain in the text area next to **Gain** (pressing the **Enter** key after entering each value) until one pair of complex poles reaches the imaginary axis.

After using the zoom tools on the GUI, you may want to store one set of zoom focus settings so that you may later return to focus on that same region of the locus.

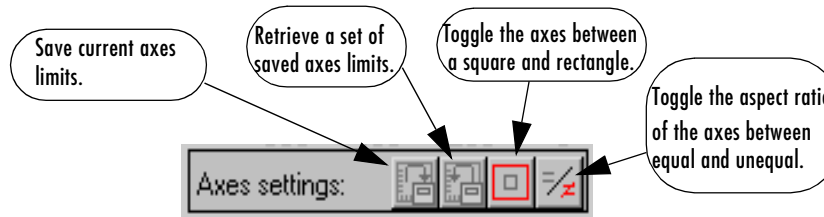
Storing and Retrieving Axes Limits

You can store and change axes limits in two ways:

- Using the **Axes settings** toolbar
- Using the **Set Axes Preferences** menu item from the **Tools** menu

For more information on square axes and/or equal axes, type `help axis` at the command line.

To use the **Axes settings** toolbar to save the current axes limits, select the left-most button shown below.

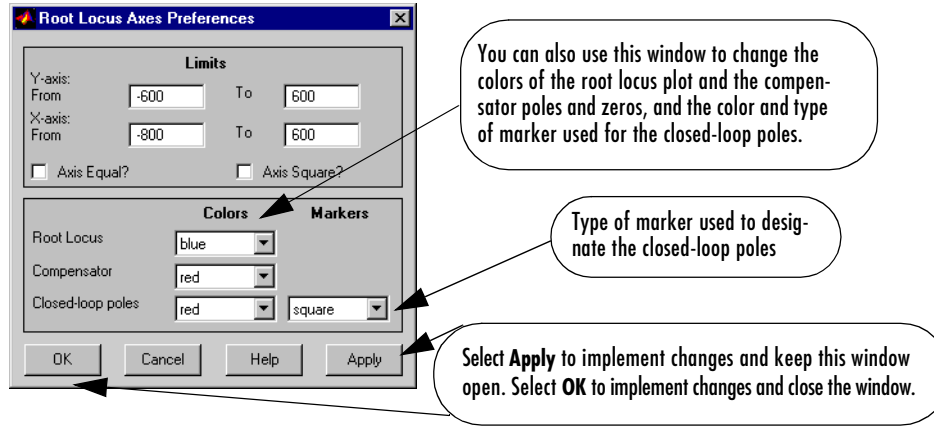


If you change the axes limits by zooming, you can always return to the saved axes limits by selecting the second **Axes settings** button.

To try out these tools:

- Select the left-most **Axes settings** button.
- Zoom in on a region of the root locus.
- Select the second **Axes settings** button.

You can also set or revise axes limits and other axes preferences in the **Root Locus Axes Preferences** window. To open this window, select **Set Axes Preferences** from the **Tools** menu.



If you have already stored axes limits using **Axes settings**, these limits appear in this window in the **Limits** field. You can reset these limits by typing in new values. To apply any changes to the entries in this window, click on **OK** after making the changes.

Displaying System Responses

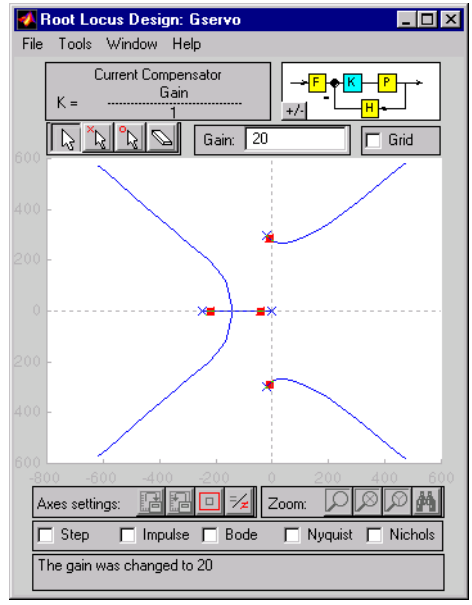
Before you design your compensator, you may want to conduct some response analysis of your closed-loop system evaluated at a fixed value of the gain.

You can access some of the system response analysis capabilities of the LTI Viewer (see Chapter 6, “The LTI Viewer”) through the checkboxes located in the lower portion of the Root Locus Design GUI. Checking one or several of these boxes opens an LTI Viewer window that displays the corresponding plots. The LTI Viewer window that opens is dynamically linked to the Root Locus Design GUI: any changes made to the system on the GUI that affect the gain set point will effect a corresponding change on the displayed LTI Viewer plots.

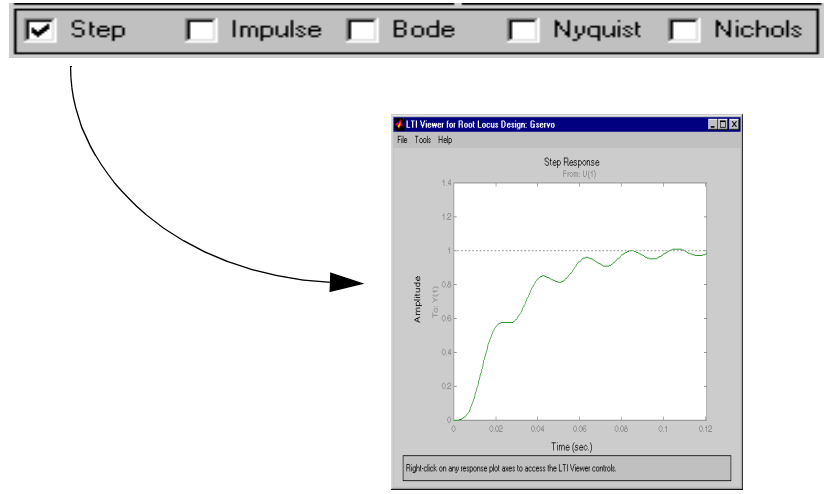
For this example, we want to know if the step response meets our design criteria. The current value for the gain set point is about 44. Clearly this value of the gain would test the limits of stability of our system. Let’s choose a reasonable value for the gain (say, 20) and look at the step response.

Change the gain to 20 by editing the text box next to **Gain**, and pressing the **Enter** key. Notice that the locations of the closed-loop poles on the root locus are recalculated for the new gain set point.

Your Root Locus Design GUI looks like this now.



Check the **Step** box, as shown below.



Once you check the **Step** box, an LTI Viewer window opens. The plot type for this LTI Viewer is the step response and this cannot be changed.

This closed-loop response does not meet the desired settling time requirement (.05 seconds or less). You can design a compensator so that you do meet the required specifications.

Designing a Compensator to Meet Specifications

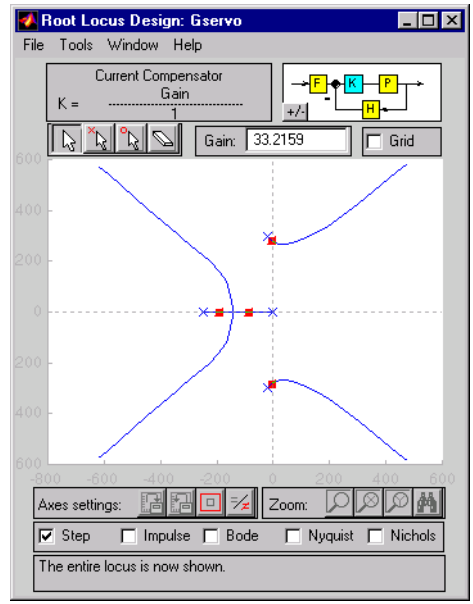
Since the current closed-loop system doesn't meet both of the required design specifications, let's first try increasing the gain to about 33. You can increase the gain in two ways:

- Edit the **Gain** textbox and press the **Enter** key.
- Move the closed loop set point around with the mouse, while increasing the gain.

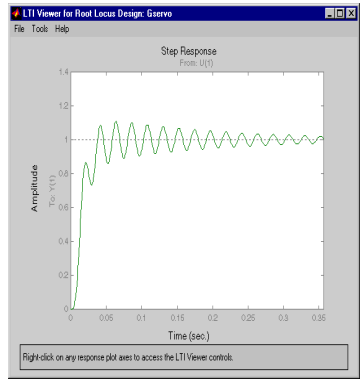
To increase the gain to about 33 using your mouse:

1. Hold your mouse button down as you click on any of the red squares.
2. Drag the square in the direction of increasing gain, without allowing any of the closed-loop poles to enter the right half of the complex plane. If, when you start to move the red square, you see the gain value in the **Gain** box decreasing, drag it in the opposite direction.
3. Release the mouse button when the gain is near 33.

Your GUI looks like this.



The step response plot on the dynamically linked LTI Viewer automatically updates when you release the mouse button.



As you may have noticed, the response time decreases with increasing gain, while the overshoot increases. Here we no longer meet the overshoot requirement. Since this gain is already relatively large, it's likely that we will not be able to meet both design requirements using only a gain for the compensator. This conjecture is supported when you specify the design region boundaries on the root locus for these design requirements. We do this in the next subsection.

Specifying Design Region Boundaries on the Root Locus

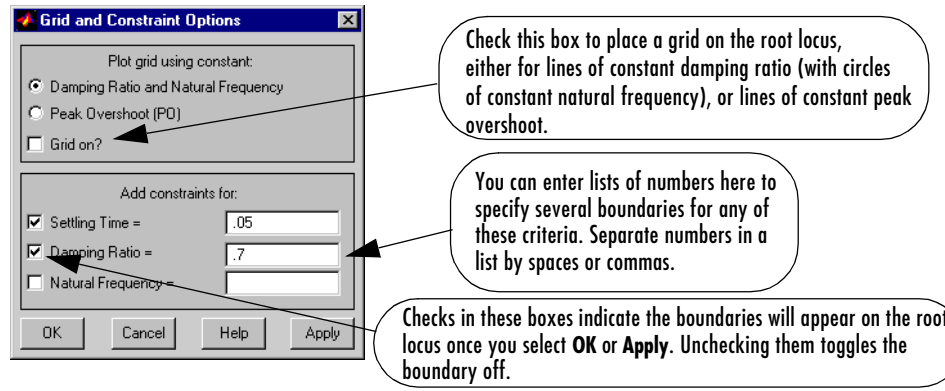
If, as in our example, your design criteria are specified in terms of step response characteristics, you may want to use the grid and boundary constraint options that are accessible from the **Add Grid/Boundary** menu item in the **Tools** menu on the Root Locus Design GUI. These options allow you to use second order system design criteria to inscribe the boundaries of design region directly on the root locus plot, or apply a grid to the plot.

Note: The boundaries you apply to the Root Locus Design GUI based on LTI system design criteria (settling time, damping ratio, natural frequency, and peak overshoot) are computed relative to second-order systems only. Therefore, for higher order systems, these boundaries provide approximations to the design region.

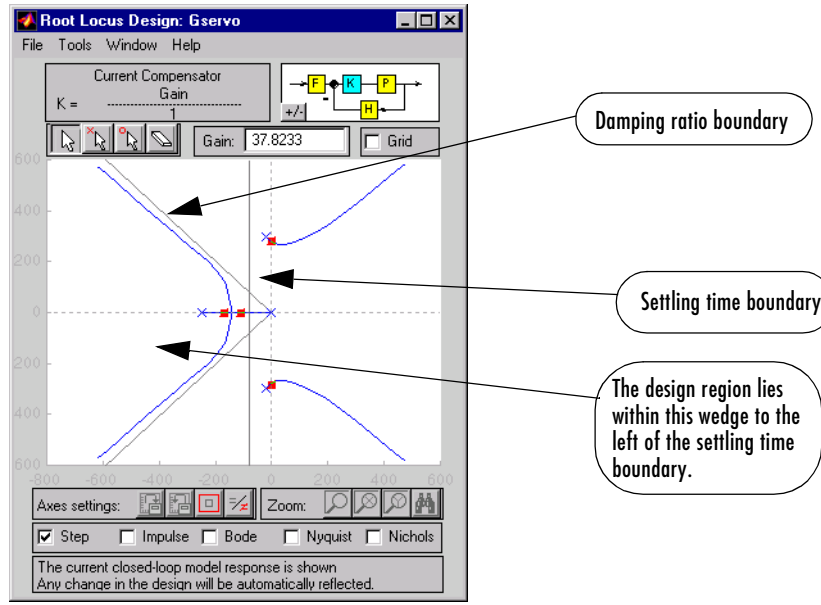
Let's place approximate design region boundaries on our root locus plot based on our design specifications. To do so, select the **Add Grid/Boundary** menu item in the **Tools** menu.

Our design specifications require that the (2 percent) settling time be less than .05 seconds, and the maximum overshoot less than 5 percent. For second-order systems, the overshoot requirement can be translated directly as a requirement on the damping ratio of about .7 (see [2]).

After you enter these values in the appropriate text fields for our specifications, your **Grid and Constraint Options** window looks like this.



After you press **OK**, the Root Locus Design GUI calculates and displays the specified boundaries.



Not all four branches of the root locus are within the design region. Let's try adding poles and zeros to our compensator to see if we can meet the design criteria.

Placing Compensator Poles and Zeros: General Information

There are three types of parameters specifying the compensator:

- Poles
- Zeros
- Gain

Once you have the gain specified, you can add poles or zeros to the compensator. You can add poles and zeros to the compensator (or remove them) in two ways:

- Use buttons on the root locus toolbar section of the GUI for pole/zero placement.
- Use the **Edit Compensator** menu item on the **Tools** menu.

The root locus toolbar is convenient for *graphically* placing compensator poles and zeros so that you meet the design specifications for a given gain set point. This method may require a certain amount of trial and error.

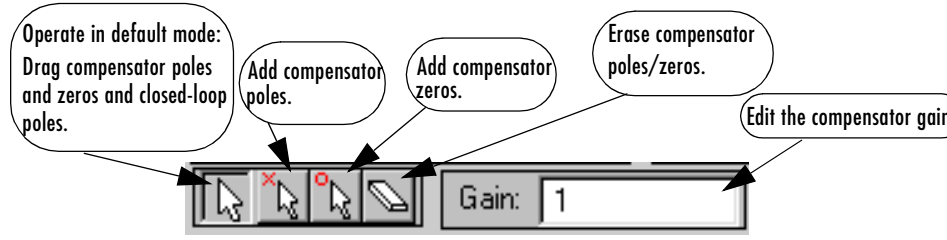
You can use the **Edit Compensator** menu item to:

- Fine-tune compensator parameter values for design implementation.
- Revise or implement an existing compensator design.

For this example we first use the root locus toolbar to place compensator poles and zeros on the root locus, and then use the **Edit Compensator** menu item to set the compensator pole and zero locations for a specific compensator solution.

Placing Compensator Poles and Zeros Using the Root Locus Toolbar

The root locus toolbar is located on the left side of the GUI, above the plot region. The figure below describes the root locus toolbar buttons.



The default mode for the toolbar is the *drag* mode. In this mode, you can:

- Click on a specific location on the root locus to place a closed-loop pole there (and consequently reassign the gain set point).
- Drag any of the closed-loop poles along its branch of the root locus (also reassigning the gain set point).
- Drag any of the compensator poles or zeros around the complex plane to change the compensator.

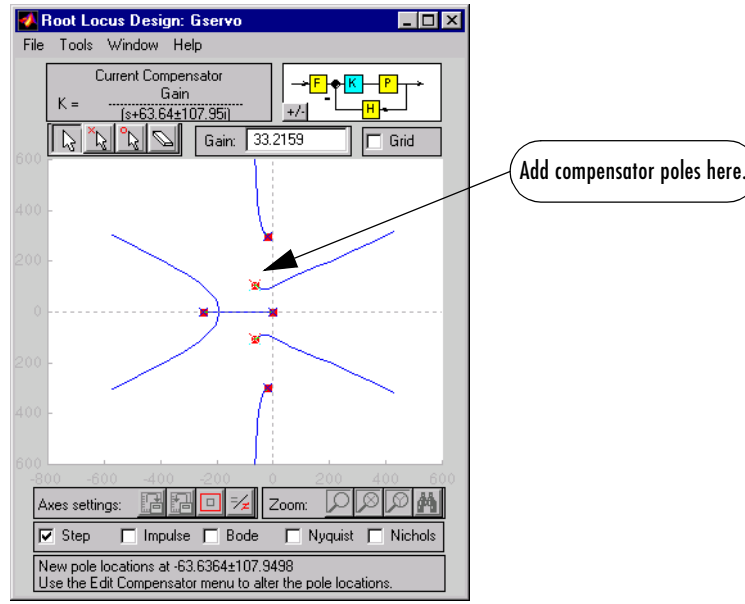
To add a complex conjugate compensator pole pair on the root locus plot:

1. Select the *add pole* button (the second button in the root locus toolbar).
2. Click on the plot region where you would like to add one of the complex poles.

Some of the features of using the root locus toolbar to add a pole are:

- While the *add pole* button is depressed, the cursor changes to an arrow with an **x** at its tip whenever it is over the plot region. This indicates the toolbar is in the "add pole" mode.
- After you add the poles, the *add pole* button pops back up and the default *drag* mode is restored. The added compensator poles appear in a different color. The default color is red.
- The LTI Viewer response plots change as soon as the pole is added.
- The text displayed in the **Current Compensator** region of the GUI now displays the new pair of poles.

Try placing a pair of complex poles just above the right-most real closed-loop pole. The resulting root locus plot looks like this.

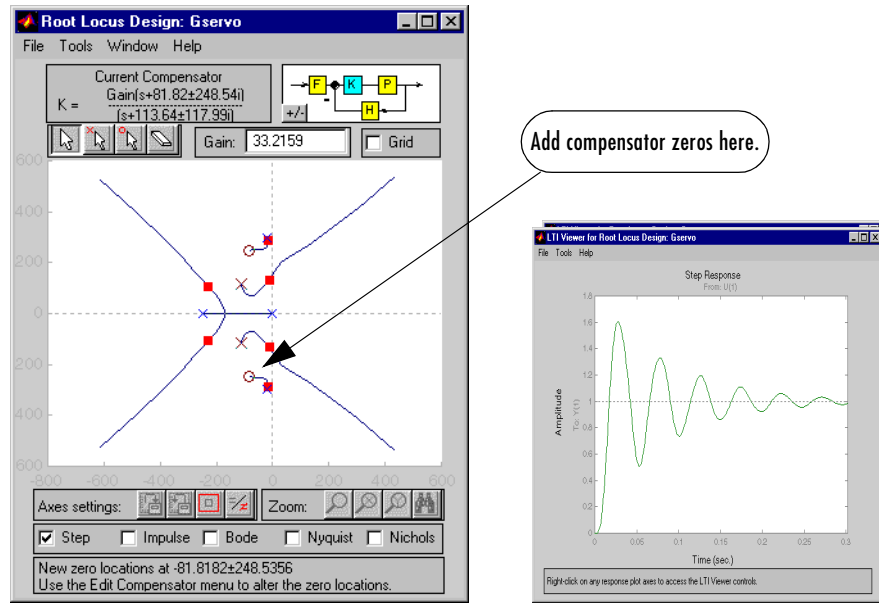


Similarly, to add a pair of complex zeros to the compensator:

1. Select the *add zero* button from the root locus toolbar (third button from the left).

2. Click on the plot region where you would like to add one of the complex zeros.

Try adding a pair of complex zeros just to the left of and a little bit below the complex closed-loop poles closest to the imaginary axis. The resulting root locus and step response plots are as follows.



If your step response is unstable, lower the gain. In this example, the resulting step response is stable, but it still doesn't meet the design criteria.

As you can see, the compensator design process can involve some trial and error. You can try dragging the compensator poles, compensator zeros, or the closed-loop poles around the root locus until you meet the design criteria. Alternatively, you can edit the compensator using the solution provided in the next section.

Note: You can follow the same procedure to add a single real pole or zero to the compensator by clicking on the real axis.

Editing Compensator Pole and Zero Locations

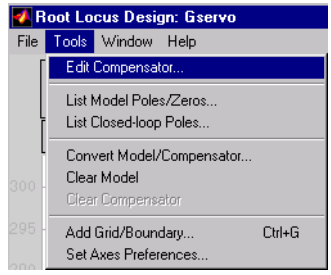
In this section we use the **Edit Compensator** window to design a compensator with the following characteristics:

- Gain: 9.7
- Poles: $-110 \pm 140i$
- Zeros: $-70 \pm 270i$

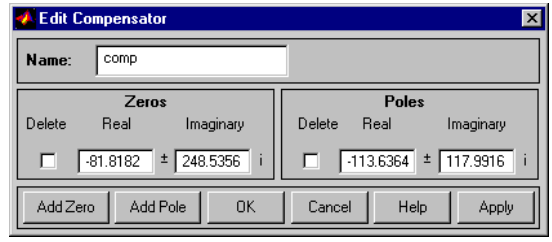
You can access the **Edit Compensator** window in one of three ways:

- Double-click on any of the **Current Compensator** text.
- Click on the (blue) compensator block in the feedback structure on the GUI.
- Select **Edit Compensator** from the **Tools** menu.

This figure shows how to choose this menu item on the GUI.



With either method, you open the **Edit Compensator** window shown in the figure below.

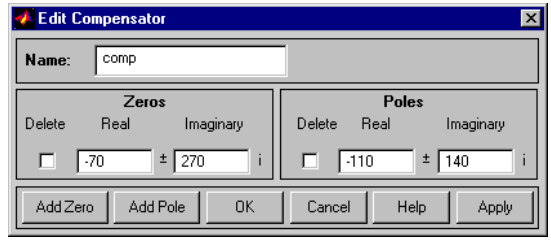


You can use the **Edit Compensator** window to:

- Edit the locations of compensator poles and zeros.
- Add compensator poles and zeros.
- Delete compensator poles and zeros.
- Change the name of the compensator (This name is used when exporting the compensator).

For this example, edit the poles and zeros to be at $-110 \pm 140i$, and $-70 \pm 270i$, respectively.

Your GUI looks like this.



If, in addition, you want to add poles to the compensator:

1 Click on **Add Pole**. A new editable text field appears.

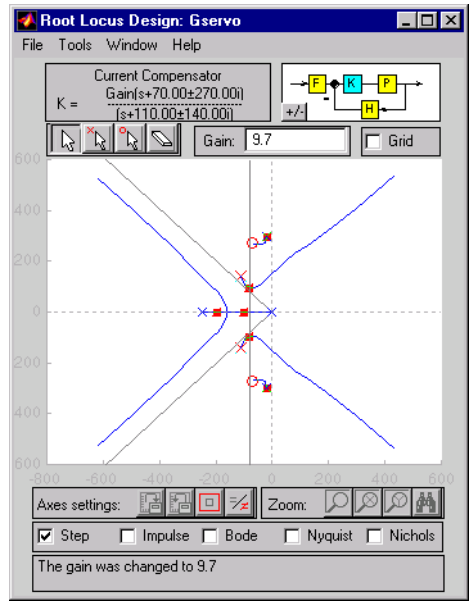
2 Enter the new pole location in the text field.

The procedure is the same for adding zeros, only use the **Add Zero** button in place of the **Add Pole** button. To erase any poles or zeros, select the **Delete** checkbox.

Before closing this window by selecting **OK**, you can also optionally change the name of the compensator.

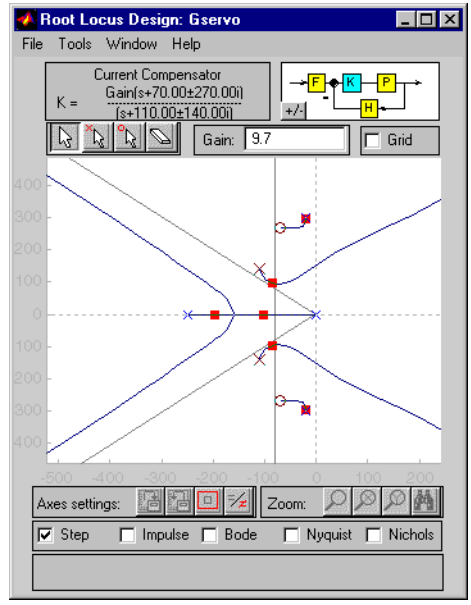
Note: You can also erase poles or zeros on the root locus using the erase button on the root locus toolbar. See “Erasing Compensator Poles and Zeros” on page 8-41

With the gain set point at 9.7, your root locus looks as follows.

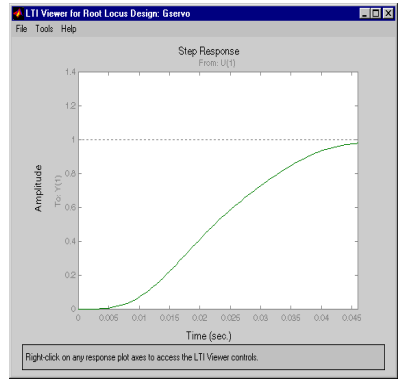


Note: Whenever the numerator or denominator are too long for the **Current Compensator** field, a default **numK** or **denK** is displayed. To display the actual numerator or denominator, resize the Root Locus Design GUI.

To check where the closed-loop roots are with respect to the boundary of the design region, let's zoom in a bit.



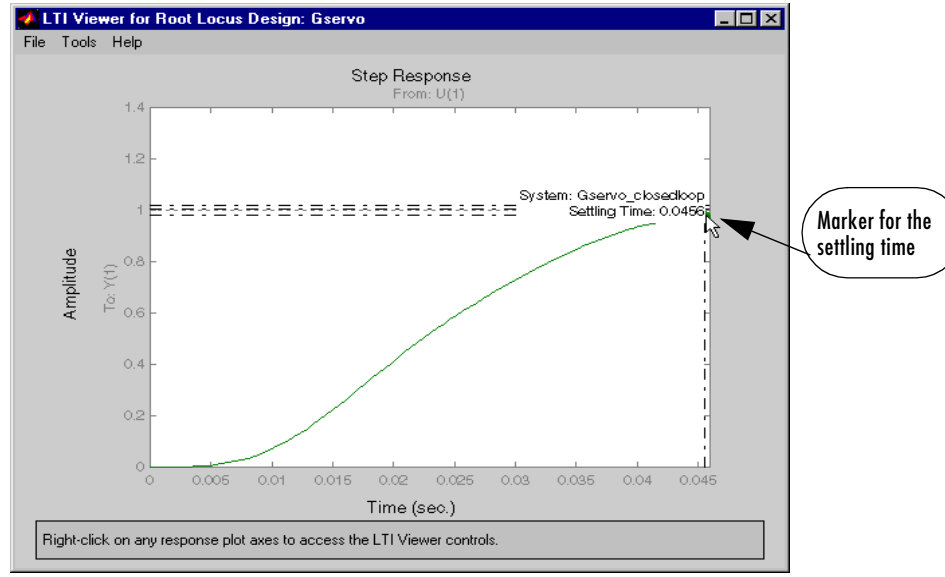
We don't have all of the closed-loop poles in the design region, but let's see if we meet the step response specifications. The updated LTI Viewer plot of the step response looks like this.



The step response looks good. However, to be certain that you've met your design specifications, you can check the settling time and peak response characteristics from the right-click menu. To do this:

1. Right-click anywhere in the plot region.
2. Select **Settling Time** from the **Characteristics** menu.
3. Set your mouse pointer to the settling time marker and click to display the value.

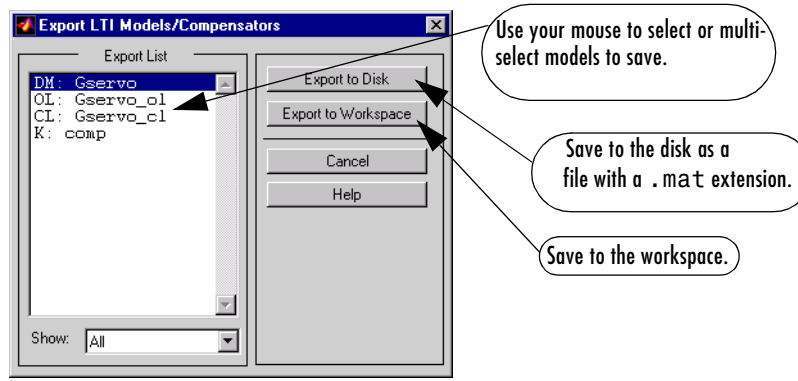
The LTI Viewer will look like this.



As you can see, the settling time is less than .05 seconds, and the overshoot is less than 5 percent. You've met the design specifications and you're done. You didn't need your closed-loop poles to be entirely in the design region, because you actually have a sixth-order system with some fast dynamics, as opposed to having only a second-order system.

Saving the Compensator and Models

Now that you have successfully designed your compensator, you may want to save your design parameters for future implementation. You can do this by selecting **Export** from the **File** menu on the Root Locus Design GUI. The window shown below opens.



The variable listed in the **Export List** on the **Export LTI Models/ Compensators** GUI are either previously named by you (on the **List Model Poles/Zeros** or the **Edit Compensator** windows) or have default names. They are coded as follows:

- `DM`: The design model
- `OL`: The open-loop model
- `CL`: The closed-loop model
- `K`: The compensator

To export your compensator to the workspace:

1. Select the compensator in the **Export List**.
2. Click on the **Export to Workspace** button.

The **Export LTI Model/Compensators** window is closed when you click on one of the export buttons. If you go to the MATLAB prompt and type

```
who
```

the compensator is now in the workspace, in the variable named `comp`.

Then type

```
comp
```

to see that this variable is stored in `zpk` format.

Additional Root Locus Design GUI Features

This section describes several features of the Root Locus Design GUI not covered in the servomechanism example. These are listed as follows:

- Specifying design models: general concepts:
  - Creating models manually within the GUI
  - Designating the model source
- Getting help with the Root Locus Design GUI
- Erasing compensator poles and zeros
- Listing poles and zeros
- Printing the root locus
- Drawing a Simulink diagram of the closed-loop model
- Converting between continuous and discrete models
- Clearing data

Specifying Design Models: General Concepts

In this section we provide general concepts for specifying the design model by using one or both of the following methods:

- Creating models manually within the GUI
- Designating the model source:
  - The MATLAB workspace
  - A MAT-file
  - An open or saved Simulink model

Creating Models Manually Within the GUI

You can create models for root locus analysis *manually* in the **Import LTI Design Model** window using `tf`, `ss`, or `zpk`. To do so, just edit the text boxes next to **P**, **F**, or **H** using any of these MATLAB expressions. You can also use a scalar number to specify **P**, **F**, or **H**.

Designating the Model Source

The source of your design model data is indicated in the **Import From** field shown below.

If you are loading your models from MAT-files or Simulink models saved on your disk, you are prompted to enter the name of the file in the editable text box below the **Import From** radio buttons. You can also select the **Browse** button to search for and select the file. The **Simulink** radio button also allows you to load SISO LTI blocks from an open Simulink model, by entering the name of the model in the text box.

When you select the model source, its contents are shown in the model listbox located in the central portion of the **Import LTI Design Model** window. The model listbox contains the following data:

- For **Workspace** or **MAT-file**: All LTI models in the workspace, or in the selected MAT-file
- For **Simulink**: All the LTI blocks in the selected Simulink diagram

---

Note: If you want to load models saved in more than one MAT-file, load these into the MATLAB workspace *before* selecting **Import Model** on the Root Locus Design GUI.

Getting Help with the Root Locus Design GUI

You can obtain instructions on how to use the Root Locus Design GUI either using the **Help** menu, from the status bar, or by using the tooltips.

Using the Help Menu

Click on the **Help** menu and you find that it contains five menu items:

- **Main Help**
- **Edit Compensator**
- **Convert Model**
- **Add Grid/Boundary**
- **Set Axis Preferences**

The first menu item, **Main Help**, opens a help window that describes how to use the controls located on the Root Locus Design GUI. The remaining menu items provide additional information on the features you can access from the **Tools** menu.

Using the Status Bar for Help

The status bar at the bottom of the Root Locus Design GUI:

- Provides you with information, hints, and error messages as you proceed through your design.
- Lets you know if you have tried to undertake an action the GUI is not capable of, or if a GUI operation you have performed has successfully been completed.
- Provides information about the location of newly placed compensator poles and zeros, as well as the damping ratio, natural frequency, and location of poles and zeros as you drag them.

Tooltips

You can obtain simple reminders (tooltips) on how to use the Root Locus Design GUI by moving your mouse and putting the cursor over one of these features. For example, if you put the cursor over the **Step** checkbox, its tooltip, **Closed-loop step response**, appears just below the button. When a tooltip is available, a small bubble containing information about the feature you selected appears. This bubble disappears when you move the mouse again.

Erasing Compensator Poles and Zeros

You can delete any compensator poles and zeros in one of two ways:

- Check the associated **Delete** box on the **Edit Compensator...** window obtained from the **Tools** menu.
- Click on the erase button (fourth from left on the root locus toolbar) and click on the pole or zero to delete.

After using the root locus toolbar erase button to delete a pole or zero, the following occurs:

- The erase button pops up and the default *drag* mode is restored.
- The zero or pole you erased (and, if applicable, its conjugate) is removed from the root locus and from the **Current Compensator** text.
- The root locus plot and any linked LTI Viewer response plots are recalculated.

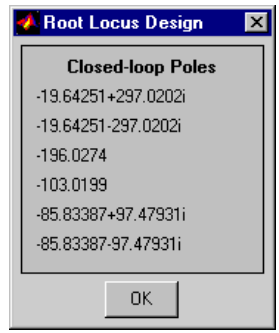
Listing Poles and Zeros

At any point when the Root Locus Design GUI is open, you can view:

- The closed-loop pole locations associated with the current **Gain** setting
- The poles, zeros, and gain of each design model component

To view the current closed-loop poles:

- Select **List Closed-Loop Poles** from the **Tools** menu. If you select this menu, a window listing the closed-loop poles associated with the current gain set point opens.



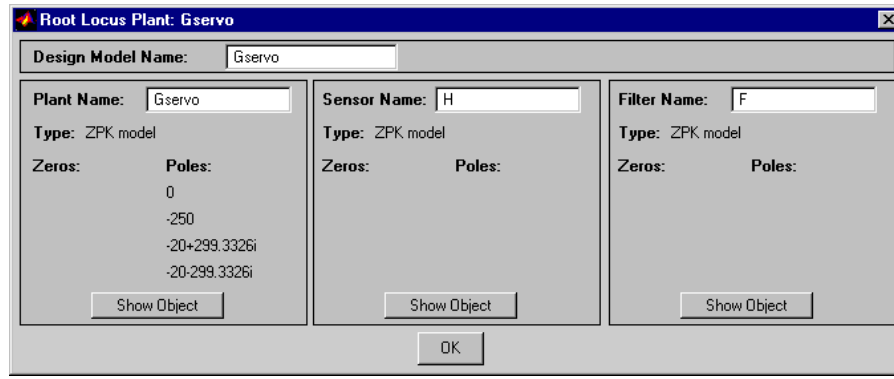
- Click on the **OK** button to close the window.

Note: You cannot edit the closed-loop pole locations listed in this window.

To view the design model poles and zeros, you can either:

- Select the **List Model Poles/Zeros** item from the **Tools** menu.
- Click on any of the design model blocks in the **Feedback Structure**. These are the yellow **F**, **P**, and **H** blocks.

The following window opens.

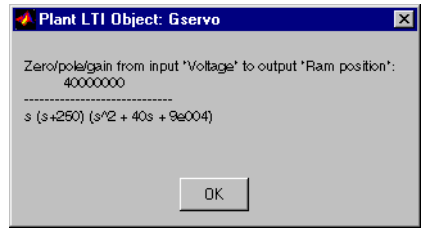


For each component of the design model, this window tells you:

- The component name
- The type of LTI model that the component is (that is, `ss`, `zpk`, or `tf`)
- The component's pole and zero locations
- The associated numerator and denominator of the model transfer function

You can edit any of the model names in the textboxes provided. Selecting **OK** closes this window and implements the changes you made to design model names.

Before closing this window, you can also use the **Show Object** buttons in this window to provide a MATLAB display of the associated LTI model. The following is the information provided for the plant.



Select **OK** to close this window.

Note: The names you enter in this window are only used when you generate a Simulink diagram of the closed-loop structure.

To list the denominator, (respectively, the numerator, including the gain) associated with a given component of the design model, in the **List Model Poles/Zeros** window, click on **Poles**, (respectively, **Zeros**) of that component. When you do, a window providing the selected information opens.

The **Show Object** button in this window opens a dialog that displays the LTI model exactly as it would be shown at the MATLAB prompt.

Printing the Root Locus

You can print the locus the GUI is displaying by:

1. Selecting **Print Locus** from the **File** menu
2. Clicking the **OK** button on the printer dialog that appears

When you select **Print Locus** a new MATLAB figure window containing the locus appears. This is the figure that is printed when you press **OK**.

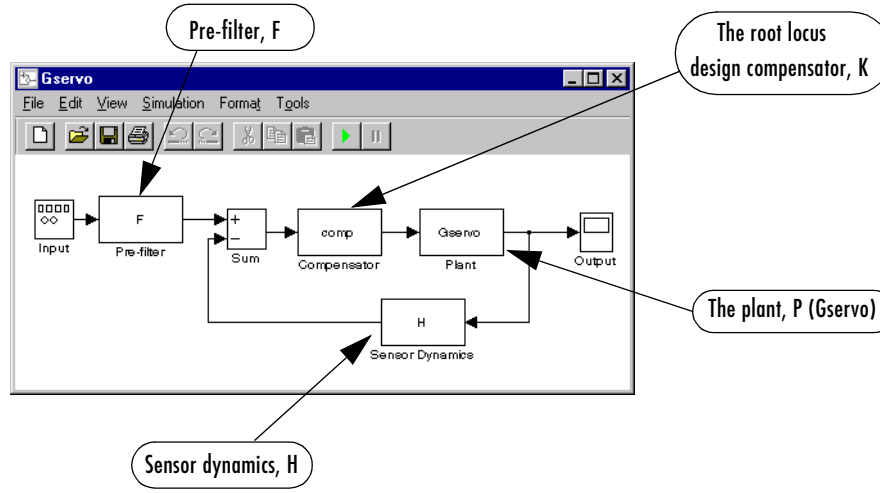
Note: You can also generate this root locus MATLAB figure window by selecting **Send Locus to Figure** from the **File** menu. This figure remains open until you close it.

Drawing a Simulink Diagram

If you have Simulink, you can use the Root Locus Design GUI to automatically draw a Simulink diagram of the closed-loop structure. To do this:

1. Select **Draw Simulink Diagram** from the **File** menu.
2. Answer **Yes** to the subsequent dialog box question to confirm that you want the design model data to be stored in the workspace using its current names, or answer **No** to this question, if you want to abort drawing the diagram.

If you answered **Yes**, a Simulink diagram such as this appears.



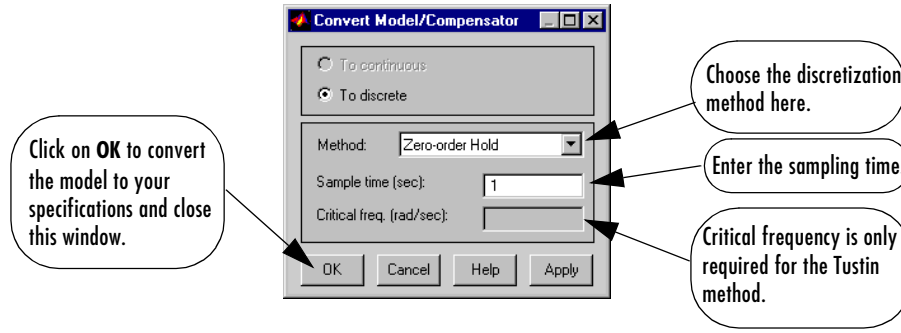
The Simulink diagram is linked to the workspace, not to the Root Locus Design GUI. If you change your compensator design in the Root Locus Design GUI, you must export it to the workspace in order to reflect these changes in the Simulink diagram.

Converting Between Continuous and Discrete Models

The Root Locus Design GUI can be applied to either discrete or continuous-time systems, and you can convert between the two types of systems at any time during the design process. However, the design model and compensator must both be either continuous or discrete.

To obtain the discrete equivalent of your current compensator design, select **Convert Model/Compensator** from the **Tools** menu.

The **Convert Model/Compensator** window, shown below, opens.



Once you select a discretization method, sampling time, and critical frequency (if required), click on the **OK** button. The **Convert Model/Compensator** window closes and the discretization is performed. At this point:

- The design model and compensator are discretized.
- The **Current Compensator** text displays a system using the complex variable 'z'.
- The root locus for the discrete-time system representing the converted continuous-time system is plotted.
- Any linked LTI Viewer response plots are updated.
- The grids and boundaries are converted into the discrete plane.

You can also use the **Convert Model/Compensator** GUI to retrieve the continuous system, or to re-sample the discrete system using a different sampling time. These options are selected from the radio buttons at the top of this GUI.

Clearing Data

You can clear the design model and/or compensator from the Root Locus Design GUI using options available in the **Tools** menu:

- **Clear Model**: removes the plant, pre-filter, and sensor dynamics and replaces them all with unity gains.
- **Clear Compensator**: removes the compensator poles and zeros, and resets the gain to one.

To test these features, select **Clear Model**. Notice that your compensator is not altered, and its poles and zeros remain plotted in the root locus plot region of the GUI.

Now, select **Clear Compensator**. The Root Locus Design GUI returns to the state it was in when you first opened it and you are ready to start a new design.

References

[1] Clark, R.N., *Control System Dynamics*, Cambridge University Press, 1996.

[2] Dorf, R.C. and R.H. Bishop, *Modern Control Systems*, 8th Edition, Addison Wesley, 1997.

9

Design Case Studies

This chapter contains four detailed case studies of control system design and analysis using the Control System Toolbox.

Case Study 1	A yaw damper for a 747 jet transport aircraft that illustrates the classical design process.
Case Study 2	A hard-disk read/write head controller that illustrates classical digital controller design.
Case Study 3	LQG regulation of the beam thickness in a steel rolling mill.
Case Study 4	Kalman filtering that illustrates both steady-state and time-varying Kalman filter design and simulation.

Demonstration files for these case studies are available as `jetdemo.m`, `diskdemo.m`, `milldemo.m`, and `kalmdemo.m`. To run any of these demonstrations, type the corresponding name at the command line, for example,

```
jetdemo
```

Yaw Damper for a 747 Jet Transport

This case study demonstrates the tools for classical control design by stepping through the design of a yaw damper for a 747 jet transport aircraft.

The jet model during cruise flight at MACH = 0.8 and H = 40,000 ft. is

```
A = [-0.0558   -0.9968    0.0802    0.0415
      0.5980   -0.1150   -0.0318         0
     -3.0500    0.3880   -0.4650         0
           0    0.0805    1.0000         0];

B = [ 0.0729    0.0000
     -4.7500    0.00775
      1.5300    0.1430
           0         0];

C = [0     1     0     0
     0     0     0     1];

D = [0     0
     0     0];
```

The following commands specify this state-space model as an LTI object and attach names to the states, inputs, and outputs.

```
states = {'beta' 'yaw' 'roll' 'phi'};
inputs = {'rudder' 'aileron'};
output = {'yaw' 'bank angle'};

sys = ss(A,B,C,D,'statename',states,...
                 'inputname',inputs,...
                 'outputname',outputs);
```

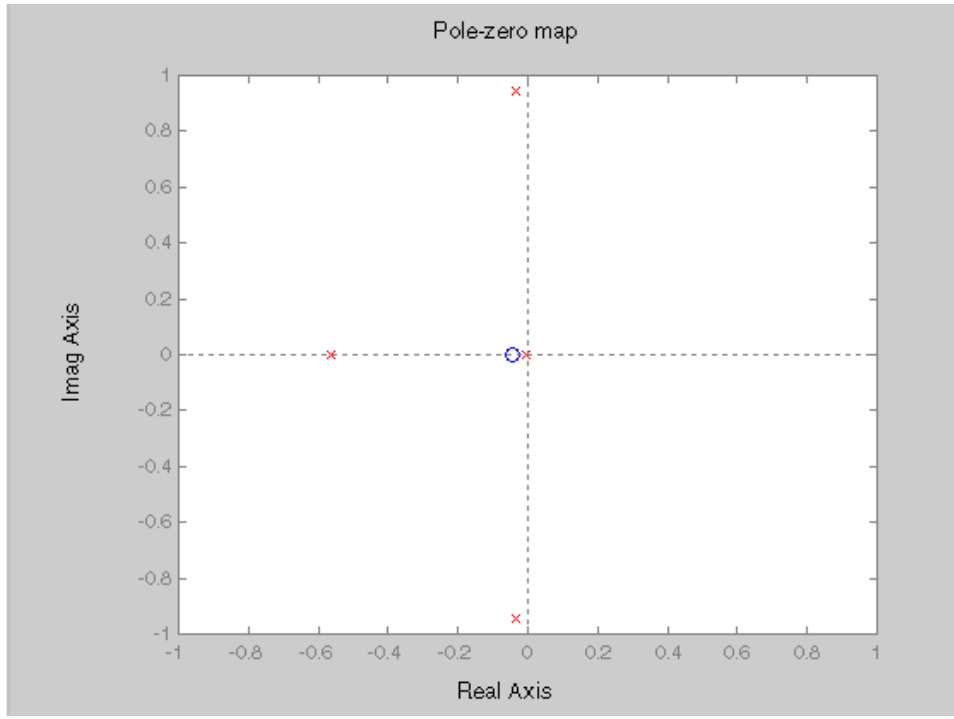
You can display the LTI model sys by typing sys. MATLAB responds with

```
a =
                beta          yaw         roll          phi
   beta      -0.0558      -0.9968       0.0802       0.0415
    yaw        0.598       -0.115      -0.0318            0
   roll        -3.05        0.388       -0.465            0
    phi            0       0.0805            1            0

b =
                rudder      aileron
   beta       0.00729            0
    yaw        -0.475      0.00775
   roll         0.153        0.143
    phi             0            0

c =
                  beta          yaw         roll          phi
    yaw              0            1            0            0
   bank angle        0            0            0            1

d =
                rudder      aileron
    yaw              0            0
   bank angle        0            0

Continuous-time model.
```

The model has two inputs and two outputs. The units are radians for `beta` (sideslip angle) and `phi` (bank angle) and radians/sec for `yaw` (yaw rate) and `roll` (roll rate). The rudder and aileron deflections are in radians as well.

Compute the open-loop eigenvalues and plot them in the *s*-plane.

```
damp(sys)

Eigenvalue                    Damping        Freq. (rad/s)

 -7.28e-003                    1.00e+000      7.28e-003
 -5.63e-001                    1.00e+000      5.63e-001
 -3.29e-002 + 9.47e-001i       3.48e-002      9.47e-001
 -3.29e-002 - 9.47e-001i       3.48e-002      9.47e-001

pzmap(sys)
```



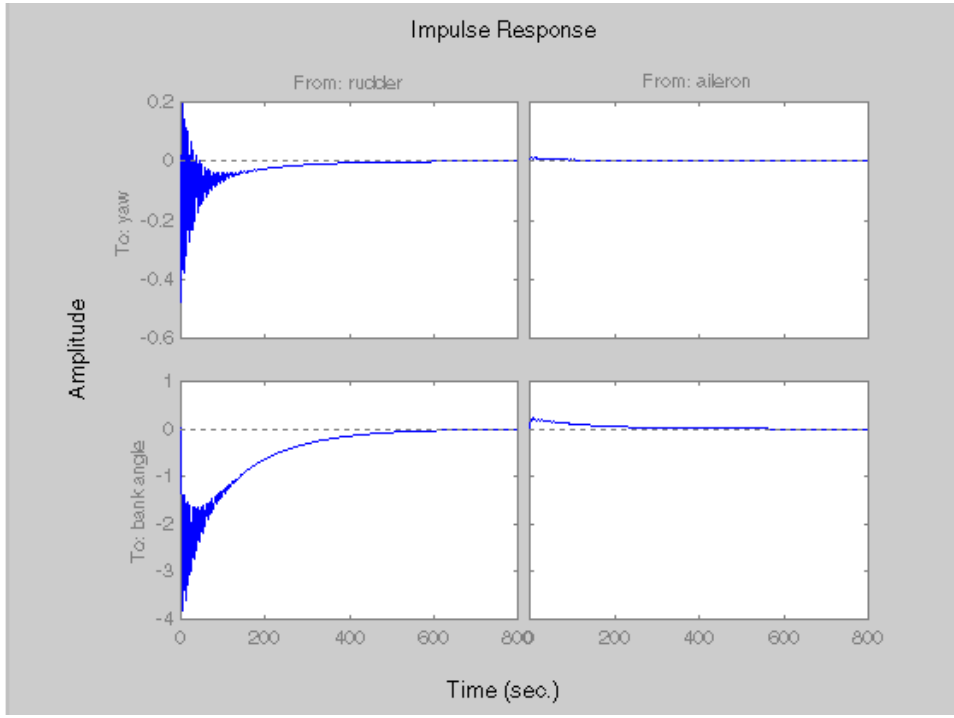
This model has one pair of lightly damped poles. They correspond to what is called the “Dutch roll mode.”

Suppose you want to design a compensator that increases the damping of these poles, so that the resulting complex poles have a damping ratio $\zeta > 0.35$ with natural frequency $\omega_n < 1$ rad/sec. You can do this using the Control System toolbox analysis tools.

Open-Loop Analysis

First perform some open-loop analysis to determine possible control strategies. Start with the time response (you could use `step` or `impulse` here).

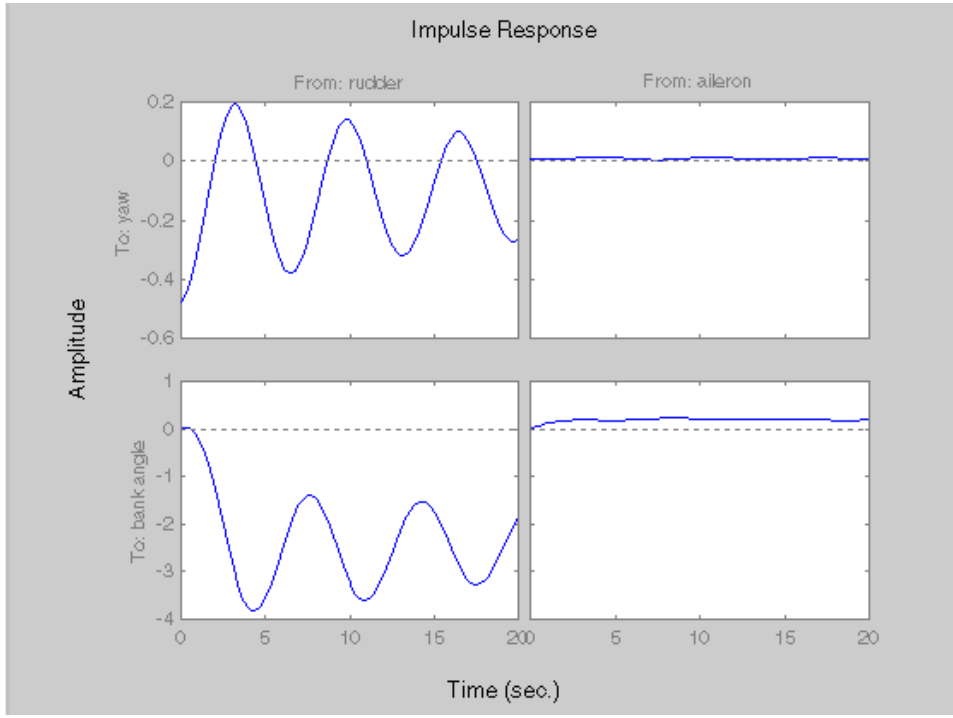
```
impulse(sys)
```



The impulse response confirms that the system is lightly damped. But the time frame is much too long because the passengers and the pilot are more

concerned about the behavior during the first few seconds rather than the first few minutes. Next look at the response over a smaller time frame of 20 seconds.

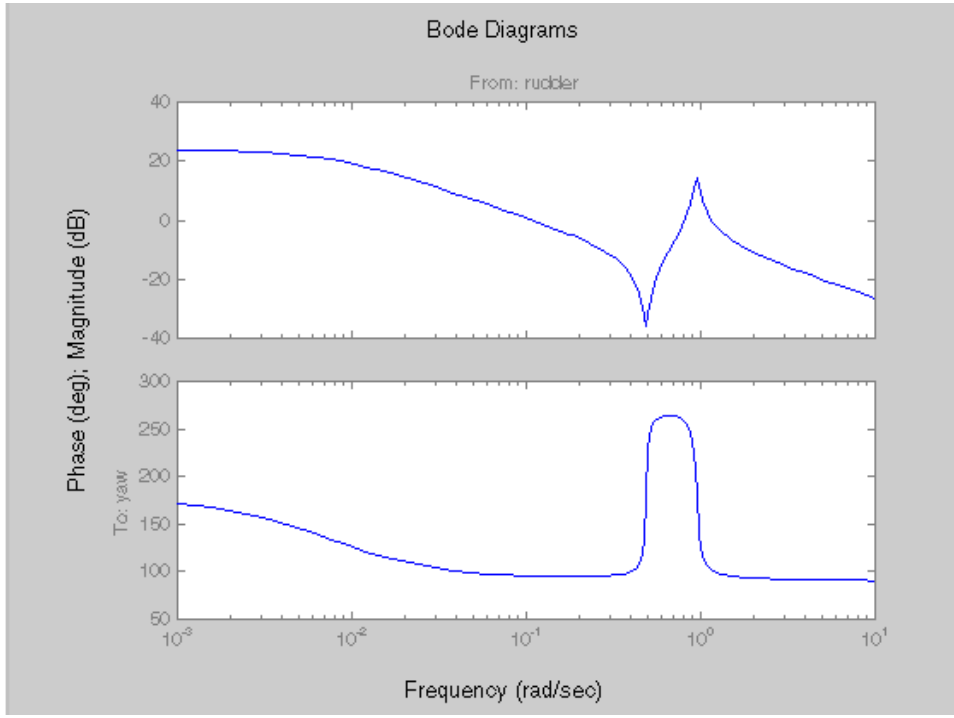
```
impulse(sys,20)
```



Look at the plot from aileron (input 2) to bank angle (output 2). The aircraft is oscillating around a nonzero bank angle. Thus, the aircraft is turning in response to an aileron impulse. This behavior will prove important later in this case study.

Typically, yaw dampers are designed using the yaw rate as sensed output and the rudder as control input. Look at the corresponding frequency response (input 1 to output 1).

```
bode(sys(1,1))
```



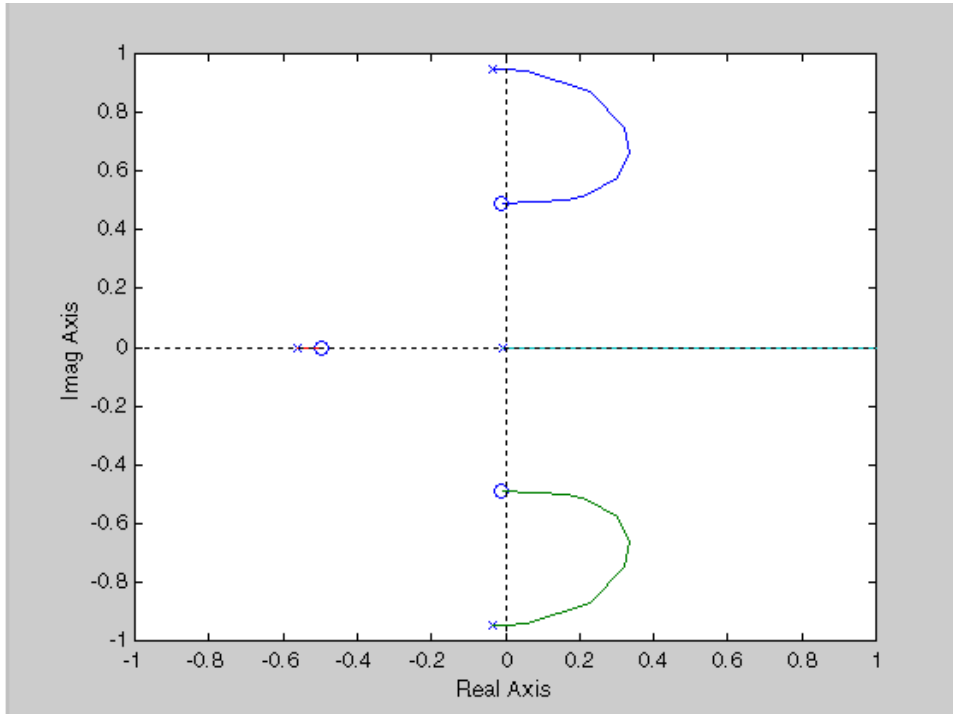
From this Bode diagram, you can see that the rudder has significant effect around the lightly damped Dutch roll mode (that is, near $\omega = 1$ rad/sec). To make the design easier, select the subsystem from rudder to yaw rate.

```
% Select system with input 1 and output 1
sys11 = sys(1,1);
```

Root Locus Design

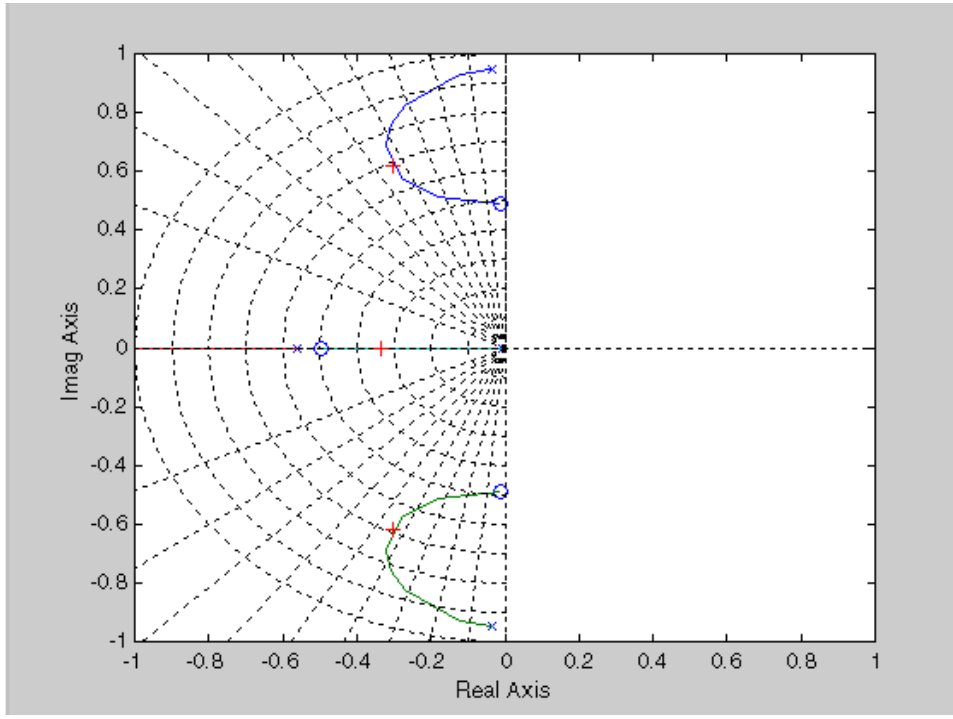
Since the simplest compensator is a static gain, first try to determine appropriate gain values using the root locus technique.

```
% Plot the root locus for the (1,1) channel
rlocus(sys11)
```



This is the root locus for negative feedback and shows that the system goes unstable almost immediately. If, instead, you use positive feedback, you may be able to keep the system stable.

```
rlocus(-sys11)
sgrid
```



This looks better. Just using simple feedback you can achieve a damping ratio of $\zeta = 0.45$. You can graphically select some pole locations and determine the corresponding gain with `rlocfind`.

```
[k,poles] = rlocfind(-sys11)
```

The '+' marks on the previous figure show a possible selection. The corresponding gain and closed-loop poles are

```
k

k =
    2.7615

damp(poles)

        Eigenvalue            Damping     Freq. (rad/s)

 -1.01e+000                  1.00e+000      1.01e+000
 -3.03e-001 + 6.18e-001i     4.41e-001      6.89e-001
 -3.03e-001 - 6.18e-001i     4.41e-001      6.89e-001
 -3.33e-001                  1.00e+000      3.33e-001
```

Next, form the closed-loop system so that you can analyze this design.

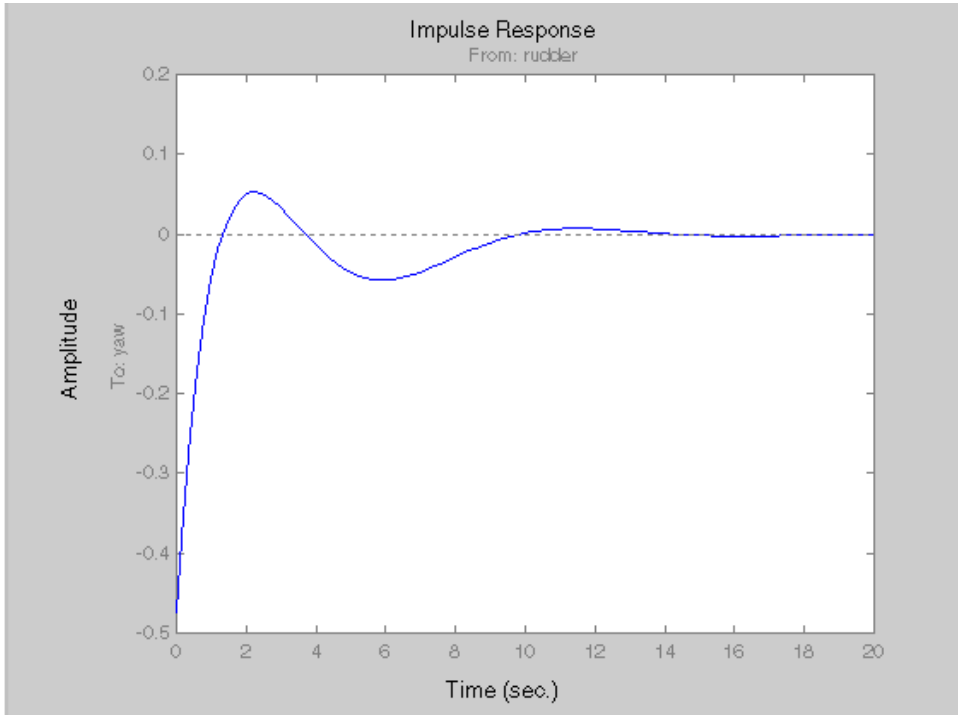
```
cl11 = feedback(sys11,-k);   % negative feedback by default
```

The closed-loop poles should match the ones chosen above (and they do).

```
damp(cl11)

        Eigenvalue            Damping     Freq. (rad/s)

 -3.33e-001                  1.00e+000      3.33e-001
 -3.03e-001 + 6.18e-001i     4.41e-001      6.89e-001
 -3.03e-001 - 6.18e-001i     4.41e-001      6.89e-001
 -1.01e+000                  1.00e+000      1.01e+000
```

Plot the closed-loop impulse response for a duration of 20 seconds.

```
impulse(cl11,20)
```



The response settles quickly and does not oscillate much.

Now close the loop on the original model and see how the response from the aileron looks. The feedback loop involves input 1 and output 1 of the plant (use

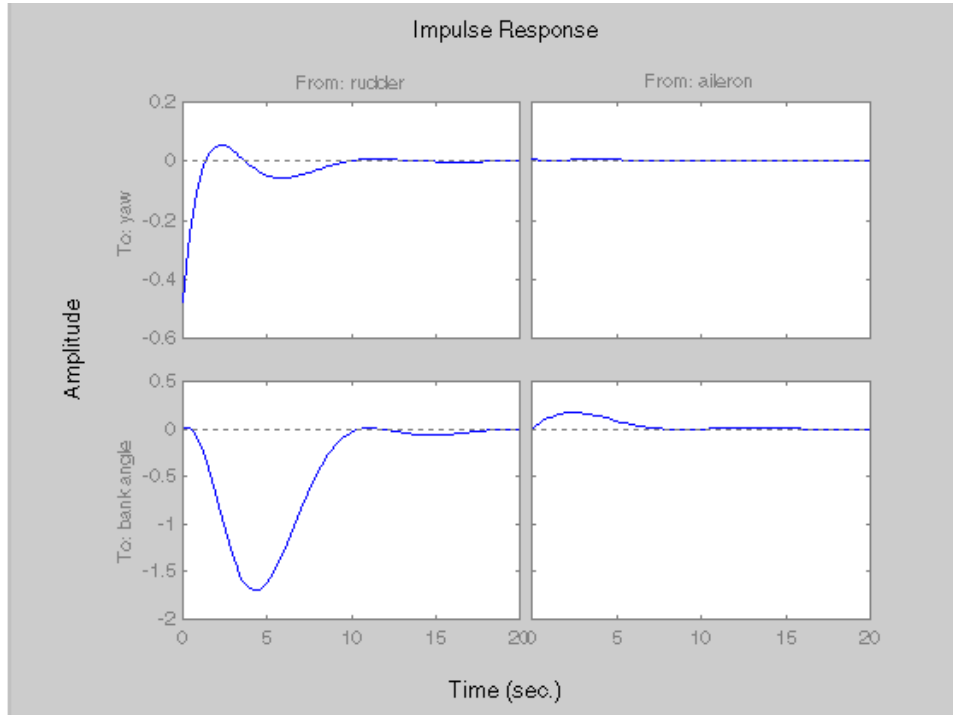
feedback with index vectors selecting this input/output pair). At the MATLAB prompt, type

```
cloop = feedback(sys,-k,1,1);
damp(cloop)    % closed-loop poles
```

```
        Eigenvalue            Damping     Freq. (rad/s)

 -3.33e-001                  1.00e+000      3.33e-001
 -3.03e-001 + 6.18e-001i     4.41e-001      6.89e-001
 -3.03e-001 - 6.18e-001i     4.41e-001      6.89e-001
 -1.01e+000                  1.00e+000      1.01e+000
```

Plot the MIMO impulse response.

```
impulse(cloop,20)
```



Look at the plot from aileron (input 2) to bank angle (output 2). When you move the aileron, the system no longer continues to bank like a normal aircraft. You have over-stabilized the spiral mode. The spiral mode is typically a very slow mode and allows the aircraft to bank and turn without constant aileron input. Pilots are used to this behavior and will not like your design if it does not allow them to fly normally. This design has moved the spiral mode so that it has a faster frequency.

Washout Filter Design

What you need to do is make sure the spiral mode does not move further into the left-half plane when you close the loop. One way flight control designers have addressed this problem is to use a washout filter $kH(s)$ where

$$H(s) = \frac{s}{s+a}$$

The washout filter places a zero at the origin, which constrains the spiral mode pole to remain near the origin. We choose $a = 0.333$ for a time constant of three seconds and use the root locus technique to select the filter gain k. First specify the fixed part $s/(s+a)$ of the washout by

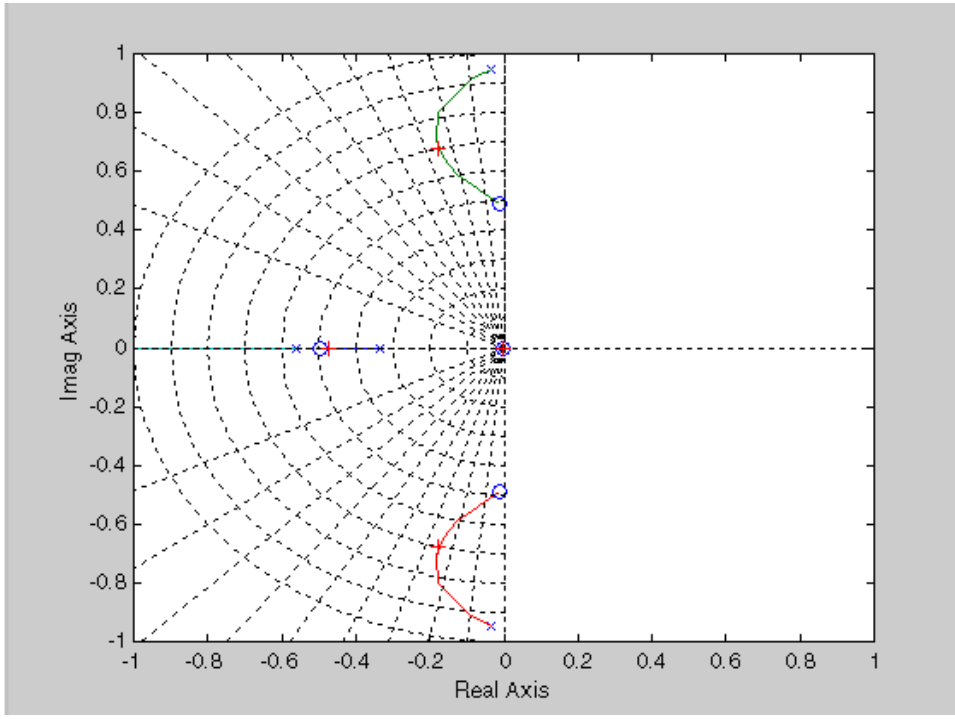
```
H = zpk(0,-0.333,1);
```

Connect the washout in series with the design model `sys11` (relation between input 1 and output 1) to obtain the open-loop model

```
oloop = H * sys11;
```

and draw another root locus for this open-loop model.

```
rlocus(-oloop)
sgrid
```



Now the maximum damping is about $\zeta = 0.25$. You can select the gain providing maximum damping graphically by

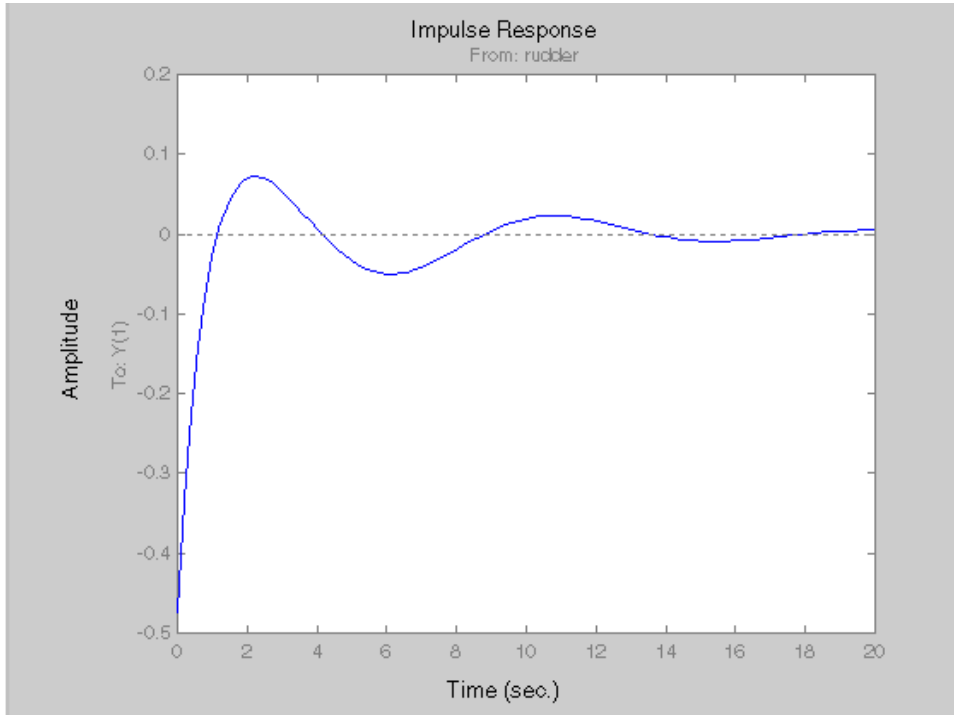
```
[k,poles] = rlocfind(-oloop)
```

The selected pole locations are marked by '+' on the root locus above. The resulting gain value and closed-loop dynamics are found by typing

```
k

k =
    2.5832


damp(poles)

        Eigenvalue            Damping     Freq. (rad/s)

 -1.37e+000                  1.00e+000      1.37e+000
 -1.76e-001 + 6.75e-001i     2.52e-001      6.98e-001
 -1.76e-001 - 6.75e-001i     2.52e-001      6.98e-001
 -4.74e-001                  1.00e+000      4.74e-001
 -3.88e-003                  1.00e+000      3.88e-003
```

Look at the closed-loop response from rudder to yaw rate.

```
cl11 = feedback(oloop,–k);
impulse(cl11,20)
```

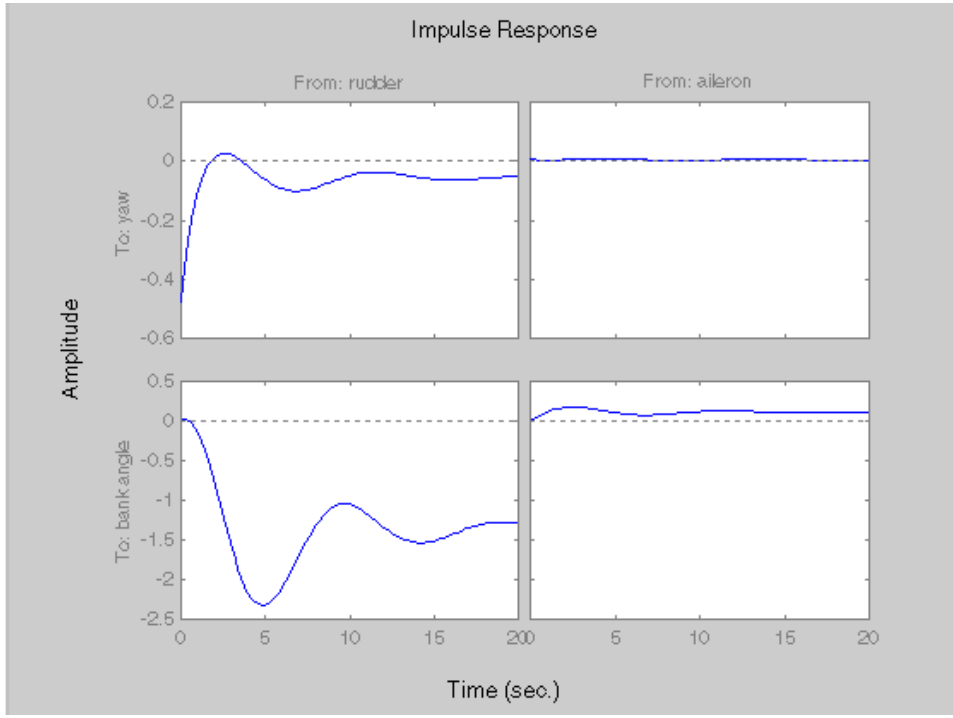


The response settles nicely but has less damping than your previous design. Finally, you can verify that the washout filter has fixed the spiral mode problem. First form the complete washout filter $kH(s)$ (washout + gain).

```
WOF = –k * H;
```

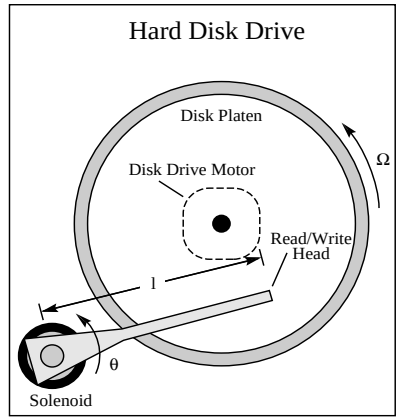
Then close the loop around the first I/O pair of the MIMO model sys and simulate the impulse response.

```
cloop = feedback(sys,WOF,1,1);

% Final closed-loop impulse response
impulse(cloop,20)
```



The bank angle response (output 2) due to an aileron impulse (input 2) now has the desired nearly constant behavior over this short time frame. Although you did not quite meet the damping specification, your design has increased the damping of the system substantially and now allows the pilot to fly the aircraft normally.

Hard-Disk Read/Write Head Controller



This case study demonstrates the ability to perform classical digital control design by going through the design of a computer hard-disk read/write head position controller.

Using Newton's law, a simple model for the read/write head is the differential equation

$$J\frac{d^2\theta}{dt^2} + C\frac{d\theta}{dt} + K\theta = K_i i$$

where J is the inertia of the head assembly, C is the viscous damping coefficient of the bearings, K is the return spring constant, K_i is the motor torque constant, θ is the angular position of the head, and i is the input current.

Taking the Laplace transform, the transfer function from i to θ is

$$H(s) = \frac{K_i}{Js^2 + Cs + K}$$

Using the values $J = 0.01$ kg m^2, $C = 0.004$ Nm/(rad/sec), $K = 10$ Nm/rad, and $K_i = 0.05$ Nm/rad, form the transfer function description of this system. At the MATLAB prompt, type

```
J = .01; C = 0.004; K = 10; Ki = .05;
num = Ki;
den = [J C K];
H = tf(num,den)
```

MATLAB responds with

```
Transfer function:
         0.05
----------------------
0.01 s^2 + 0.004 s + 10
```

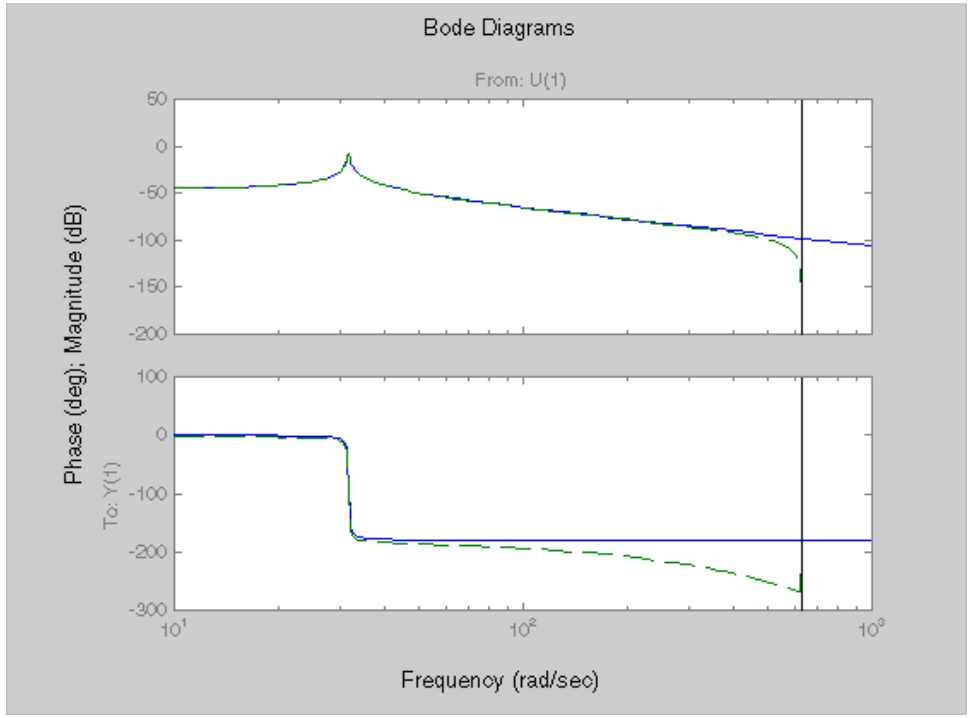
The task here is to design a digital controller that provides accurate positioning of the read/write head. The design is performed in the digital domain. First, discretize the continuous plant. Because our plant will be equipped with a digital-to-analog converter (with a zero-order hold) connected to its input, use `c2d` with the `'zoh'` discretization method. Type

```
Ts = 0.005;     % sampling period = 0.005 second
Hd = c2d(H,Ts,'zoh')
```

```
Transfer function:
6.233e-05 z + 6.229e-05
-----------------------
 z^2 - 1.973 z + 0.998

Sampling time: 0.005
```

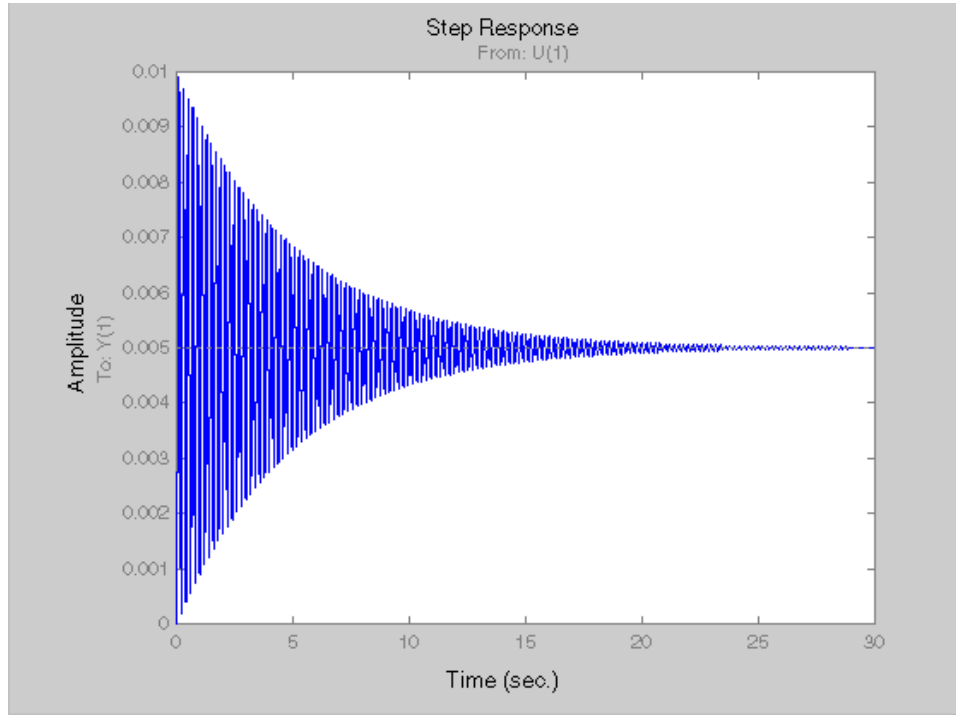
You can compare the Bode plots of the continuous and discretized models with

```
bode(H,'-',Hd,'--')
```



To analyze the discrete system, plot its step response, type

```
step(Hd)
```



The system oscillates quite a bit. This is probably due to very light damping. You can check this by computing the open-loop poles. Type

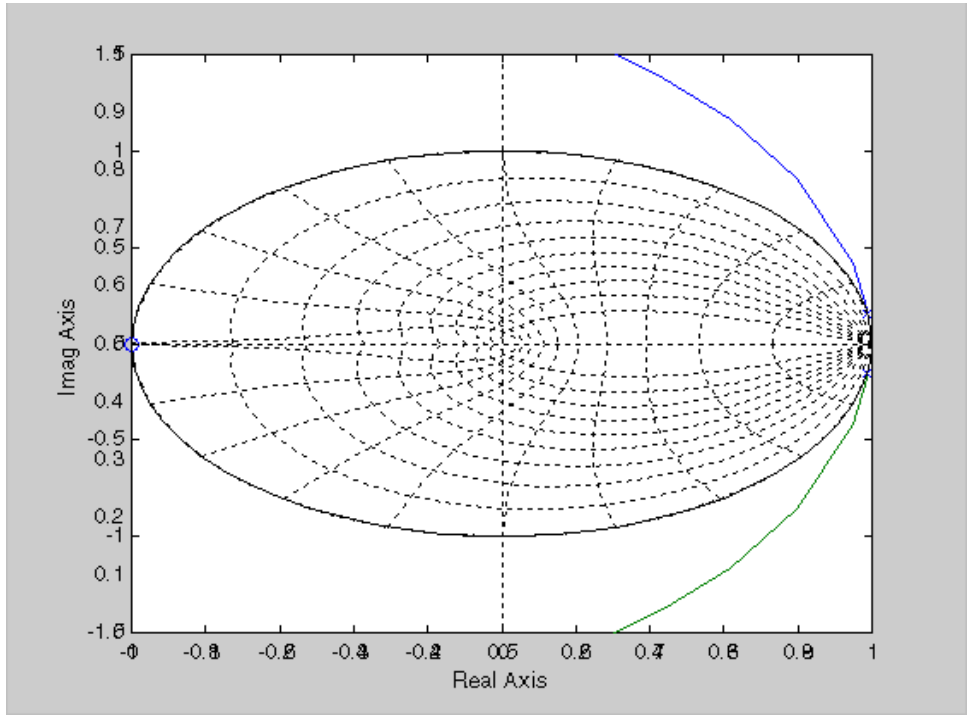
```
% Open-loop poles of discrete model
damp(Hd)
```

```
   Eigenvalue          Magnitude    Equiv. Damping    Equiv. Freq.

9.87e-01 + 1.57e-01i   9.99e-01       6.32e-03         3.16e+01
9.87e-01 - 1.57e-01i   9.99e-01       6.32e-03         3.16e+01
```

The poles have very light equivalent damping and are near the unit circle. You need to design a compensator that increases the damping of these poles.

The simplest compensator is just a gain, so try the root locus technique to select an appropriate feedback gain.

```
rlocus(Hd)
```

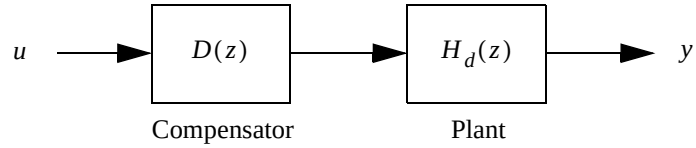


As shown in the root locus, the poles quickly leave the unit circle and go unstable. You need to introduce some lead or a compensator with some zeros. Try the compensator

$$D(z) = \frac{z + a}{z + b}$$

with $a = -0.85$ and $b = 0$.

The corresponding open-loop model



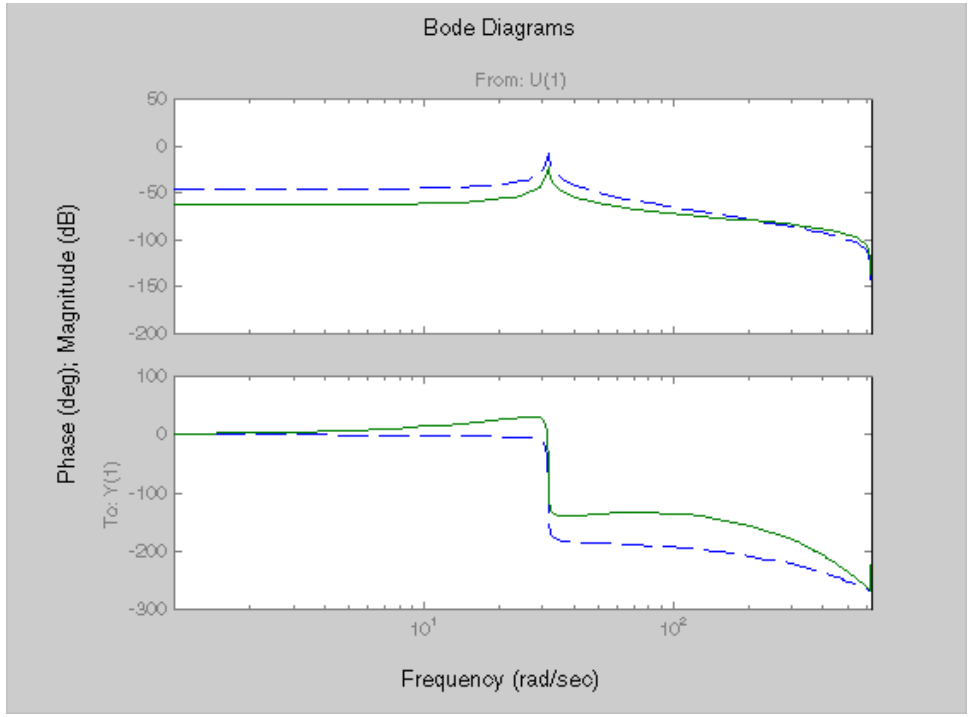
is obtained by the series connection

```
D = zpk(0.85,0,1,Ts)
oloop = Hd * D
```

Now see how this compensator modifies the open-loop frequency response.

```
bode(Hd,'--',oloop,'-')
```

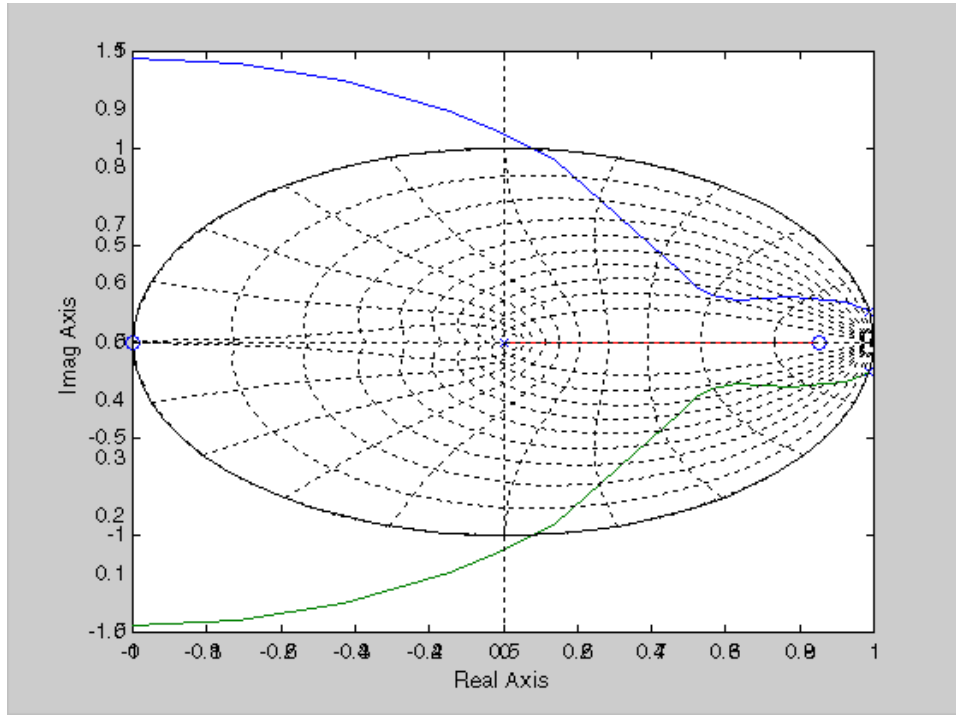
The plant response is the dashed line and the open-loop response with the compensator is the solid line.



The plot above shows that the compensator has shifted up the phase plot (added lead) in the frequency range $\omega > 10$ rad/sec.

Now try the root locus again with the plant and compensator as open loop.

```
rlocus(oloop)
zgrid
```



This time, the poles stay within the unit circle for some time (the lines drawn by `zgrid` show the damping ratios from $\zeta = 0$ to 1 in steps of 0.1). This plot shows a set of poles `'+'` selected using `rlocfind`. At the MATLAB prompt, type

```
[k,poles] = rlocfind(oloop)
k

k =
   4.1179e+03
```

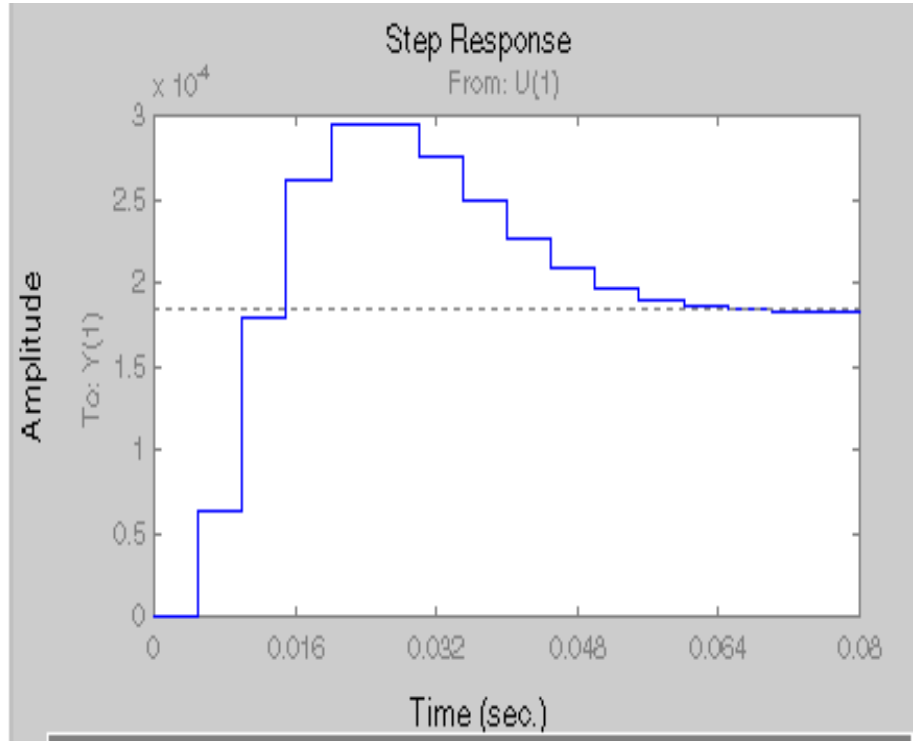
Type

```
ddamp(poles,Ts)
```

to see the equivalent damping and natural frequency for each of the eigenvalues.

To analyze this design, form the closed-loop system and plot the closed-loop step response.

```
cloop = feedback(oloop,k);
step(cloop)
```

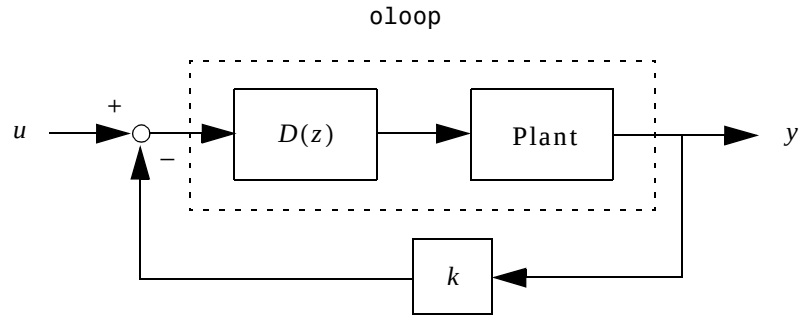


This response depends on your closed loop set point. The one shown here is relatively fast and settles in about 0.07 seconds. Therefore, this closed loop disk drive system has a seek time of about 0.07 seconds. This is slow by today's standards, but you also started with a very lightly damped system.

Now look at the robustness of your design. The most common classical robustness criteria are the gain and phase margins. Use the function `margin` to determine these margins. With output arguments, `margin` returns the gain and phase margins as well as the corresponding crossover frequencies. Without output argument, `margin` plots the Bode response and displays the margins graphically.

To compute the margins, first form the unity-feedback open loop by connecting the compensator $D(z)$, plant model, and feedback gain k in series.

```
olk = k * oloop;
```



Next apply `margin` to this open-loop model. Type

```
[Gm,Pm,Wcg,Wcp] = margin(olk);
Margins = [Gm Wcg Pm Wcp]
```

```
Margins =
    3.7809  295.3172   43.1686  106.4086
```

To obtain the gain margin in dB, type

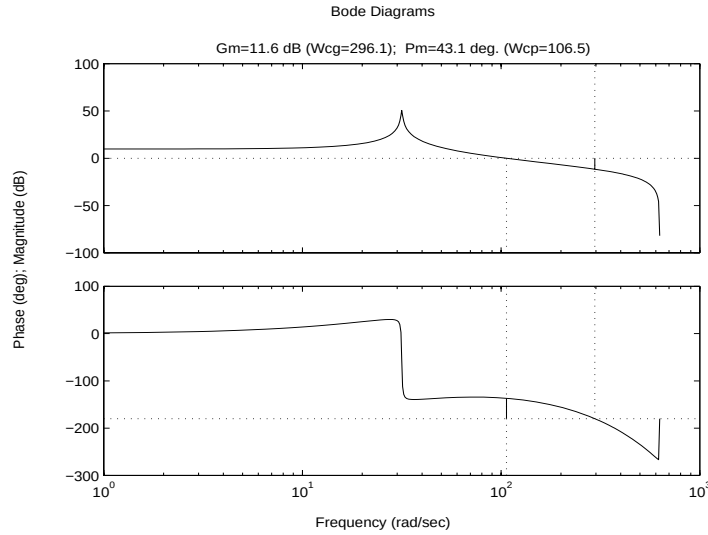
```
20*log10(Gm)
```

```
ans =
   11.5760
```

You can also display the margins graphically by typing

```
margin(olk)
```

The command produces the plot shown below.



This design is robust and can tolerate a 11 dB gain increase or a 40 degree phase lag in the open-loop system without going unstable. By continuing this design process, you may be able to find a compensator that stabilizes the open-loop system and allows you to reduce the seek time.

LQG Regulation

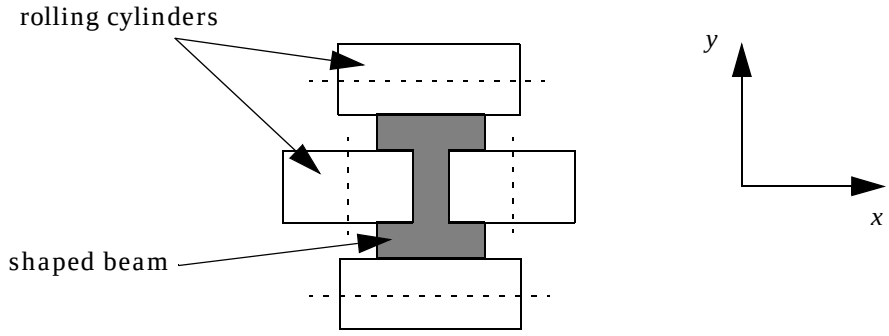
This case study demonstrates the use of the LQG design tools in a process control application. The goal is to regulate the horizontal and vertical thickness of the beam produced by a hot steel rolling mill. This example is adapted from [1]. The full plant model is MIMO and the example shows the advantage of direct MIMO LQG design over separate SISO designs for each axis. Type

```
milldemo
```

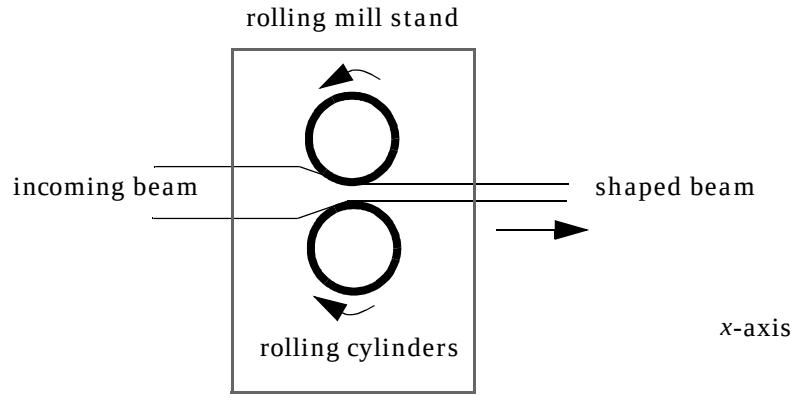
at the command line to run this demonstration interactively.

Process and Disturbance Models

The rolling mill is used to shape rectangular beams of hot metal. The desired outgoing shape is sketched below.



This shape is impressed by two pairs of rolling cylinders (one per axis) positioned by hydraulic actuators. The gap between the two cylinders is called the *roll gap*.



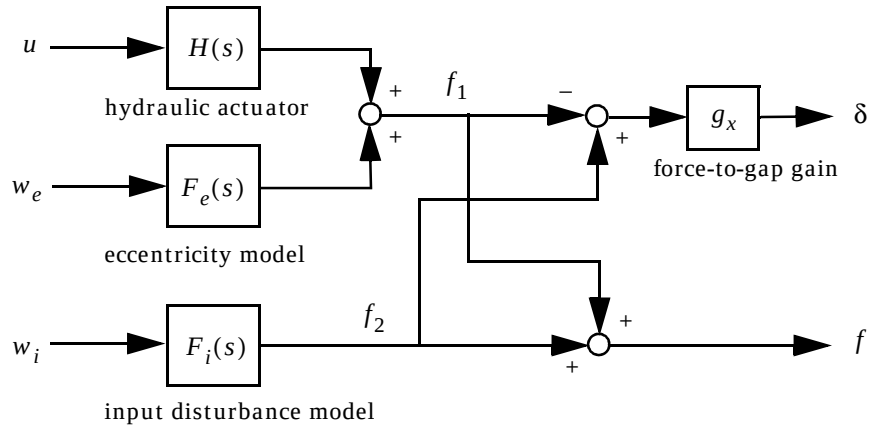
The objective is to maintain the beam thickness along the *x*- and *y*-axes within the quality assurance tolerances. Variations in output thickness can arise from the following:

- Variations in the thickness/hardness of the incoming beam
- Eccentricity in the rolling cylinders

Feedback control is necessary to reduce the effect of these disturbances. Because the roll gap cannot be measured close to the mill stand, the rolling force is used instead for feedback.

The input thickness disturbance is modeled as a low pass filter driven by white noise. The eccentricity disturbance is approximately periodic and its frequency is a function of the rolling speed. A reasonable model for this disturbance is a second-order bandpass filter driven by white noise.

This leads to the following generic model for each axis of the rolling process.



u	command
δ	thickness gap (in mm)
f	incremental rolling force
w_i, w_e	driving white noise for disturbance models

Figure 9-1: Open-loop model for x- or y-axis

The measured rolling force variation f is a combination of the incremental force delivered by the hydraulic actuator and of the disturbance forces due to eccentricity and input thickness variation. Note that:

- The outputs of $H(s), F_e(s)$, and $F_i(s)$ are the incremental forces delivered by each component.
- An increase in hydraulic or eccentricity force *reduces* the output thickness gap δ.
- An increase in input thickness *increases* this gap.

The model data for each axis is summarized below.

Model Data for the x-Axis

$$H_x(s) = \frac{2.4 \times 10^8}{s^2 + 72s + 90^2}$$

$$F_{ix}(s) = \frac{10^4}{s + 0.05}$$

$$F_{ex}(s) = \frac{3 \times 10^4 \ s}{s^2 + 0.125s + 6^2}$$

$$g_x = 10^{-6}$$

Model Data for the y-Axis

$$H_y(s) = \frac{7.8 \times 10^8}{s^2 + 71s + 88^2}$$

$$F_{iy}(s) = \frac{2 \times 10^4}{s + 0.05}$$

$$F_{ey}(s) = \frac{10^5 \ s}{s^2 + 0.19s + 9.4^2}$$

$$g_y = 0.5 \times 10^{-6}$$

LQG Design for the x-Axis

As a first approximation, ignore the cross-coupling between the *x*- and *y*-axes and treat each axis independently. That is, design one SISO LQG regulator for each axis. The design objective is to reduce the thickness variations δ_x and δ_y due to eccentricity and input thickness disturbances.

Start with the *x*-axis. First specify the model components as transfer function objects.

```
% Hydraulic actuator (with input "u-x")
Hx = tf(2.4e8,[1 72 90^2],'inputname','u-x')

% Input thickness/hardness disturbance model
Fix = tf(1e4,[1 0.05],'inputn','w-ix')

% Rolling eccentricity model
Fex = tf([3e4 0],[1 0.125 6^2],'inputn','w-ex')

% Gain from force to thickness gap
gx = 1e-6;
```

Next build the open-loop model shown in Figure 9-1 above. You could use the function `connect` for this purpose, but it is easier to build this model by elementary `append` and `series` connections.

```
% I/O map from inputs to forces f1 and f2
Px = append([ss(Hx) Fex],Fix)

% Add static gain from f1,f2 to outputs "x-gap" and "x-force"
Px = [-gx gx;1 1] * Px

% Give names to the outputs:
set(Px,'outputn',{'x-gap' 'x-force'})
```

Note: To obtain minimal state-space realizations, always convert transfer function models to state space *before* connecting them. Combining transfer functions and then converting to state space may produce nonminimal state-space models.

The variable Px now contains an open-loop state-space model complete with input and output names.

```
Px.inputname

ans =
    'u-x'
    'w-ex'
    'w-ix'

Px.outputname

ans =
    'x-gap'
    'x-force'
```

The second output `'x-force'` is the rolling force measurement. The LQG regulator will use this measurement to drive the hydraulic actuator and reduce disturbance-induced thickness variations δ_x.

The LQG design involves two steps:

1. Design a full-state-feedback gain that minimizes an LQ performance measure of the form

$$J(u_x) = \int_0^\infty \left\{ q\delta_x^2 + ru_x^2 \right\} dt$$

2. Design a Kalman filter that estimates the state vector given the force measurements `'x-force'`.

The performance criterion $J(u_x)$ penalizes low and high frequencies equally. Because low-frequency variations are of primary concern, eliminate the

high-frequency content of δ_x with the low-pass filter $30/(s+30)$ and use the filtered value in the LQ performance criterion.

```
lpf = tf(30,[1 30])

% Connect low-pass filter to first output of Px
Pxdes = append(lpf,1) * Px
set(Pxdes,'outputn',{'x-gap*' 'x-force'})

% Design the state-feedback gain using LQRY and q=1, r=1e-4
kx = lqry(Pxdes(1,1),1,1e-4)
```

Note: `lqry` expects all inputs to be commands and all outputs to be measurements. Here the command `'u-x'` and the measurement `'x-gap*'` (filtered gap) are the first input and first output of `Pxdes`. Hence, use the syntax `Pxdes(1,1)` to specify just the I/O relation between `'u-x'` and `'x-gap*'`.

Next, design the Kalman estimator with the function `kalman`. The process noise

$$w_x = \begin{bmatrix} w_{ex} \\ w_{ix} \end{bmatrix}$$

has unit covariance by construction. Set the measurement noise covariance to 1000 to limit the high frequency gain, and keep only the measured output `'x-force'` for estimator design.

```
estx = kalman(Pxdes(2,:),eye(2),1000)
```

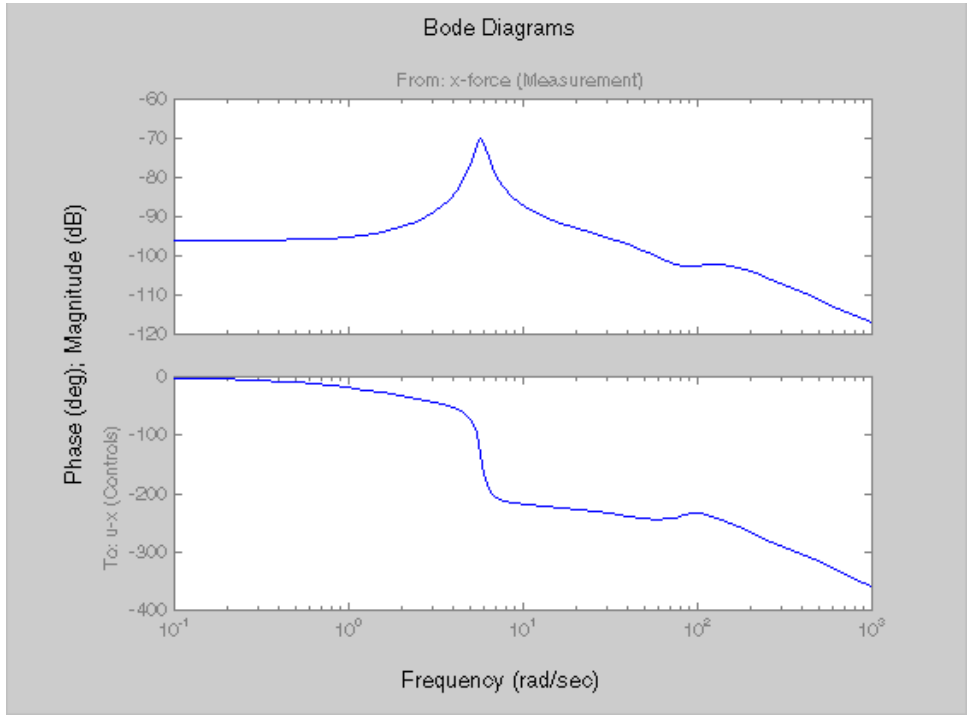
Finally, connect the state-feedback gain `kx` and state estimator `estx` to form the LQG regulator.

```
Regx = lqgreg(estx,kx)
```

This completes the LQG design for the x-axis.

Let's look at the regulator Bode response between 0.1 and 1000 rad/sec.

```
bode(Regx,{0.1 1000})
```



The phase response has an interesting physical interpretation. First, consider an increase in input thickness. This low-frequency disturbance boosts both output thickness and rolling force. Because the regulator phase is approximately 0^o at low frequencies, the feedback loop then adequately reacts by increasing the hydraulic force to offset the thickness increase. Now consider the effect of eccentricity. Eccentricity causes fluctuations in the roll gap (gap between the rolling cylinders). When the roll gap is minimal, the rolling force increases and the beam thickness diminishes. The hydraulic force must then be reduced (negative force feedback) to restore the desired thickness. This is exactly what the LQG regulator does as its phase drops to -180^o near the natural frequency of the eccentricity disturbance (6 rad/sec).

Next, compare the open- and closed-loop responses from disturbance to thickness gap. Use `feedback` to close the loop. To help specify the feedback connection, look at the I/O names of the plant `Px` and regulator `Regx`.

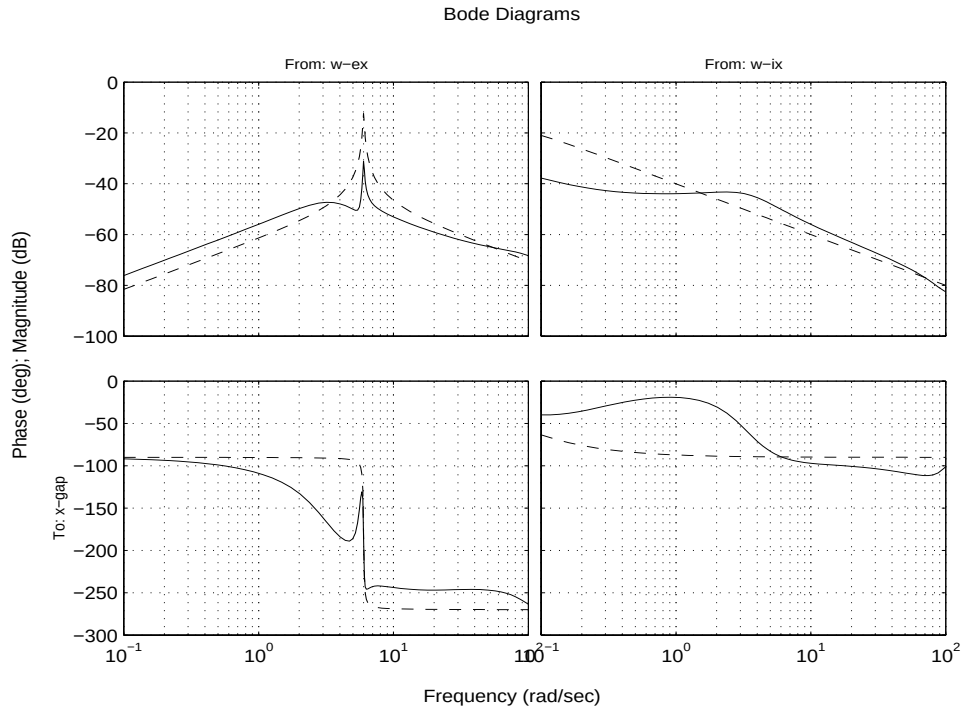
```
Px.inputname
ans =
    'u-x'
    'w-ex'
    'w-ix'

Regx.outputname
ans =
    'u-x'

Px.outputname
ans =
    'x-gap'
    'x-force'

Regx.inputname
ans =
    'x-force'
```

This indicates that you must connect the first input and second output of `Px` to the regulator.

```
clx = feedback(Px,Regx,1,2,+1)    % Note: +1 for positive feedback
```

You are now ready to compare the open- and closed-loop Bode responses from disturbance to thickness gap.

```
bode(Px(1,2:3),'--',clx(1,2:3),'-',{0.1 100})
```

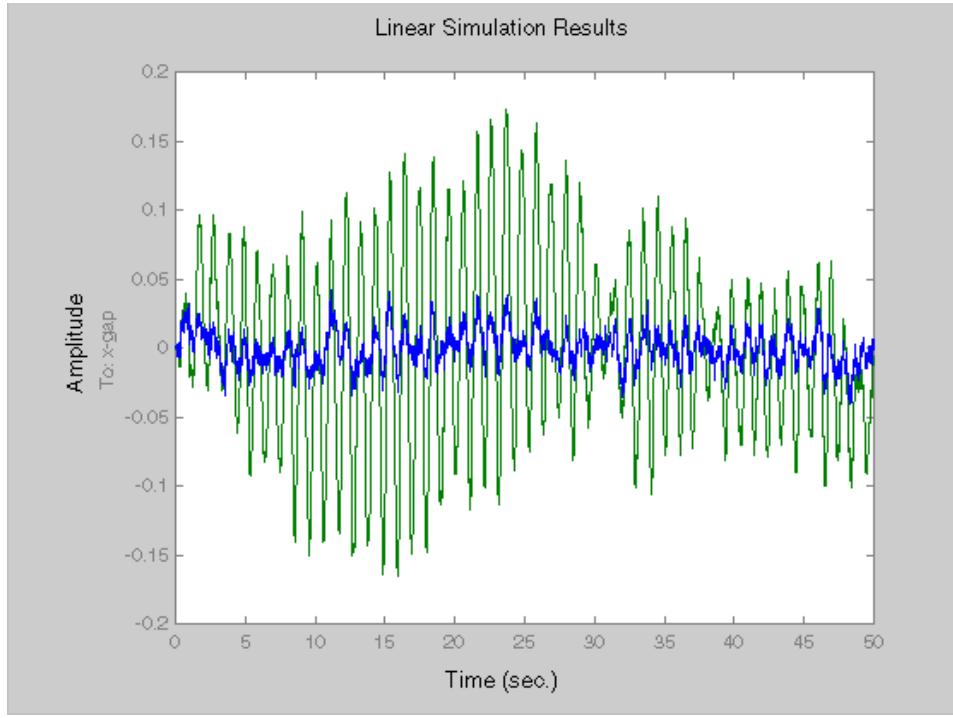


The dashed lines show the open-loop response. Note that the peak gain of the eccentricity-to-gap response and the low-frequency gain of the input-thickness-to-gap response have been reduced by about 20 dB.

Finally, use `lsim` to simulate the open- and closed-loop time responses to the white noise inputs w_{ex} and w_{ix}. Choose `dt=0.01` as sampling period for the

simulation, and derive equivalent discrete white noise inputs for this sampling rate.

```
dt = 0.01
t = 0:dt:50   % time samples

% Generate unit-covariance driving noise wx = [w-ex;w-ix].
% Equivalent discrete covariance is 1/dt
wx = sqrt(1/dt) * randn(2,length(t))

lsim(Px(1,2:3),':',clx(1,2:3),'-',wx,t)
```



The dotted lines correspond to the open-loop response. In this simulation, the LQG regulation reduces the peak thickness variation by a factor 4.

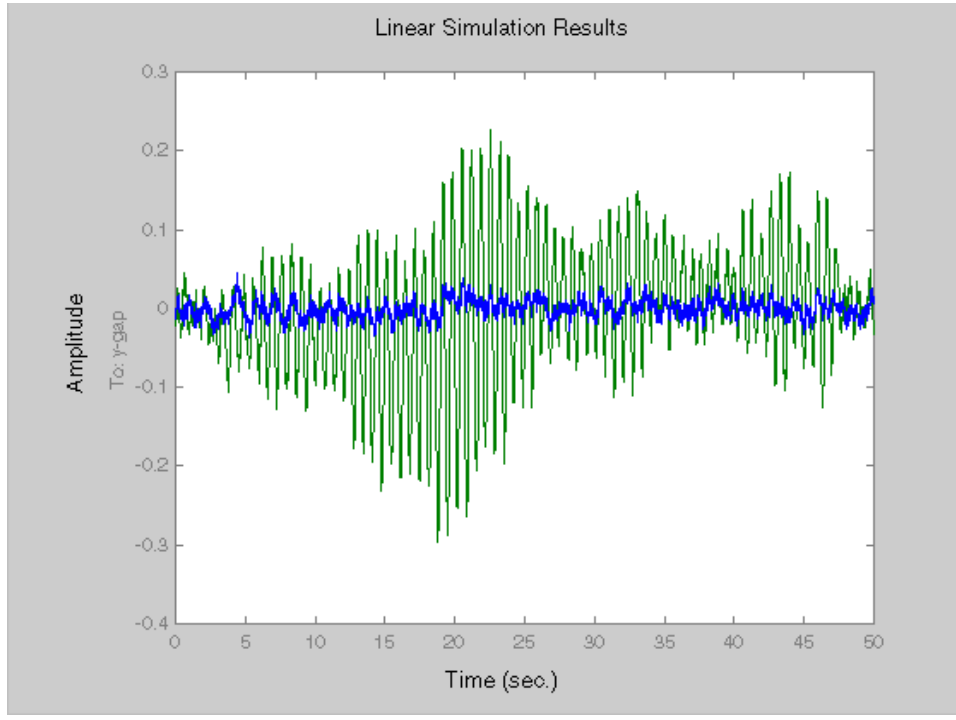
LQG Design for the y-Axis

The LQG design for the y-axis (regulation of the y thickness) follows the exact same steps as for the x-axis.

```
% Specify model components
Hy = tf(7.8e8,[1 71 88^2],'inputn','u-y')
Fiy = tf(2e4,[1 0.05],'inputn','w-iy')
Fey = tf([1e5 0],[1 0.19 9.4^2],'inputn','w-ey')
gy = 0.5e-6    % force-to-gap gain

% Build open-loop model
Py = append([ss(Hy) Fey],Fiy)
Py = [-gy gy;1 1] * Py
set(Py,'outputn',{'y-gap' 'y-force'})

% State-feedback gain design
Pydes = append(lpf,1) * Py     % Add low-freq. weigthing
set(Pydes,'outputn',{'y-gap*' 'y-force'})
ky = lqry(Pydes(1,1),1,1e-4)

% Kalman estimator design
esty = kalman(Pydes(2,:),eye(2),1e3)

% Form SISO LQG regulator for y-axis and close the loop
Regy = lqgreg(esty,ky)
cly = feedback(Py,Regy,1,2,+1)
```

Compare the open- and closed-loop response to the white noise input disturbances.

```
dt = 0.01
t = 0:dt:50
wy = sqrt(1/dt) * randn(2,length(t))

lsim(Py(1,2:3),':',cly(1,2:3),'-',wy,t)
```



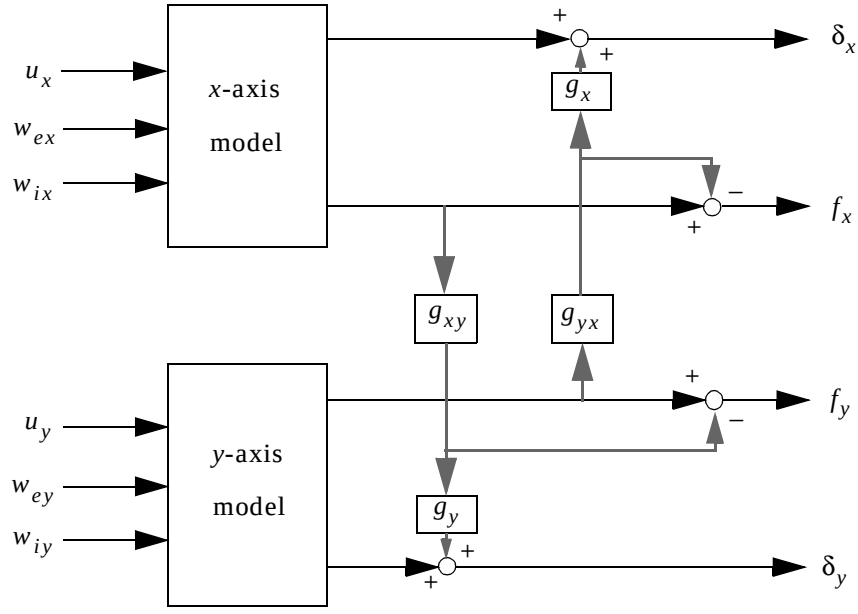
The dotted lines correspond to the open-loop response. The simulation results are comparable to those for the x-axis.

Cross-Coupling Between Axes

The x/y thickness regulation, is a MIMO problem. So far you have treated each axis separately and closed one SISO loop at a time. This design is valid as long as the two axes are fairly decoupled. Unfortunately, this rolling mill

process exhibits some degree of cross-coupling between axes. Physically, an increase in hydraulic force along the x-axis compresses the material, which in turn boosts the repelling force on the y-axis cylinders. The result is an increase in y-thickness and an equivalent (relative) decrease in hydraulic force along the y-axis.

The coupling between axes is as follows.



$g_{xy} = 0.1$

$g_{yx} = 0.4$

Figure 9-2: Coupling between the x- and y-axes

Accordingly, the thickness gaps and rolling forces are related to the outputs $\bar{\delta}_x$, $\bar{f}_x$, ... of the *x*- and *y*-axis models by

$$\begin{bmatrix} \delta_x \\ \delta_y \\ f_x \\ f_y \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & g_{yx}g_x \\ 0 & 1 & g_{xy}g_y & 0 \\ 0 & 0 & 1 & -g_{yx} \\ 0 & 0 & -g_{xy} & 1 \end{bmatrix}}_{\textbf{cross-coupling matrix}} \begin{bmatrix} \bar{\delta}_x \\ \bar{\delta}_y \\ \bar{f}_x \\ \bar{f}_y \end{bmatrix}$$

Let's see how the previous "decoupled" LQG design fares when cross-coupling is taken into account. To build the two-axes model shown in Figure 9-2, append the models `Px` and `Py` for the *x*- and *y*-axes.

```
P = append(Px,Py)
```

For convenience, reorder the inputs and outputs so that the commands and thickness gaps appear first.

```
P = P([1 3 2 4],[1 4 2 3 5 6])
P.outputname

ans =
    'x-gap'
    'y-gap'
    'x-force'
    'y-force'
```

Finally, place the cross-coupling matrix in series with the outputs.

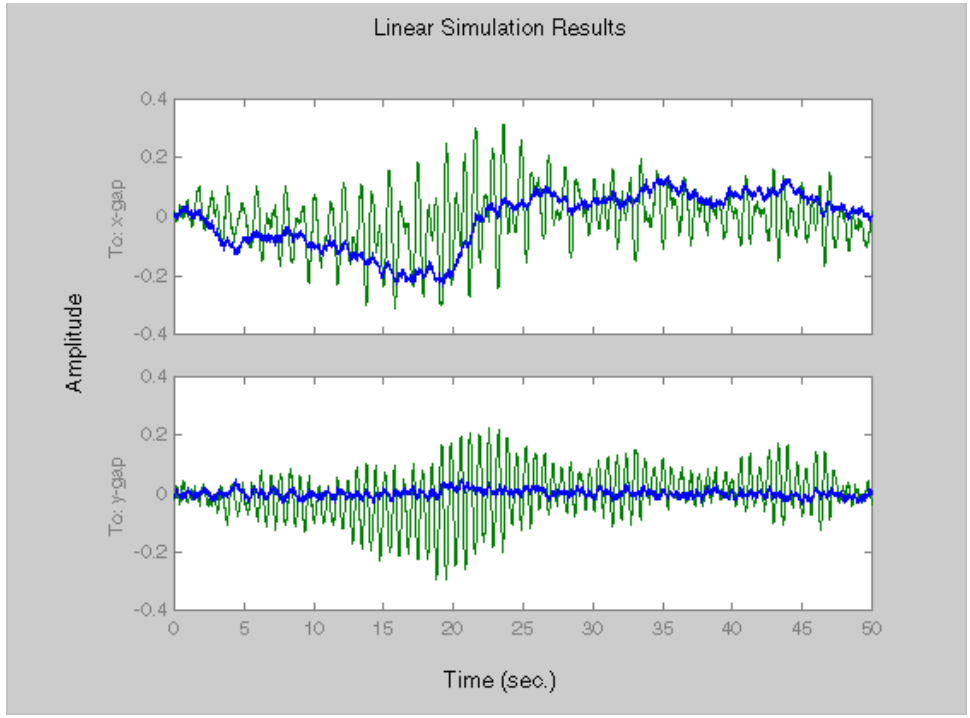
```
gxy = 0.1; gyx = 0.4;
CCmat = [eye(2) [0 gyx*gx;gxy*gy 0] ; zeros(2) [1 -gyx;-gxy 1]]
Pc = CCmat * P
Pc.outputname = P.outputname
```

To simulate the closed-loop response, also form the closed-loop model by

```
feedin = 1:2   % first two inputs of Pc are the commands
feedout = 3:4  % last two outputs of Pc are the measurements
cl = feedback(Pc,append(Regx,Regy),feedin,feedout,+1)
```

You are now ready to simulate the open- and closed-loop responses to the driving white noises wx (for the *x*-axis) and wy (for the *y*-axis).

```
wxy = [wx ; wy]
lsim(Pc(1:2,3:6),':',cl(1:2,3:6),'-',wxy,t)
```



The response reveals a severe deterioration in regulation performance along the *x*-axis (the peak thickness variation is about four times larger than in the simulation without cross-coupling). Hence, designing for one loop at a time is inadequate for this level of cross-coupling, and you must perform a joint-axis MIMO design to correctly handle coupling effects.

MIMO LQG Design

Start with the complete two-axis state-space model `Pc` derived above. The model inputs and outputs are

```
Pc.inputname

ans =
    'u-x'
    'u-y'
    'w-ex'
    'w-ix'
    'w_ey'
    'w_iy'

P.outputname

ans =
    'x-gap'
    'y-gap'
    'x-force'
    'y-force'
```

As earlier, add low-pass filters in series with the `'x-gap'` and `'y-gap'` outputs to penalize only low-frequency thickness variations.

```
Pdes = append(lpf,lpf,eye(2)) * Pc
Pdes.outputn = Pc.outputn
```

Next, design the LQ gain and state estimator as before (there are now two commands and two measurements).

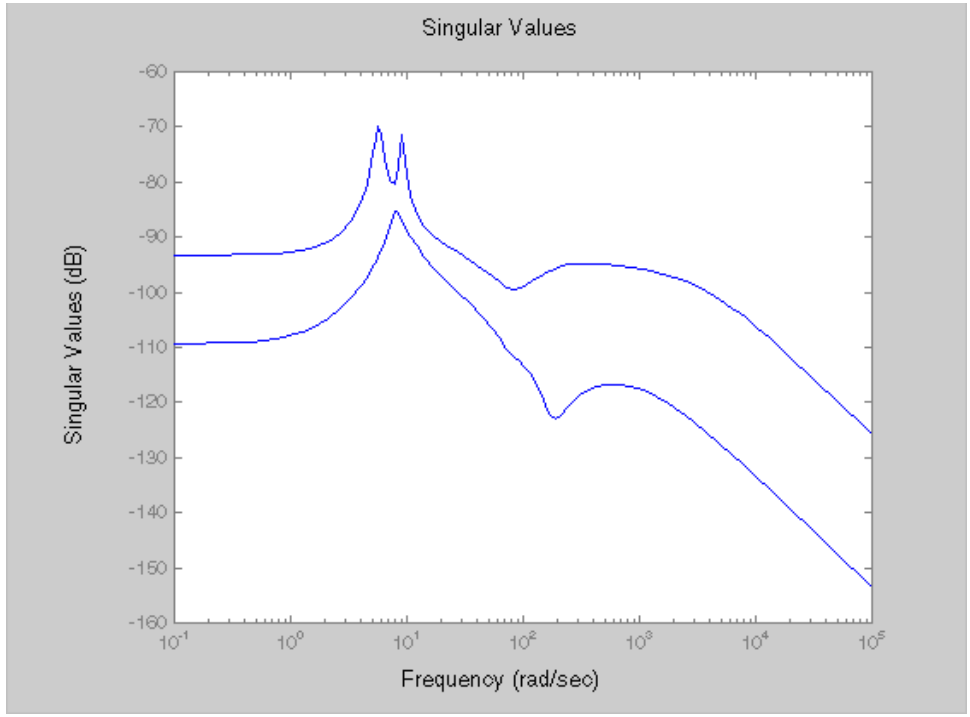
```
k = lqry(Pdes(1:2,1:2),eye(2),1e-4*eye(2))    % LQ gain
est = kalman(Pdes(3:4,:),eye(4),1e3*eye(2))   % Kalman estimator

RegMIMO = lqgreg(est,k)   % form MIMO LQG regulator
```

The resulting LQG regulator `RegMIMO` has two inputs and two outputs.

```
RegMIMO.inputname

ans =
    'x-force'
    'y-force'

RegMIMO.outputname

ans =
    'u-x'
    'u-y'
```

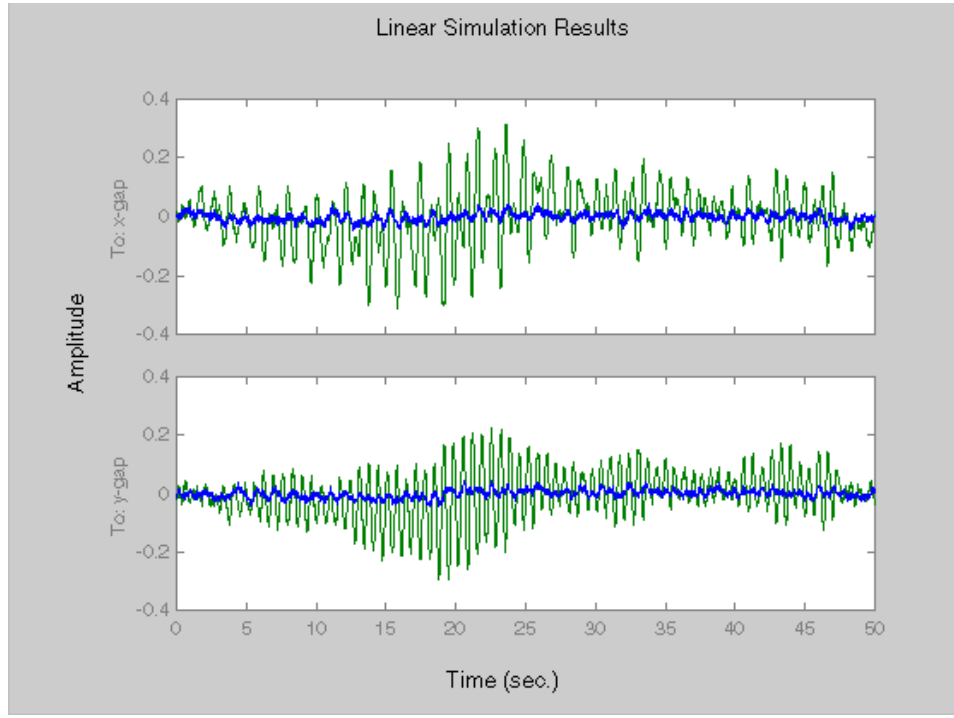
Plot its singular value response (principal gains).

```
sigma(RegMIMO)
```



Next, plot the open- and closed-loop time responses to the white noise inputs (using the MIMO LQG regulator for feedback).

```
% Form the closed-loop model
cl = feedback(Pc,RegMIMO,1:2,3:4,+1);

% Simulate with LSIM using same noise inputs
lsim(Pc(1:2,3:6),':',cl(1:2,3:6),'-',wxy,t)
```



The MIMO design is a clear improvement over the separate SISO designs for each axis. In particular, the level of x/y thickness variation is now comparable to that obtained in the decoupled case. This example illustrates the benefits of direct MIMO design for multivariable systems.

Kalman Filtering

This final case study illustrates the use of the Control System Toolbox for Kalman filter design and simulation. Both steady-state and time-varying Kalman filters are considered.

Consider the discrete plant

$$x[n+1] = Ax[n] + B(u[n] + w[n])$$
$$y[n] = Cx[n]$$

with additive Gaussian noise $w[n]$ on the input $u[n]$ and data

```
A = [1.1269   -0.4940    0.1129
     1.0000         0         0
          0    1.0000         0];

B = [-0.3832
      0.5919
      0.5191];

C = [1 0 0];
```

Our goal is to design a Kalman filter that estimates the output $y[n]$ given the inputs $u[n]$ and the noisy output measurements

$$y_v[n] = Cx[n] + v[n]$$

where $v[n]$ is some Gaussian white noise.

Discrete Kalman Filter

The equations of the steady-state Kalman filter for this problem are given as follows.

Measurement update

$$\hat{x}[n|n] = \hat{x}[n|n-1] + M(y_v[n] - C\hat{x}[n|n-1])$$

Time update

$$\hat{x}[n+1|n] = A\hat{x}[n|n] + Bu[n]$$

In these equations:

- $\hat{x}[n|n-1]$ is the estimate of $x[n]$ given past measurements up to $y_v[n-1]$
- $\hat{x}[n|n]$ is the updated estimate based on the last measurement $y_v[n]$

Given the current estimate $\hat{x}[n|n]$, the time update predicts the state value at the next sample $n+1$ (one-step-ahead predictor). The measurement update then adjusts this prediction based on the new measurement $y_v[n+1]$. The correction term is a function of the *innovation*, that is, the discrepancy.

$$y_v[n+1] - C\hat{x}[n+1|n] = C(x[n+1] - \hat{x}[n+1|n])$$

between the measured and predicted values of $y[n+1]$. The innovation gain M is chosen to minimize the steady-state covariance of the estimation error given the noise covariances

$$E(w[n]w[n]^T) = Q, \qquad E(v[n]v[n]^T) = R$$

You can combine the time and measurement update equations into one state-space model (the Kalman filter).

$$\hat{x}[n+1|n] = A(I-MC)\,\hat{x}[n|n-1] + \begin{bmatrix} B & AM \end{bmatrix} \begin{bmatrix} u[n] \\ y_v[n] \end{bmatrix}$$

$$\hat{y}[n|n] = C(I-MC)\,\hat{x}[n|n-1] + CM\,y_v[n]$$

This filter generates an optimal estimate $\hat{y}[n|n]$ of $y[n]$. Note that the filter state is $\hat{x}[n|n-1]$.

Steady-State Design

You can design the steady-state Kalman filter described above with the function `kalman`. First specify the plant model with the process noise.

$$x[n+1] = Ax[n] + Bu[n] + Bw[n] \qquad \text{(state equation)}$$
$$y[n] = Cx[n] \qquad \text{(measurement equation)}$$

This is done by

```
% Note: set sample time to -1 to mark model as discrete
Plant = ss(A,[B B],C,0,-1,'inputname',{'u' 'w'},...
                          'outputname','y');
```

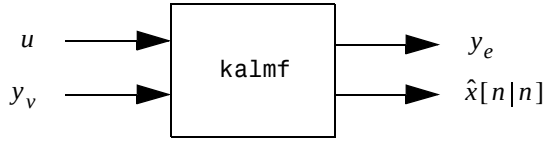
Assuming that $Q = R = 1$, you can now design the discrete Kalman filter by

```
Q = 1; R = 1;
[kalmf,L,P,M] = kalman(Plant,Q,R);
```

This returns a state-space model `kalmf` of the filter as well as the innovation gain

```
M

M =
   3.7980e-01
   8.1732e-02
  -2.5704e-01
```

The inputs of `kalmf` are u and y_v, and its outputs are the plant output and state estimates $y_e = \hat{y}[n|n]$ and $\hat{x}[n|n]$.



Kalman filter

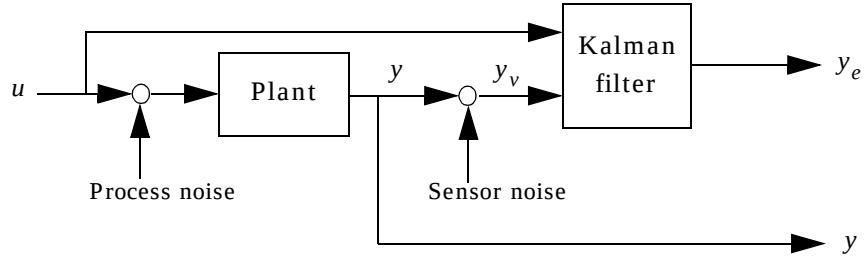
Because you are interested in the output estimate y_e, keep only the first output of `kalmf`. Type

```
kalmf = kalmf(1,:);
kalmf
```

```
a =
                    x1_e          x2_e          x3_e
         x1_e      0.76830      -0.49400       0.11290
         x2_e      0.62020             0             0
         x3_e     -0.08173       1.00000             0

b =
                        u             y
         x1_e     -0.38320       0.35860
         x2_e      0.59190       0.37980
         x3_e      0.51910       0.08173

c =
                    x1_e          x2_e          x3_e
          y_e      0.62020             0             0

d =
                        u             y
          y_e           0       0.37980

Sampling time: unspecified
Discrete-time system.
```

To see how the filter works, generate some input data and random noise and compare the filtered response y_e with the true response y. You can either generate each response separately, or generate both together. To simulate each response separately, use `lsim` with the plant alone first, and then with the plant and filter hooked up together. The joint simulation alternative is detailed next.

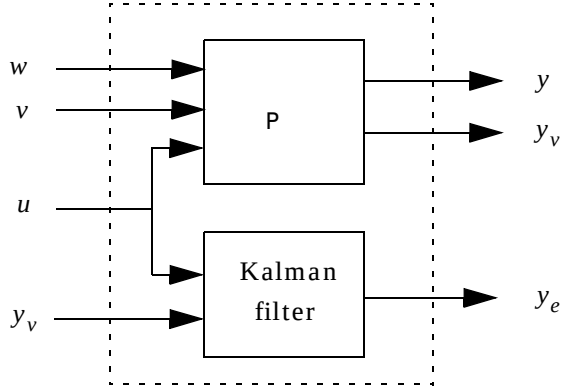
The block diagram below shows how to generate both true and filtered outputs.



You can construct a state-space model of this block diagram with the functions `parallel` and `feedback`. First build a complete plant model with u, w, v as inputs and y and y_v (measurements) as outputs.

```
a = A;
b = [B B 0*B];
c = [C;C];
d = [0 0 0;0 0 1];
P = ss(a,b,c,d,-1,'inputname',{'u' 'w' 'v'},...
                  'outputname',{'y' 'yv'});
```

Then use `parallel` to form the following parallel connection.




```
sys = parallel(P,kalmf,1,1,[],[])
```

Finally, close the sensor loop by connecting the plant output y_v to the filter input y_v with positive feedback.

```
% Close loop around input #4 and output #2
SimModel = feedback(sys,1,4,2,1)
% Delete yv from I/O list
SimModel = SimModel([1 3],[1 2 3])
```

The resulting simulation model has w, v, u as inputs and y, y_e as outputs.

```
SimModel.input

ans =
    'w'
    'v'
    'u'

SimModel.output

ans =
    'y'
    'y_e'
```

You are now ready to simulate the filter behavior. Generate a sinusoidal input u and process and measurement noise vectors w and v.

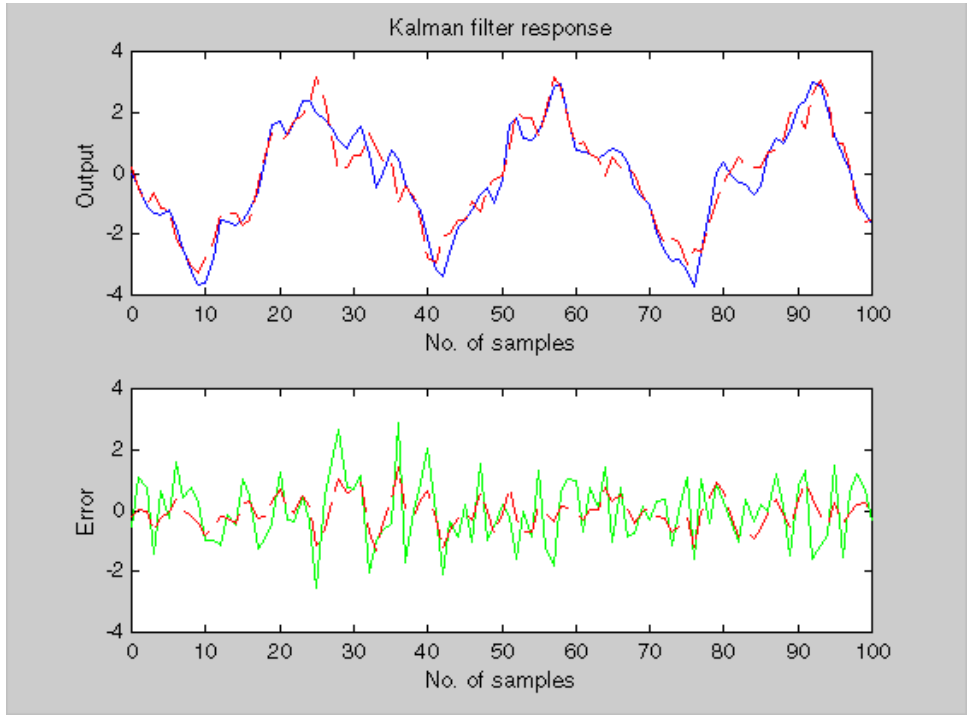
```
t = [0:100]';
u = sin(t/5);

n = length(t)
randn('seed',0)
w = sqrt(Q)*randn(n,1);
v = sqrt(R)*randn(n,1);
```

Now simulate with `lsim`.

```
[out,x] = lsim(SimModel,[w,v,u]);

y = out(:,1);   % true response
ye = out(:,2);  % filtered response
yv = y + v;     % measured response
```

and compare the true and filtered responses graphically.

```
subplot(211), plot(t,y,'--',t,ye,'-'),
xlabel('No. of samples'), ylabel('Output')
subplot(212), plot(t,y–yv,'-.',t,y–ye,'-'),
xlabel('No. of samples'), ylabel('Error')
```



The first plot shows the true response y (dashed line) and the filtered output y_e (solid line). The second plot compares the measurement error (dash-dot) with the estimation error (solid). This plot shows that the noise level has been significantly reduced. This is confirmed by the following error covariance computations.

```
MeasErr = y–yv;
MeasErrCov = sum(MeasErr.*MeasErr)/length(MeasErr);
EstErr = y–ye;
EstErrCov = sum(EstErr.*EstErr)/length(EstErr);
```

The error covariance before filtering (measurement error) is

```
MeasErrCov

MeasErrCov =
    1.1138
```

while the error covariance after filtering (estimation error) is only

```
EstErrCov

EstErrCov =
    0.2722
```

Time-Varying Kalman Filter

The time-varying Kalman filter is a generalization of the steady-state filter for time-varying systems or LTI systems with nonstationary noise covariance. Given the plant state and measurement equations

$$x[n+1] = Ax[n] + Bu[n] + Gw[n]$$
$$y_v[n] = Cx[n] + v[n]$$

the time-varying Kalman filter is given by the recursions

Measurement update

$$\hat{x}[n|n] = \hat{x}[n|n-1] + M[n](y_v[n] - C\hat{x}[n|n-1])$$
$$M[n] = P[n|n-1]C^T(R[n] + CP[n|n-1]C^T)^{-1}$$
$$P[n|n] = (I - M[n]C)\ P[n|n-1]$$

Time update

$$\hat{x}[n+1|n] = A\hat{x}[n|n] + Bu[n]$$
$$P[n+1|n] = AP[n|n]A^T + GQ[n]G^T$$

with $\hat{x}[n|n-1]$ and $\hat{x}[n|n]$ as defined on page 9-50, and in the following.

$$Q[n] = E(w[n]w[n]^T)$$
$$R[n] = E(v[n]v[n]^T)$$
$$P[n|n] = E(\{x[n]-x[n|n]\}\{x[n]-x[n|n]\}^T)$$
$$P[n|n-1] = E(\{x[n]-x[n|n-1]\}\{x[n]-x[n|n-1]\}^T)$$

For simplicity, we have dropped the subscripts indicating the time dependence of the state-space matrices.

Given initial conditions $x[1|0]$ and $P[1|0]$, you can iterate these equations to perform the filtering. Note that you must update both the state estimates $x[n|.]$ and error covariance matrices $P[n|.]$ at each time sample.

Time-Varying Design

Although the Control System Toolbox does not offer specific commands to perform time-varying Kalman filtering, it is easy to implement the filter recursions in MATLAB. This section shows how to do this for the stationary plant considered above.

First generate noisy output measurements

```
% Use process noise w and measurement noise v generated above
sys = ss(A,B,C,0,-1);
y = lsim(sys,u+w);     % w = process noise
yv = + v;            % v = measurement noise
```

Given the initial conditions

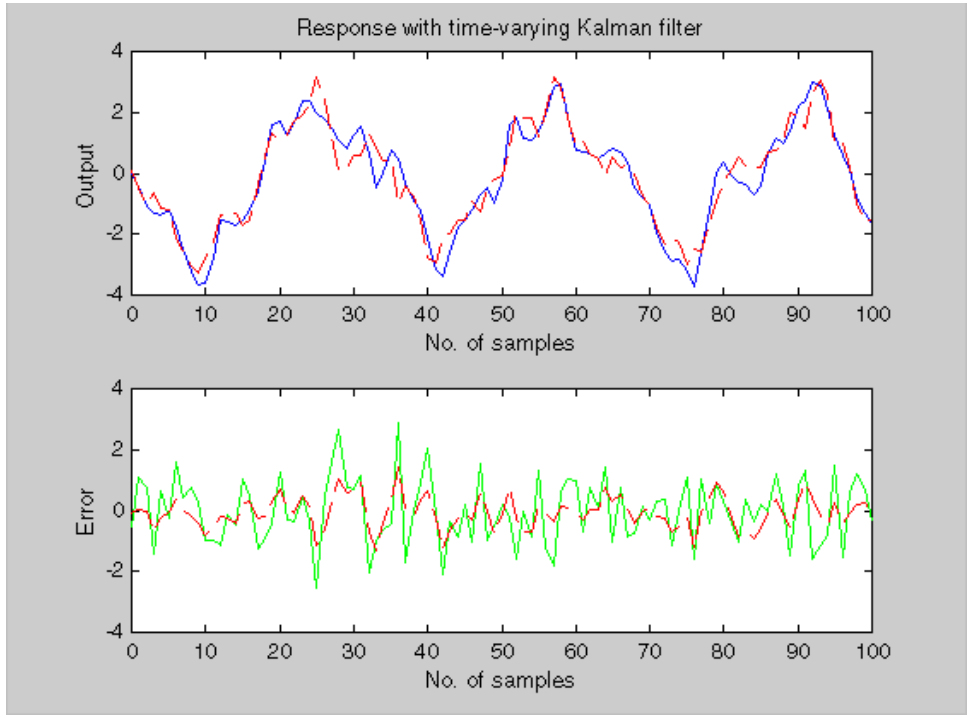
$$x[1|0] = 0\,, \qquad P[1|0] = BQB^T$$

you can implement the time-varying filter with the following `for` loop.

```
P = B*Q*B';         % Initial error covariance
x = zeros(3,1);     % Initial condition on the state
ye = zeros(length(t),1);
ycov = zeros(length(t),1);

for i=1:length(t)
  % Measurement update
  Mn = P*C'/(C*P*C'+R);
  x = x + Mn*(yv(i)-C*x);   % x[n|n]
  P = (eye(3)-Mn*C)*P;      % P[n|n]

  ye(i) = C*x;
  errcov(i) = C*P*C';

  % Time update
  x = A*x + B*u(i);        % x[n+1|n]
  P = A*P*A' + B*Q*B';     % P[n+1|n]
end
```

You can now compare the true and estimated output graphically.

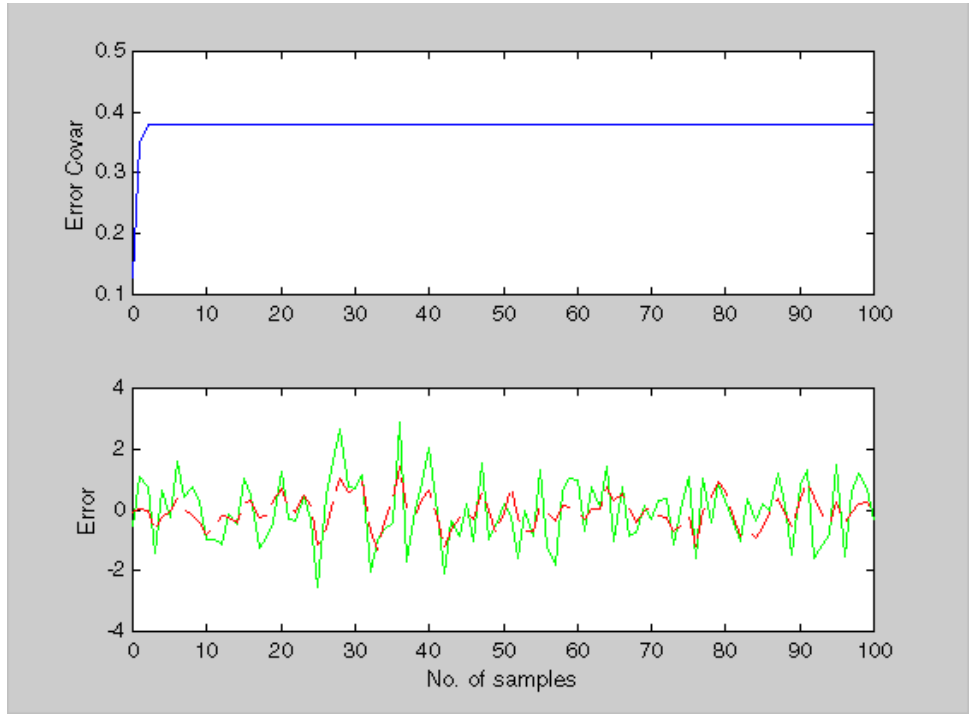
```
subplot(211), plot(t,y,'--',t,ye,'-')
subplot(212), plot(t,y-yv,'-.',t,y-ye,'-')
```



The first plot shows the true response y (dashed line) and the filtered response y_e (solid line). The second plot compares the measurement error (dash-dot) with the estimation error (solid).

The time-varying filter also estimates the covariance `errcov` of the estimation error $y - y_e$ at each sample. Plot it to see if your filter reached steady state (as you expect with stationary input noise).

```
subplot(211)
plot(t,errcov), ylabel('Error covar')
```



From this covariance plot, you can see that the output covariance did indeed reach a steady state in about five samples. From then on, your time-varying filter has the same performance as the steady-state version.

Compare with the estimation error covariance derived from the experimental data. Type

```
EstErr = y–ye;
EstErrCov = sum(EstErr.*EstErr)/length(EstErr)

EstErrCov =
    0.2718
```

This value is smaller than the theoretical value `errcov` and close to the value obtained for the steady-state design.

Finally, note that the final value $M[n]$ and the steady-state value M of the innovation gain matrix coincide.

```
Mn, M

Mn =
    0.3798
    0.0817
   –0.2570

M =
    0.3798
    0.0817
   –0.2570
```

References

[1] Grimble, M.J., *Robust Industrial Control: Optimal Design Approach for Polynomial Systems*, Prentice Hall, 1994, p. 261 and pp. 443–456.

10

Reliable Computations

When working with low-order SISO models (less than five states), computers are usually quite forgiving and insensitive to numerical problems. You generally won't encounter any numerical difficulties and MATLAB will give you accurate answers regardless of the model or conversion method you choose. For high order SISO models and MIMO models, however, the finite-precision arithmetic of a computer is not so forgiving and you must exercise caution.

In general, to get a numerically accurate answer from a computer, you need:

- A well-conditioned problem
- An algorithm that is numerically stable in finite-precision arithmetic
- A good software implementation of the algorithm

A problem is said to be well-conditioned if small changes in the data cause only small corresponding changes in the solution. If small changes in the data have the potential to induce large changes in the solution, the problem is said to be ill-conditioned. An algorithm is numerically stable if it does not introduce any more sensitivity to perturbation than is already inherent in the problem. Many numerical linear algebra algorithms can be shown to be backward stable; i.e., the computed solution can be shown to be (near) the exact solution of a slightly perturbed original problem. The solution of a slightly perturbed original problem will be close to the true solution if the problem is well-conditioned.

Thus, a stable algorithm cannot be expected to solve an ill-conditioned problem any more accurately than the data warrant, but an unstable algorithm can produce poor solutions even to well-conditioned problems. For further details and references to the literature see [5].

While most of the tools in the Control System Toolbox use reliable algorithms, some of the tools do not use stable algorithms and some solve ill-conditioned problems. These unreliable tools work quite well on some problems (low-order systems) but can encounter numerical difficulties, often severe, when pushed on higher-order problems. These tools are provided because:

- They are quite useful for low-order systems, which form the bulk of real-world engineering problems.
- Many control engineers think in terms of these tools.
- A more reliable alternative tool is usually available in this toolbox.
- They are convenient for pedagogical purposes.

At the same time, it is important to appreciate the limitations of computer analyses. By following a few guidelines, you can avoid certain tools and models when they are likely to get you into trouble. The following sections try to illustrate, through examples, some of the numerical pitfalls to be avoided. We also encourage you to get the most out of the good algorithms by ensuring, if possible, that your models give rise to problems that are well-conditioned.

Conditioning and Numerical Stability

Two of the key concepts in numerical analysis are the conditioning of problems and the stability of algorithms.

Conditioning

Consider the linear system $Ax = b$ given by

```
A =
    0.7800    0.5630
    0.9130    0.6590
b =
    0.2170
    0.2540
```

The true solution is `x = [1, -1]'` and you can calculate it approximately using MATLAB.

```
x = A\b
x =
     1.0000
    -1.0000

format long, x
x =
     0.99999999991008
    -0.99999999987542
```

Of course, in real problems you almost never have the luxury of knowing the true solution. This problem is very ill-conditioned. To see this, add a small perturbation to `A`

```
E =
     0.0010     0.0010
    -0.0020    -0.0010
```

and solve the perturbed system $(A + E)x = b$

```
xe = (A+E)\b
xe =
    -5.0000
     7.3085
```

Notice how much the small change in the data is magnified in the solution.

One way to measure the magnification factor is by means of the quantity

$$\|A\| \, \|A^{-1}\|$$

called the condition number of A with respect to inversion. The condition number determines the loss in precision due to roundoff errors in Gaussian elimination and can be used to estimate the accuracy of results obtained from matrix inversion and linear equation solution. It arises naturally in perturbation theories that compare the perturbed solution $(A + E)^{-1}b$ with the true solution $A^{-1}b$.

In MATLAB, the function `cond` calculates the condition number in 2-norm. `cond(A)` is the ratio of the largest singular value of `A` to the smallest. Try it for the example above. The usual rule is that the exponent `log10(cond(A))` on the condition number indicates the number of decimal places that the computer can lose to roundoff errors.

IEEE standard double precision numbers have about 16 decimal digits of accuracy, so if a matrix has a condition number of 10^{10}, you can expect only six digits to be accurate in the answer. If the condition number is much greater than `1/sqrt(eps)`, caution is advised for subsequent computations. For IEEE arithmetic, the machine precision, `eps`, is about 2.2×10^{-16}, and `1/sqrt(eps)` = 6.7×10^{8}.

Another important aspect of conditioning is that, in general, residuals are reliable indicators of accuracy only if the problem is well-conditioned. To illustrate, try computing the residual vector $r = Ax - b$ for the two candidate solutions `x = [0.999 -1.001]'` and `x = [0.341 -0.087]'`. Notice that the second, while clearly a much less accurate solution, gives a far smaller residual. The conclusion is that residuals are unreliable indicators of relative solution accuracy for ill-conditioned problems. This is a good reason to be concerned with computing or estimating accurately the condition of your problem.

Another simple example of an ill-conditioned problem is the n-by-n matrix with ones on the first upper-diagonal.

```
A = diag(ones(1,n-1),1);
```

This matrix has n eigenvalues at 0. Now consider a small perturbation of the data consisting of adding the number 2^{-n} to the first element in the last (n th)

row of A. This perturbed matrix has n distinct eigenvalues $\lambda_1, ..., \lambda_n$ with $\lambda_k = 1/2 \exp(j2\pi k/n)$. Thus, you can see that this small perturbation in the data has been magnified by a factor on the order of 2^n to result in a rather large perturbation in the solution (the eigenvalues of A). Further details and related examples are to be found in [7].

It is important to realize that a matrix can be ill-conditioned with respect to inversion but have a well-conditioned eigenproblem, and vice versa. For example, consider an upper triangular matrix of ones (zeros below the diagonal) given by

```
A = triu(ones(n));
```

This matrix is ill-conditioned with respect to its eigenproblem (try small perturbations in A(n,1) for, say, n=20), but is well-conditioned with respect to inversion (check its condition number). On the other hand, the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1+\delta \end{bmatrix}$$

has a well-conditioned eigenproblem, but is ill-conditioned with respect to inversion for small δ.

Numerical Stability

Numerical stability is somewhat more difficult to illustrate meaningfully. Consult the references in [5], [6], and [7] for further details. Here is one small example to illustrate the difference between stability and conditioning.

Gaussian elimination with no pivoting for solving the linear system $Ax = b$ is known to be numerically unstable. Consider

$$A = \begin{bmatrix} 0.001 & 1.000 \\ 1.000 & -1.000 \end{bmatrix} \qquad b = \begin{bmatrix} 1.000 \\ 0.000 \end{bmatrix}$$

All computations are carried out in three-significant-figure decimal arithmetic. The true answer $x = A^{-1}b$ is approximately

$$x = \begin{bmatrix} 0.999 \\ 0.999 \end{bmatrix}$$

Using row 1 as the pivot row (i.e., subtracting 1000 times row 1 from row 2) you arrive at the equivalent triangular system.

$$\begin{bmatrix} 0.001 & 1.000 \\ 0 & -1000 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.000 \\ -1000 \end{bmatrix}$$

Note that the coefficient multiplying x_2 in the second equation should be -1001, but because of roundoff, becomes -1000. As a result, the second equation yields $x_2 = 1.000$, a good approximation, but now back-substitution in the first equation

$$0.001x_1 = 1.000 - (1.000)(1.000)$$

yields $x_1 = 0.000$. This extremely bad approximation of x_1 is the result of numerical instability. The problem itself can be shown to be quite well-conditioned. Of course, MATLAB implements Gaussian elimination with pivoting.

Choice of LTI Model

Now turn to the implications of the results in the last section on the linear modeling techniques used for control engineering. The Control System Toolbox includes the following types of LTI models that are applicable to discussions of computational reliability:

- State space
- Transfer function, polynomial form
- Transfer function, factored zero-pole-gain form

The following subsections show that state space is most preferable for numerical computations.

State Space

The state-space representation is the most reliable LTI model to use for computer analysis. This is one of the reasons for the popularity of “modern” state-space control theory. Stable computer algorithms for eigenvalues, frequency response, time response, and other properties of the (A, B, C, D) quadruple are known [5] and implemented in this toolbox. The state-space model is also the most natural model in MATLAB's matrix environment.

Even with state-space models, however, accurate results are not guaranteed, because of the problems of finite-word-length computer arithmetic discussed in the last section. A well-conditioned problem is usually a prerequisite for obtaining accurate results and makes it important to have reasonable scaling of the data. Scaling is discussed further in the “Scaling” section later in this chapter.

Transfer Function

Transfer function models, when expressed in terms of expanded polynomials, tend to be inherently ill-conditioned representations of LTI systems. For systems of order greater than 10, or with very large/small polynomial coefficients, difficulties can be encountered with functions like `roots`, `conv`, `bode`, `step`, or conversion functions like `ss` or `zpk`.

A major difficulty is the extreme sensitivity of the roots of a polynomial to its coefficients. This example is adapted from Wilkinson, [6] as an illustration. Consider the transfer function

$$H(s) = \frac{1}{(s+1)(s+2)...(s+20)} = \frac{1}{s^{20} + 210s^{19} + ... + 20!}$$

The A matrix of the companion realization of $H(s)$ is

$$A = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ : & : & . & . & : \\ 0 & 0 & ... & . & 1 \\ -20! & . & ... & . & -210 \end{bmatrix}$$

Despite the benign looking poles of the system (at –1,–2,..., –20) you are faced with a rather large range in the elements of A, from 1 to $20! \approx 2.4 \times 10^{18}$. But the difficulties don't stop here. Suppose the coefficient of s^{19} in the transfer function (or $A(n, n)$) is perturbed from 210 to $210 + 2^{-23}$ ($2^{-23} \approx 1.2 \times 10^{-7}$). Then, computed on a VAX (IEEE arithmetic has enough mantissa for only $n = 17$), the poles of the perturbed transfer function (equivalently, the eigenvalues of A) are

```
eig(A)'

ans =

Columns 1 through 7
-19.9998 -19.0019 -17.9916 -17.0217 -15.9594 -15.0516 -13.9504

Columns 8 through 14
-13.0369 -11.9805 -11.0081  -9.9976  -9.0005  -7.9999  -7.0000

Columns 15 through 20
 -6.0000  -5.0000  -4.0000  -3.0000  -2.0000  -1.0000
```

The problem here is not roundoff. Rather, high-order polynomials are simply intrinsically very sensitive, even when the zeros are well separated. In this case, a relative perturbation of the order of 10^{-9} induced relative perturbations of the order of 10^{-2} in some roots. But some of the roots changed

very little. This is true in general. Different roots have different sensitivities to different perturbations. Computed roots may then be quite meaningless for a polynomial, particularly high-order, with imprecisely known coefficients.

Finding all the roots of a polynomial (equivalently, the poles of a transfer function or the eigenvalues of a matrix in controllable or observable canonical form) is often an intrinsically sensitive problem. For a clear and detailed treatment of the subject, including the tricky numerical problem of deflation, consult [6].

It is therefore preferable to work with the factored form of polynomials when available. To compute a state-space model of the transfer function $H(s)$ defined above, for example, you could expand the denominator of H, convert the transfer function model to state space, and extract the state-space data by

```
H1 = tf(1,poly(1:20))
H1ss = ss(H1)
[a1,b1,c1] = ssdata(H1)
```

However, you should rather keep the denominator in factored form and work with the zero-pole-gain representation of $H(s)$.

```
H2 = zpk([],1:20,1)
H2ss = ss(H2)
[a2,b2,c2] = ssdata(H2)
```

Indeed, the resulting state matrix `a2` is better conditioned.

```
[cond(a1)  cond(a2)]

ans =
   2.7681e+03   8.8753e+01
```

and the conversion from zero-pole-gain to state space incurs no loss of accuracy in the poles.

```
format long e
[sort(eig(a1))  sort(eig(a2))]

ans =
     9.999999999998792e-01     1.000000000000000e+00
     2.000000000001984e+00     2.000000000000000e+00
     3.000000000475623e+00     3.000000000000000e+00
     3.999999981263996e+00     4.000000000000000e+00
     5.000000270433721e+00     5.000000000000000e+00
     5.999998194359617e+00     6.000000000000000e+00
     7.000004542844700e+00     7.000000000000000e+00
     8.000013753274901e+00     8.000000000000000e+00
     8.999848908317270e+00     9.000000000000000e+00
     1.000059459550623e+01     1.000000000000000e+01
     1.099854678336595e+01     1.100000000000000e+01
     1.200255822210095e+01     1.200000000000000e+01
     1.299647702454549e+01     1.300000000000000e+01
     1.400406940833612e+01     1.400000000000000e+01
     1.499604787386921e+01     1.500000000000000e+01
     1.600304396718421e+01     1.600000000000000e+01
     1.699828695210055e+01     1.700000000000000e+01
     1.800062935148728e+01     1.800000000000000e+01
     1.899986934359322e+01     1.900000000000000e+01
     2.000001082693916e+01     2.000000000000000e+01
```

There is another difficulty with transfer function models when realized in state-space form with ss. They may give rise to badly conditioned eigenvector matrices, even if the eigenvalues are well separated. For example, consider the normal matrix

```
A = [5 4 1 1
     4 5 1 1
     1 1 4 2
     1 1 2 4]
```

Its eigenvectors and eigenvalues are given as follows.

```
[v,d] = eig(A)

v =
    0.7071   -0.0000   -0.3162    0.6325
   -0.7071    0.0000   -0.3162    0.6325
    0.0000    0.7071    0.6325    0.3162
   -0.0000   -0.7071    0.6325    0.3162

d =
    1.0000         0         0         0
         0    2.0000         0         0
         0         0    5.0000         0
         0         0         0   10.0000
```

The condition number (with respect to inversion) of the eigenvector matrix is

```
cond(v)

ans =
    1.000
```

Now convert a state-space model with the above A matrix to transfer function form, and back again to state-space form.

```
b = [1 ; 1 ; 0 ; -1];
c = [0 0 2 1];
H = tf(ss(A,b,c,0));      % transfer function
[Ac,bc,cc] = ssdata(H)    % convert back to state space
```

The new A matrix is

```
Ac =
   18.0000   -6.0625    2.8125   -1.5625
   16.0000         0         0         0
         0    4.0000         0         0
         0         0    1.0000         0
```

Note that Ac is not a standard companion matrix and has already been balanced as part of the ss conversion (see ssbal for details).

Note also that the eigenvectors have changed.

```
[vc,dc] = eig(Ac)

vc =
   -0.5017    0.2353    0.0510    0.0109
   -0.8026    0.7531    0.4077    0.1741
   -0.3211    0.6025    0.8154    0.6963
   -0.0321    0.1205    0.4077    0.6963

dc =
   10.0000         0         0         0
         0    5.0000         0         0
         0         0    2.0000         0
         0         0         0    1.0000
```

The condition number of the new eigenvector matrix

```
cond(vc)

ans =
    34.5825
```

is thirty times larger.

The phenomenon illustrated above is not unusual. Matrices in companion form or controllable/observable canonical form (like `Ac`) typically have worse-conditioned eigensystems than matrices in general state-space form (like `A`). This means that their eigenvalues and eigenvectors are more sensitive to perturbation. The problem generally gets far worse for higher-order systems. Working with high-order transfer function models and converting them back and forth to state space is numerically risky.

In summary, the main numerical problems to be aware of in dealing with transfer function models (and hence, calculations involving polynomials) are:

- The potentially large range of numbers leads to ill-conditioned problems, especially when such models are linked together giving high-order polynomials.

- The pole locations are very sensitive to the coefficients of the denominator polynomial.
- The balanced companion form produced by ss, while better than the standard companion form, often results in ill-conditioned eigenproblems, especially with higher-order systems.

The above statements hold even for systems with distinct poles, but are particularly relevant when poles are multiple.

Zero-Pole-Gain Models

The third major representation used for LTI models in MATLAB is the factored, or zero-pole-gain (ZPK) representation. It is sometimes very convenient to describe a model in this way although most major design methodologies tend to be oriented towards either transfer functions or state-space.

In contrast to polynomials, the ZPK representation of systems can be more reliable. At the very least, the ZPK representation tends to avoid the extraordinary arithmetic range difficulties of polynomial coefficients, as illustrated in the “Transfer Function” section. The transformation from state space to zero-pole-gain is stable, although the handling of infinite zeros can sometimes be tricky, and repeated roots can cause problems.

If possible, avoid repeated switching between different model representations. As discussed in the previous sections, when transformations between models are not numerically stable, roundoff errors are amplified.

Scaling

State space is the preferred model for LTI systems, especially with higher order models. Even with state-space models, however, accurate results are not guaranteed, because of the finite-word-length arithmetic of the computer. A well-conditioned problem is usually a prerequisite for obtaining accurate results.

You should generally normalize or scale the (A, B, C, D) matrices of a system to improve their conditioning. An example of a poorly scaled problem might be a dynamic system where two states in the state vector have units of light years and millimeters. You would expect the A matrix to contain both very large and very small numbers. Matrices containing numbers widely spread in value are often poorly conditioned both with respect to inversion and with respect to their eigenproblems, and inaccurate results can ensue.

Normalization also allows meaningful statements to be made about the degree of controllability and observability of the various inputs and outputs.

A set of (A, B, C, D) matrices can be normalized using diagonal scaling matrices N_u, N_x, and N_y to scale u, x, and y.

$$u = N_u u_n \qquad x = N_x x_n \qquad y = N_y y_n$$

so the normalized system is

$$\dot{x}_n = A_n x_n + B_n u_n$$
$$y_n = C_n x_n + D_n u_n$$

where

$$A_n = N_x^{-1} A N_x \qquad B_n = N_x^{-1} B N_u$$
$$C_n = N_y^{-1} C N_x \qquad D_n = N_y^{-1} D N_u$$

Choose the diagonal scaling matrices according to some appropriate normalization procedure. One criterion is to choose the maximum range of each of the input, state, and output variables. This method originated in the days of analog simulation computers when u_n, x_n, and y_n were forced to be between ± 10 Volts. A second method is to form scaling matrices where the diagonal entries are the smallest deviations that are significant to each variable. An

excellent discussion of scaling is given in the introduction to the *LINPACK Users' Guide*, [1].

Choose scaling based upon physical insight to the problem at hand. If you choose not to scale, and for many small problems scaling is not necessary, be aware that this choice affects the accuracy of your answers.

Finally, note that the function `ssbal` performs automatic scaling of the state vector. Specifically, it seeks to minimize the norm of

$$\begin{bmatrix} N_x^{-1} A N_x & N_x^{-1} B \\ C N_x & 0 \end{bmatrix}$$

by using diagonal scaling matrices N_x. Such diagonal scaling is an economical way to compress the numerical range and improve the conditioning of subsequent state-space computations.

Summary

This chapter has described numerous things that can go wrong when performing numerical computations. You won't encounter most of these difficulties when you solve practical lower-order problems. The problems described here pertain to all computer analysis packages. MATLAB has some of the best algorithms available, and, where possible, notifies you when there are difficulties. The important points to remember are:

- State-space models are, in general, the most reliable models for subsequent computations.
- Scaling model data can improve the accuracy of your results.
- Numerical computing is a tricky business, and virtually all computer tools can fail under certain conditions.

References

[1] Dongarra, J.J., J.R. Bunch, C.B. Moler, and G.W. Stewart, *LINPACK Users Guide*, SIAM Publications, Philadelphia, PA, 1978.

[2] Franklin, G.F. and J.D. Powell, *Digital Control of Dynamic Systems*, Addison-Wesley, 1980.

[3] Kailath, T., *Linear Systems*, Prentice-Hall, 1980.

[4] Laub, A.J., "Numerical Linear Algebra Aspects of Control Design Computations," *IEEE Transactions on Automatic Control*, Vol. AC-30, No. 2, February 1985, pp. 97-108.

[5] Wilkinson, J.H., *Rounding Errors in Algebraic Processes*, Prentice-Hall, 1963.

[6] Wilkinson, J.H., *The Algebraic Eigenvalue Problem*, Oxford University Press, 1965.

11

Reference

This chapter contains detailed descriptions of all Control System Toolbox functions. It begins with a list of functions grouped by subject area and continues with the reference entries in alphabetical order. Information is also available through the online Help facility.

Category Tables

Table 11-1: LTI Models

Function Name	Description
drss	Generate random discrete state-space model.
dss	Create descriptor state-space model.
filt	Create discrete filter with DSP convention.
frd	Create a frequency response data (FRD) model.
frdata	Retrieve data from an FRD model.
get	Query LTI model properties.
ltimodels	Information on LTI models
ltiprops	Information on all LTI properties.
set	Set LTI or response object properties.
rss	Generate random continuous state-space model.
ss	Create state-space model.
ssdata, dssdata	Retrieve state-space data or covert data to cell array format.
tf	Create transfer function.
tfdata	Retrieve transfer function data.
totaldelay	Provide the aggregate delay for an LTI model.
zpk	Create zero-pole-gain model.
zpkdata	Retrieve zero-pole-gain data.

Table 11-2: Model Characteristics

Function Name	Description
`class`	Display model type (`'tf'`, `'zpk'`, `'ss'`, or `'frd'`).
`hasdelay`	Test true if LTI model has any type of delay.
`isa`	Test true if LTI model is of specified type.
`isct`	Test true for continuous-time models.
`isdt`	Test true for discrete-time models.
`isempty`	Test true for empty LTI models.
`isproper`	Test true for proper LTI models.
`issiso`	Test true for SISO models.
`ndims`	Get the number of model/array dimensions.
`size`	Get output/input/array dimensions or model order.

Table 11-3: Model Conversion

Function Name	Description
`c2d`	Convert from continuous- to discrete-time models.
`chgunits`	Convert the `units` property for FRD models.
`d2c`	Convert from discrete- to continuous-time models.
`d2d`	Resample discrete-time models.
`delay2z`	Convert delays in discrete-time models or FRD models.
`frd`	Convert to a frequency response data model.
`pade`	Compute the Padé approximation of delays.
`reshape`	Change the shape of an LTI array

Table 11-3: Model Conversion (Continued)

Function Name	Description
`residue`	Provide partial fraction expansion (see *Using MATLAB*).
`ss`	Convert to a state space model.
`tf`	Convert to a transfer function model.
`zpk`	Convert to a zero-pole-gain model.

Table 11-4: Model Order Reduction

Function Name	Description
`balreal`	Calculate an I/O balanced realization.
`minreal`	Calculate minimal realization or pole/zero cancellation.
`modred`	Delete states in I/O balanced realization.
`sminreal`	Calculate structured model reduction.

Table 11-5: State-Space Realizations

Function Name	Description
`canon`	Canonical state-space realizations.
`ctrb`	Controllability matrix.
`ctrbf`	Controllability staircase form.
`gram`	Controllability and observability gramians.
`obsv`	Observability matrix.
`obsvf`	Observability staircase form.

Table 11-5: State-Space Realizations (Continued)

Function Name	Description
ss2ss	State coordinate transformation.
ssbal	Diagonal balancing of state-space realizations.

Table 11-6: Model Dynamics

Function Name	Description
damp	Calculate natural frequency and damping.
dcgain	Calculate low-frequency (DC) gain.
covar	Calculate covariance of response to white noise.
dsort	Sort discrete-time poles by magnitude.
esort	Sort continuous-time poles by real part.
norm	Calculate norms of LTI models (H_2 and L_∞).
pole, eig	Calculate the poles of an LTI model.
pzmap	Plot the pole/zero map of an LTI model.
roots	Calculate roots of polynomial (see *Using MATLAB*).
zero	Calculate zeros of an LTI model.

Table 11-7: Model Building

Function Name	Description
append	Append models in a block diagonal configuration.
augstate	Augment output by appending states.

Table 11-7: Model Building (Continued)

Function Name	Description
`connect`	Connect the subsystems of a block-diagonal model according to an interconnection scheme of your choice.
`conv`	Convolve two polynomials (see *Using MATLAB*).
`drmodel, drss`	Generate random discrete-time model.
`feedback`	Calculate the feedback connection of models.
`lft`	Calculate the star product (LFT interconnection).
`ord2`	Generate second-order model.
`pade`	Compute the Padé approximation of time delays.
`parallel`	Create a generalized parallel connection.
`rmodel, rss`	Generate random continuous model.
`series`	Create a generalized series connection.
`stack`	Concatenate LTI models along array dimensions.

Table 11-8: Time Response

Function Name	Description
`filter`	Simulate discrete SISO filter (see *Using MATLAB*).
`gensig`	Generate an input signal.
`impulse`	Calculate impulse response.
`initial`	Calculate initial condition response.
`lsim`	Simulate response of LTI model to arbitrary inputs.

Table 11-8: Time Response (Continued)

Function Name	Description
`ltiview`	Open the LTI Viewer for linear response analysis.
`step`	Calculate step response.

Table 11-9: Frequency Response

Function Name	Description
`bode`	Calculate bode plot.
`evalfr`	Evaluate response at single complex frequency.
`freqresp`	Evaluate frequency response for selected frequencies.
`linspace`	Create a vector of evenly spaced frequencies.
`logspace`	Create a vector of logarithmically spaced frequencies.
`ltiview`	Open the LTI Viewer for linear response analysis.
`margin`	Calculate gain and phase margins.
`ngrid`	Superimpose grid lines on a Nichols plot.
`nichols`	Calculate Nichols plot.
`nyquist`	Calculate Nyquist plot.
`pzmap`	Calculate pole/zero map.
`rlocus`	Calculate root locus.
`rlocfind`	Find gain/pole on root locus.
`rltool`	Open Root Locus Design GUI.
`sgrid`	Superimpose s-plane grid on root locus or pole/zero map.

Table 11-9: Frequency Response (Continued)

Function Name	Description
`sigma`	Calculate singular value plot.
`zgrid`	Superimpose z-plane grid on root locus or pole/zero map.

Table 11-10: Pole Placement

Function Name	Description
`acker`	Calculate SISO pole placement design.
`place`	Calculate MIMO pole placement design.
`estim`	Form state estimator given estimator gain.
`reg`	Form output-feedback compensator given state-feedback and estimator gains.
`rltool`	Open Root Locus Design GUI

Table 11-11: LQG Design

Function Name	Description
`lqr`	Calculate the LQ-optimal gain for continuous models.
`dlqr`	Calculate the LQ-optimal gain for discrete models.
`lqry`	Calculate the LQ-optimal gain with output weighting.
`lqrd`	Calculate the discrete LQ gain for continuous models.
`kalman`	Calculate the Kalman estimator.

Table 11-11: LQG Design (Continued)

Function Name	Description
`kalmd`	Calculate the discrete Kalman estimator for continuous models.
`lqgreg`	Form LQG regulator given LQ gain and Kalman filter.

Table 11-12: Equation Solvers

Function Name	Description
`care`	Solve continuous-time algebraic Riccati equations.
`dare`	Solve discrete-time algebraic Riccati equations.
`lyap`	Solve continuous-time Lyapunov equations.
`dlyap`	Solve discrete-time Lyapunov equations.

Table 11-13: Graphical User Interfaces for Model Analysis and Design

Function Name	Description
`ltiview`	Open the LTI Viewer for linear response analysis.
`rltool`	Open the Root Locus Design GUI.

Purpose

Pole placement design for single-input systems

Syntax

```
k = acker(A,b,p)
```

Description

Given the single-input system

$$\dot{x} = Ax + bu$$

and a vector `p` of desired closed-loop pole locations, `acker(A,b,p)` uses Ackermann's formula [1] to calculate a gain vector `k` such that the state feedback $u = -kx$ places the closed-loop poles at the locations `p`. In other words, the eigenvalues of $A - bk$ match the entries of `p` (up to ordering). Here `A` is the state transmitter matrix and `b` is the input to state transmission vector.

You can also use `acker` for estimator gain selection by transposing the matrix `A` and substituting `c'` for `b` when $y = cx$ is a single output.

```
l = acker(a',c',p).'
```

Limitations

`acker` is limited to single-input systems and the pair (A, b) must be controllable.

Note that this method is not numerically reliable and starts to break down rapidly for problems of order greater than 5 or for weakly controllable systems. See `place` for a more general and reliable alternative.

See Also

`lqr`	Optimal LQ regulator
`place`	Pole placement design
`rlocus`, `rlocfind`	Root locus design

References

[1] Kailath, T., *Linear Systems*, Prentice-Hall, 1980, p. 201.

Purpose

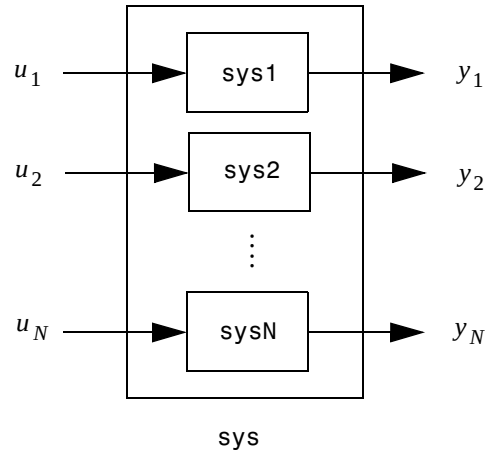
Group LTI models by appending their inputs and outputs

Syntax

```
sys = append(sys1,sys2,...,sysN)
```

Description

append appends the inputs and outputs of the LTI models sys1,...,sysN to form the augmented model sys depicted below.



For systems with transfer functions $H_1(s)$,...,$H_N(s)$, the resulting system sys has the block-diagonal transfer function

$$\begin{bmatrix} H_1(s) & 0 & .. & 0 \\ 0 & H_2(s) & . & : \\ : & . & . & 0 \\ 0 & .. & 0 & H_N(s) \end{bmatrix}$$

For state-space models sys1 and sys2 with data (A_1, B_1, C_1, D_1) and (A_2, B_2, C_2, D_2), append(sys1,sys2) produces the following state-space model.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}A_1 & 0\\ 0 & A_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}B_1 & 0\\ 0 & B_2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}C_1 & 0\\ 0 & C_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}D_1 & 0\\ 0 & D_2\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

Arguments

The input arguments `sys1`,..., `sysN` can be LTI models of any type. Regular matrices are also accepted as a representation of static gains, but there should be at least one LTI object in the input list. The LTI models should be either all continuous, or all discrete with the same sample time. When appending models of different types, the resulting type is determined by the precedence rules (see "Precedence Rules" on page 2-5 for details).

There is no limitation on the number of inputs.

Example

The commands

```
sys1 = tf(1,[1 0])
sys2 = ss(1,2,3,4)
sys = append(sys1,10,sys2)
```

produce the state-space model

```
sys

a =
                    x1          x2
           x1        0           0
           x2        0     1.00000

b =
                    u1          u2          u3
           x1  1.00000           0           0
           x2        0           0     2.00000
```

```
c =
                x1          x2
   y1      1.00000           0
   y2            0           0
   y3            0     3.00000

d =
                u1          u2          u3
   y1            0           0           0
   y2            0    10.00000           0
   y3            0           0     4.00000

Continuous-time system.
```

See Also

`connect`	Modeling of block diagram interconnections
`feedback`	Feedback connection
`parallel`	Parallel connection
`series`	Series connection

Purpose

Append the state vector to the output vector

Syntax

```
asys = augstate(sys)
```

Description

Given a state-space model sys with equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

(or their discrete-time counterpart), augstate appends the states x to the outputs y to form the model

$$\dot{x} = Ax + Bu$$
$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} x + \begin{bmatrix} D \\ 0 \end{bmatrix} u$$

This command prepares the plant so that you can use the feedback command to close the loop on a full-state feedback $u = -Kx$.

Limitation

Because augstate is only meaningful for state-space models, it cannot be used with TF, ZPK or FRD models.

See Also

feedback	Feedback connection
parallel	Parallel connection
series	Series connection

balreal

Purpose

Input/output balancing of state-space realizations

Syntax

```
sysb = balreal(sys)
[sysb,g,T,Ti] = balreal(sys)
```

Description

`sysb = balreal(sys)` produces a balanced realization `sysb` of the LTI model `sys` with equal and diagonal controllability and observability gramians (see `gram` for a definition of gramian). `balreal` handles both continuous and discrete systems. If `sys` is not a state-space model, it is first and automatically converted to state space using `ss`.

`[sysb,g,T,Ti] = balreal(sys)` also returns the vector `g` containing the diagonal of the balanced gramian, the state similarity transformation $x_b = Tx$ used to convert `sys` to `sysb`, and the inverse transformation `Ti` = T^{-1}.

If the system is normalized properly, the diagonal `g` of the joint gramian can be used to reduce the model order. Because `g` reflects the combined controllability and observability of individual states of the balanced model, you can delete those states with a small `g(i)` while retaining the most important input-output characteristics of the original system. Use `modred` to perform the state elimination.

Example

Consider the zero-pole-gain model

```
sys = zpk([-10 -20.01],[-5 -9.9 -20.1],1)

Zero/pole/gain:
   (s+10) (s+20.01)
----------------------
(s+5) (s+9.9) (s+20.1)
```

A state-space realization with balanced gramians is obtained by

```
[sysb,g] = balreal(sys)
```

The diagonal entries of the joint gramian are

```
g'

ans =
   1.0062e-01   6.8039e-05   1.0055e-05
```

which indicates that the last two states of sysb are weakly coupled to the input and output. You can then delete these states by

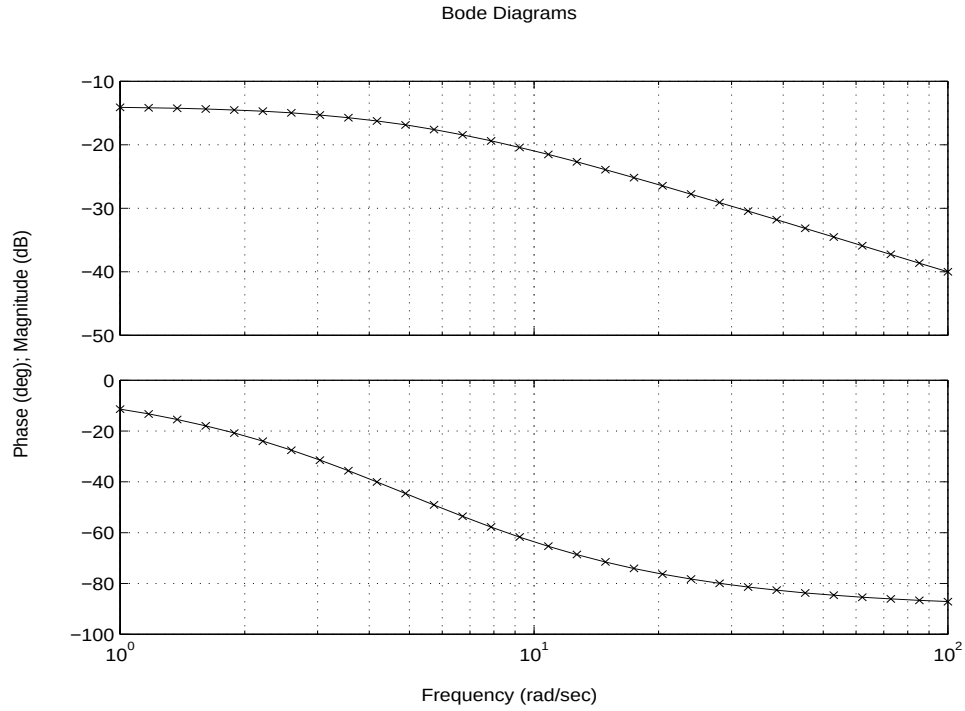
```
sysr = modred(sysb,[2 3],'del')
```

to obtain the following first-order approximation of the original system.

```
zpk(sysr)

Zero/pole/gain:
 1.0001
--------
(s+4.97)
```

Compare the Bode responses of the original and reduced-order models.

```
bode(sys,'-',sysr,'x')
```



Algorithm

Consider the model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

with controllability and observability gramians W_c and W_o. The state coordinate transformation $\bar{x} = Tx$ produces the equivalent model

$$\dot{\bar{x}} = TAT^{-1}\bar{x} + TBu$$
$$y = CT^{-1}\bar{x} + Du$$

and transforms the gramians to

$$\overline{W}_c = TW_cT^T, \qquad \overline{W}_o = T^{-T}W_oT^{-1}$$

The function `balreal` computes a particular similarity transformation T such that

$$\overline{W}_c = \overline{W}_o = diag(g)$$

See [1,2] for details on the algorithm.

Limitations

The LTI model `sys` must be stable. In addition, controllability and observability are required for state-space models.

See Also

`gram`	Controllability and observability gramians
`minreal`	Minimal realizations
`modred`	Model order reduction

References

[1] Laub, A.J., M.T. Heath, C.C. Paige, and R.C. Ward, "Computation of System Balancing Transformations and Other Applications of Simultaneous Diagonalization Algorithms," *IEEE Trans. Automatic Control*, AC-32 (1987), pp. 115–122.

[2] Moore, B., "Principal Component Analysis in Linear Systems: Controllability, Observability, and Model Reduction," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 17–31.

[3] Laub, A.J., "Computation of Balancing Transformations," *Proc. ACC*, San Francisco, Vol.1, paper FA8-E, 1980.

Purpose

Compute the Bode frequency response of LTI models

Syntax

```
bode(sys)
bode(sys,w)

bode(sys1,sys2,...,sysN)
bode(sys1,sys2,...,sysN,w)
bode(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[mag,phase,w] = bode(sys)
```

Description

`bode` computes the magnitude and phase of the frequency response of LTI models. When invoked without left-hand arguments, `bode` produces a Bode plot on the screen. The magnitude is plotted in decibels (dB), and the phase in degrees. The decibel calculation for `mag` is computed as `20*log10(abs(frsys))`, where `frsys` is the frequency response of `sys`. Bode plots are used to analyze system properties such as the gain margin, phase margin, DC gain, bandwidth, disturbance rejection, and stability.

`bode(sys)` plots the Bode response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. In the MIMO case, `bode` produces an array of Bode plots, each plot showing the Bode response of one particular I/O channel. The frequency range is determined automatically based on the system poles and zeros.

`bode(sys,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set `w = {wmin,wmax}`. To use particular frequency points, set `w` to the vector of desired frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. All frequencies should be specified in radians/sec.

`bode(sys1,sys2,...,sysN)` or `bode(sys1,sys2,...,sysN,w)` plots the Bode responses of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but may otherwise be a mix of continuous and discrete systems. This syntax is useful to compare the Bode responses of multiple systems.

`bode(sys1,'PlotStyle1',...,sysN,'PlotStyleN')` specifies which color, linestyle, and/or marker should be used to plot each system. For example,

```
bode(sys1,'r--',sys2,'gx')
```

uses red dashed lines for the first system `sys1` and green `'x'` markers for the second system `sys2`.

When invoked with left-hand arguments

```
[mag,phase,w] = bode(sys)
[mag,phase] = bode(sys,w)
```

return the magnitude and phase (in degrees) of the frequency response at the frequencies `w` (in rad/sec). The outputs `mag` and `phase` are 3-D arrays with the frequency as the last dimension (see "Arguments" below for details). You can convert the magnitude to decibels by

```
magdb = 20*log10(mag)
```

Remark

If `sys` is an FRD model, `bode(sys,w)`, `w` can only include frequencies in `sys.frequency`.

Arguments

The output arguments `mag` and `phase` are 3-D arrays with dimensions

(number of outputs) × (number of inputs) × (length of `w`)

For SISO systems, `mag(1,1,k)` and `phase(1,1,k)` give the magnitude and phase of the response at the frequency ω_k = `w(k)`.

$$\texttt{mag(1,1,k)} = \left|h(j\omega_k)\right|$$
$$\texttt{phase(1,1,k)} = \angle h(j\omega_k)$$

MIMO systems are treated as arrays of SISO systems and the magnitudes and phases are computed for each SISO entry h_{ij} independently (h_{ij} is the transfer function from input j to output i). The values `mag(i,j,k)` and `phase(i,j,k)` then characterize the response of h_{ij} at the frequency `w(k)`.

$$\texttt{mag(i,j,k)} = \left|h_{ij}(j\omega_k)\right|$$
$$\texttt{phase(i,j,k)} = \angle h_{ij}(j\omega_k)$$

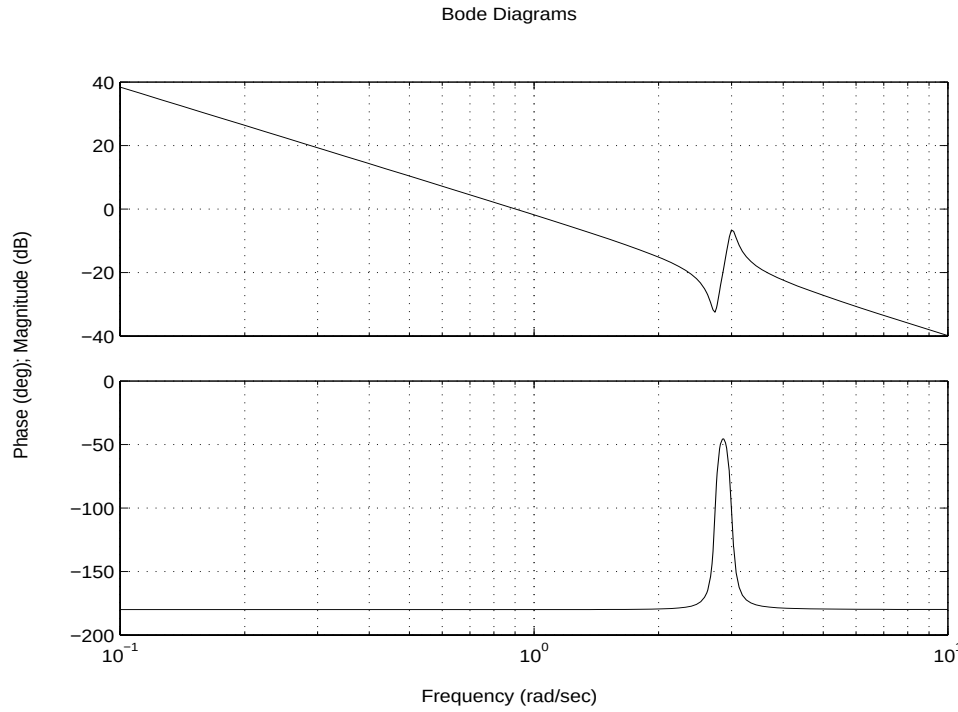
Example

You can plot the Bode response of the continuous SISO system

$$H(s) = \frac{s^2 + 0.1s + 7.5}{s^4 + 0.12s^3 + 9s^2}$$

by typing

```
g = tf([1 0.1 7.5],[1 0.12 9 0 0]);
bode(g)
```

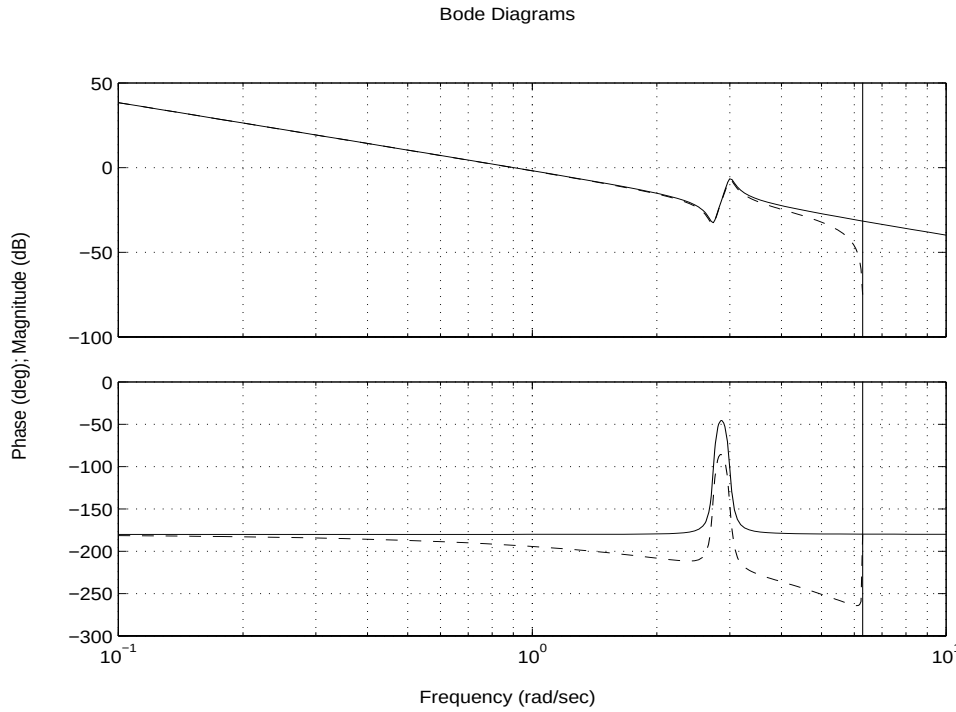


To plot the response on a wider frequency range, for example, from 0.1 to 100 rad/sec, type

```
bode(g,{0.1 , 100})
```

You can also discretize this system using zero-order hold and the sample time $T_s = 0.5$ second, and compare the continuous and discretized responses by typing

```
gd = c2d(g,0.5)
bode(g,'r',gd,'b--')
```



Algorithm

For continuous-time systems, bode computes the frequency response by evaluating the transfer function $H(s)$ on the imaginary axis $s = j\omega$. Only positive frequencies ω are considered. For state-space models, the frequency response is $D + C(j\omega - A)^{-1}B$, $\omega \geq 0$

When numerically safe, A is diagonalized for maximum speed. Otherwise, A is reduced to upper Hessenberg form and the linear equation $(j\omega - A)X = B$ is solved at each frequency point, taking advantage of the Hessenberg

structure. The reduction to Hessenberg form provides a good compromise between efficiency and reliability. See [1] for more details on this technique.

For discrete-time systems, the frequency response is obtained by evaluating the transfer function $H(z)$ on the unit circle. To facilitate interpretation, the upper-half of the unit circle is parametrized as

$$z = e^{j\omega T_s}, \qquad 0 \le \omega \le \omega_N = \frac{\pi}{T_s}$$

where T_s is the sample time. ω_N is called the *Nyquist frequency*. The equivalent "continuous-time frequency" ω is then used as the x-axis variable. Because

$$H(e^{j\omega T_s})$$

is periodic with period $2\omega_N$, `bode` plots the response only up to the Nyquist frequency ω_N. If the sample time is unspecified, the default value $T_s = 1$ is assumed.

Diagnostics

If the system has a pole on the $j\omega$ axis (or unit circle in the discrete case) and `w` happens to contain this frequency point, the gain is infinite, $j\omega I - A$ is singular, and `bode` produces the warning message

```
Singularity in freq. response due to jw-axis or unit circle pole.
```

See Also

`evalfr`	Response at single complex frequency
`freqresp`	Frequency response computation
`ltiview`	LTI system viewer
`nichols`	Nichols plot
`nyquist`	Nyquist plot
`sigma`	Singular value plot

References

[1] Laub, A.J., "Efficient Multivariable Frequency Response Computations," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 407–408.

Purpose

Discretize continuous-time systems

Syntax

```
sysd = c2d(sys,Ts)
sysd = c2d(sys,Ts,method)
```

Description

`sysd = c2d(sys,Ts)` discretizes the continuous-time LTI model `sys` using zero-order hold on the inputs and a sample time of `Ts` seconds.

`sysd = c2d(sys,Ts,method)` gives access to alternative discretization schemes. The string *`method`* selects the discretization method among the following:

`'zoh'`	Zero-order hold. The control inputs are assumed piecewise constant over the sampling period `Ts`.
`'foh'`	Triangle approximation (modified first-order hold, see [1], p. 151). The control inputs are assumed piecewise linear over the sampling period `Ts`.
`'tustin'`	Bilinear (Tustin) approximation.
`'prewarp'`	Tustin approximation with frequency prewarping.
`'matched'`	Matched pole-zero method. See [1], p. 147.

Refer to “Continuous/Discrete Conversions of LTI Models” in Chapter 3 for more detail on these discretization methods.

`c2d` supports MIMO systems (except for the `'matched'` method) as well as LTI models with input delays (`'zoh'` and `'foh'` methods only).

Example

Consider the system

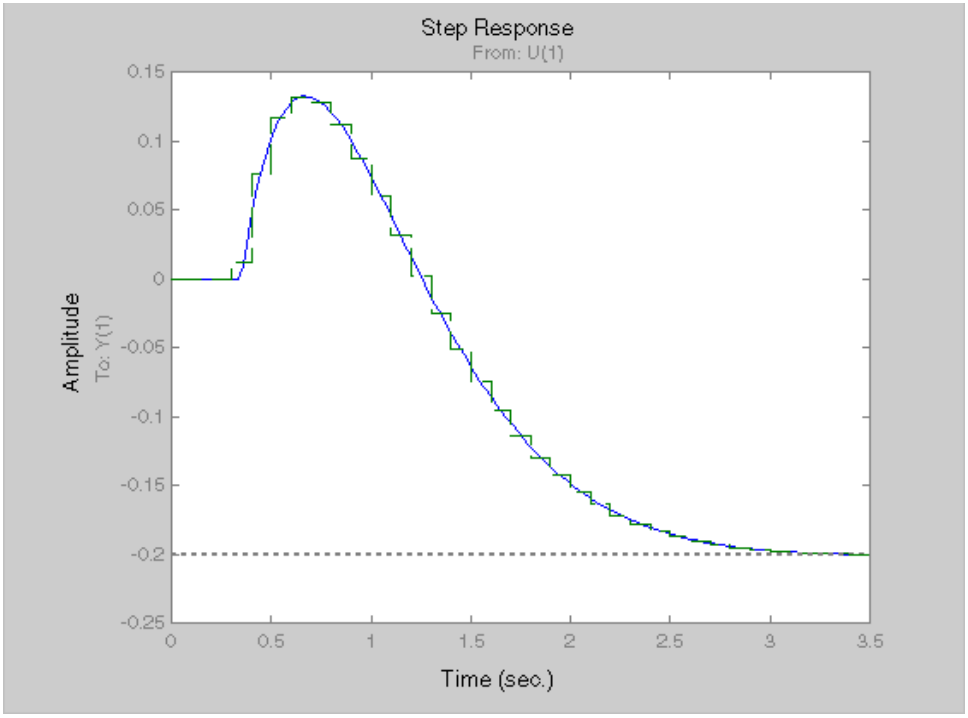
$$H(s) = \frac{s-1}{s^2+4s+5}$$

with input delay $T_d = 0.35$ second. To discretize this system using the triangle approximation with sample time $T_s = 0.1$ second, type

```
H = tf([1 -1],[1 4 5],'inputdelay',0.35)

Transfer function:
                   s - 1
exp(-0.35*s) * -------------
               s^2 + 4 s + 5

Hd = c2d(H,0.1,'foh')

Transfer function:
0.0115 z^3 + 0.0456 z^2 - 0.0562 z - 0.009104
-----------------------------------------------
        z^6 - 1.629 z^5 + 0.6703 z^4

Sampling time: 0.1
```

The next command compares the continuous and discretized step responses.

```
step(H,'-',Hd,'--')
```



See Also

`d2c`	Discrete to continuous conversion
`d2d`	Resampling of discrete systems

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

Purpose

Compute canonical state-space realizations

Syntax

```
csys = canon(sys,'type')
[csys,T] = canon(sys,'type')
```

Description

canon computes a canonical state-space model for the continuous or discrete LTI system sys. Two types of canonical forms are supported.

Modal Form

`csys = canon(sys,'modal')` returns a realization csys in modal form, that is, where the real eigenvalues appear on the diagonal of the A matrix and the complex conjugate eigenvalues appear in 2-by-2 blocks on the diagonal of A. For a system with eigenvalues $(\lambda_1, \sigma \pm j\omega, \lambda_2)$, the modal A matrix is of the form

$$\begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \sigma & \omega & 0 \\ 0 & -\omega & \sigma & 0 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix}$$

Companion Form

`csys = canon(sys,'companion')` produces a companion realization of sys where the characteristic polynomial of the system appears explicitly in the rightmost column of the A matrix. For a system with characteristic polynomial

$$p(s) = s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n$$

the corresponding companion A matrix is

$$A = \begin{bmatrix} 0 & 0 & .. & .. & 0 & -a_n \\ 1 & 0 & 0 & .. & 0 & -a_{n-1} \\ 0 & 1 & 0 & . & : & : \\ : & 0 & . & . & : & : \\ 0 & . & . & 1 & 0 & -a_2 \\ 0 & .. & .. & 0 & 1 & -a_1 \end{bmatrix}$$

For state-space models sys,

```
[csys,T] = canon(a,b,c,d,'type')
```

also returns the state coordinate transformation T relating the original state vector x and the canonical state vector x_c.

$$x_c = Tx$$

This syntax returns T=[] when sys is not a state-space model.

Algorithm

Transfer functions or zero-pole-gain models are first converted to state space using ss.

The transformation to modal form uses the matrix P of eigenvectors of the A matrix. The modal form is then obtained as

$$\dot{x_c} = P^{-1}APx_c + P^{-1}Bu$$
$$y = CPx_c + Du$$

The state transformation T returned is the inverse of P.

The reduction to companion form uses a state similarity transformation based on the controllability matrix [1].

Limitations

The modal transformation requires that the A matrix be diagonalizable. A sufficient condition for diagonalizability is that A has no repeated eigenvalues.

The companion transformation requires that the system be controllable from the first input. The companion form is often poorly conditioned for most state-space computations; avoid using it when possible.

See Also

ctrb	Controllability matrix
ctrbf	Controllability canonical form
ss2ss	State similarity transformation

References

[1] Kailath, T. *Linear Systems*, Prentice-Hall, 1980.

Purpose

Solve continuous-time algebraic Riccati equations (CARE)

Syntax

```
[X,L,G,rr] = care(A,B,Q)
[X,L,G,rr] = care(A,B,Q,R,S,E)

[X,L,G,report] = care(A,B,Q,...,'report')
[X1,X2,L,report] = care(A,B,Q,...,'implicit')
```

Description

`[X,L,G,rr] = care(A,B,Q)` computes the unique solution X of the algebraic Riccati equation

$$Ric(X) = A^T X + XA - XBB^T X + Q = 0$$

such that $A - BB^T X$ has all its eigenvalues in the open left-half plane. The matrix X is symmetric and called the *stabilizing* solution of $Ric(X) = 0$. `[X,L,G,rr] = care(A,B,Q)` also returns:

- The eigenvalues `L` of $A - BB^T X$
- The gain matrix $G = B^T X$
- The relative residual `rr` defined by $rr = \dfrac{\|Ric(X)\|_F}{\|X\|_F}$

`[X,L,G,rr] = care(A,B,Q,R,S,E)` solves the more general Riccati equation

$$Ric(X) = A^T XE + E^T XA - (E^T XB + S)R^{-1}(B^T XE + S^T) + Q = 0$$

Here the gain matrix is $G = R^{-1}(B^T XE + S^T)$ and the "closed-loop" eigenvalues are `L = eig(A-B*G,E)`.

Two additional syntaxes are provided to help develop applications such as H_∞-optimal control design.

`[X,L,G,report] = care(A,B,Q,...,'report')` turns off the error messages when the solution X fails to exist and returns a failure report instead.

The value of `report` is:

- –1 when the associated Hamiltonian pencil has eigenvalues on or very near the imaginary axis (failure)
- –2 when there is no finite solution, i.e., $X = X_2 X_1^{-1}$ with X_1 singular (failure)
- The relative residual rr defined above when the solution exists (success)

Alternatively, `[X1,X2,L,report] = care(A,B,Q,...,'implicit')` also turns off error messages but now returns X in implicit form.

$$X = X_2 X_1^{-1}$$

Note that this syntax returns `report = 0` when successful.

Examples

Example 1

Given

$$A = \begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad R = 3$$

you can solve the Riccati equation

$$A^T X + XA - XBR^{-1}B^T X + C^T C = 0$$

by

```
a = [-3 2;1 1]
b = [0 ; 1]
c = [1 -1]
r = 3
[x,l,g] = care(a,b,c'*c,r)
```

This yields the solution

```
x

x =
    0.5895    1.8216
    1.8216    8.8188
```

You can verify that this solution is indeed stabilizing by comparing the eigenvalues of `a` and `a-b*g`.

```
[eig(a)  eig(a-b*g)]

ans =
   -3.4495   -3.5026
    1.4495   -1.4370
```

Finally, note that the variable `l` contains the closed-loop eigenvalues `eig(a–b*g)`.

```
l

l =
   –3.5026
   –1.4370
```

Example 2

To solve the H_∞-like Riccati equation

$$A^T X + XA + X(\gamma^{-2} B_1 B_1^T - B_2 B_2^T)X + C^T C = 0$$

rewrite it in the `care` format as

$$A^T X + XA - X \underbrace{[B_1, B_2]}_{B} \underbrace{\begin{bmatrix} -\gamma^{-2} I & 0 \\ 0 & I \end{bmatrix}}_{R}^{-1} \begin{bmatrix} B_1^T \\ B_2^T \end{bmatrix} X + C^T C = 0$$

You can now compute the stabilizing solution X by

```
B = [B1 , B2]
m1 = size(B1,2)
m2 = size(B2,2)
R = [–g^2*eye(m1) zeros(m1,m2) ; zeros(m2,m1) eye(m2)]

X = care(A,B,C'*C,R)
```

Algorithm

`care` implements the algorithms described in [1]. It works with the Hamiltonian matrix when R is well-conditioned and $E = I$; otherwise it uses the extended Hamiltonian pencil and QZ algorithm.

Limitations

The (A, B) pair must be stabilizable (that is, all unstable modes are controllable). In addition, the associated Hamiltonian matrix or pencil must have no eigenvalue on the imaginary axis. Sufficient conditions for this to hold are (Q, A) detectable when $S = 0$ and $R > 0$, or

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

See Also

`dare`	Solve discrete-time Riccati equations
`lyap`	Solve continuous-time Lyapunov equations

References

[1] Arnold, W.F., III and A.J. Laub, "Generalized Eigenproblem Algorithms and Software for Algebraic Riccati Equations," *Proc. IEEE*, 72 (1984), pp. 1746–1754.

Purpose

Convert the frequency units of an FRD model

Syntax

```
sys = chgunits(sys,units)
```

Description

`sys = chgunits(sys,units)` converts the units of the frequency points stored in an FRD model, `sys` to `units`, where `units` is either of the strings `'Hz'` or `'rad/s'`. This operation changes the assigned frequencies by applying the appropriate (`2*pi`) scaling factor, and the `'Units'` property is updated.

If the `'Units'` field already matches `units`, no conversion is made.

Example

```
w = logspace(1,2,2);
sys = rss(3,1,1);
sys = frd(sys,w)

From input 'input 1' to:

  Frequency(rad/s)        output 1
  ----------------        --------

           10         0.293773+0.001033i

          100         0.294404+0.000109i

Continuous-time frequency response data.

sys = chgunits(sys,'Hz')
sys.freq

ans =
    1.5915
   15.9155
```

See Also

`frd`	Create or convert to an FRD model
`get`	Get the properties of an LTI model
`set`	Set the properties of an LTI model

Purpose

Derive state-space model from block diagram description

Syntax

```
sysc = connect(sys,Q,inputs,outputs)
```

Description

Complex dynamical systems are often given in block diagram form. For systems of even moderate complexity, it can be quite difficult to find the state-space model required in order to bring certain analysis and design tools into use. Starting with a block diagram description, you can use `append` and `connect` to construct a state-space model of the system.

First, use

```
sys = append(sys1,sys2,...,sysN)
```

to specify each block `sysj` in the diagram and form a block-diagonal, *unconnected* LTI model `sys` of the diagram.

Next, use

```
sysc = connect(sys,Q,inputs,outputs)
```

to connect the blocks together and derive a state-space model `sysc` for the overall interconnection. The arguments `Q`, `inputs`, and `outputs` have the following purpose:

- The matrix `Q` indicates how the blocks on the diagram are connected. It has a row for each input of `sys`, where the first element of each row is the input number. The subsequent elements of each row specify where the block input gets its summing inputs; negative elements indicate minus inputs to the summing junction. For example, if input 7 gets its inputs from the outputs 2, 15, and 6, where the input from output 15 is negative, the corresponding row of `Q` is `[7  2  -15  6]`. Short rows can be padded with trailing zeros (see example below).
- Given `sys` and `Q`, `connect` computes a state-space model of the interconnection with the same inputs and outputs as `sys` (that is, the concatenation of all block inputs and outputs). The index vectors `inputs` and `outputs` then indicate which of the inputs and outputs in the large unconnected system are external inputs and outputs of the block diagram. For example, if the external inputs are inputs 1, 2, and 15 of `sys`, and the

external outputs are outputs 2 and 7 of sys, then inputs and outputs should be set to

```
inputs = [1 2 15];
outputs = [2 7];
```

The final model sysc has these particular inputs and outputs.

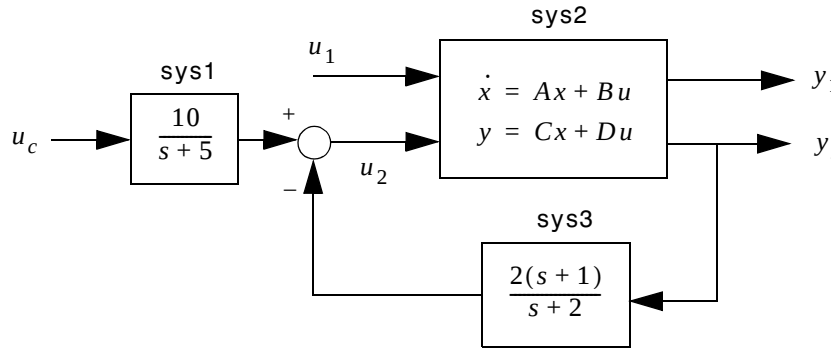
Since it is easy to make a mistake entering all the data required for a large model, be sure to verify your model in as many ways as you can. Here are some suggestions:

- Make sure the poles of the unconnected model sys match the poles of the various blocks in the diagram.
- Check that the final poles and DC gains are reasonable.
- Plot the step and bode responses of sysc and compare them with your expectations.

If you need to work extensively with block diagrams, Simulink is a much easier and more comprehensive tool for model building.

Example

Consider the following block diagram



Given the matrices of the state-space model sys2

```
A = [ −9.0201  17.7791
      −1.6943  3.2138 ];
B = [ −.5112  .5362
      −.002  −1.8470];
C = [ −3.2897  2.4544
      −13.5009  18.0745];
D = [−.5476  −.1410
     −.6459  .2958 ];
```

Define the three blocks as individual LTI models.

```
sys1 = tf(10,[1 5],'inputname','uc')
sys2 = ss(A,B,C,D,'inputname',{'u1' 'u2'},...
                  'outputname',{'y1' 'y2'})
sys3 = zpk(−1,−2,2)
```

Next append these blocks to form the unconnected model sys.

```
sys = append(sys1,sys2,sys3)
```

This produces the block-diagonal model

```
sys
```

```
a =
                    x1          x2          x3          x4
          x1        -5           0           0           0
          x2         0     -9.0201      17.779           0
          x3         0     -1.6943      3.2138           0
          x4         0           0           0          -2


b =
                    uc          u1          u2           ?
          x1         4           0           0           0
          x2         0     -0.5112      0.5362           0
          x3         0      -0.002      -1.847           0
          x4         0           0           0      1.4142
```

```
c =
                x1          x2          x3          x4
 ?         2.5           0           0           0
y1           0     -3.2897      2.4544           0
y2           0     -13.501      18.075           0
 ?           0           0           0     -1.4142

d =
                uc          u1          u2           ?
 ?           0           0           0           0
y1           0     -0.5476      -0.141           0
y2           0     -0.6459      0.2958           0
 ?           0           0           0           2

Continuous-time system.
```

Note that the ordering of the inputs and outputs is the same as the block ordering you chose. Unnamed inputs or outputs are denoted by `?`.

To derive the overall block diagram model from `sys`, specify the interconnections and the external inputs and outputs. You need to connect outputs 1 and 4 into input 3 (`u2`), and output 3 (`y2`) into input 4. The interconnection matrix `Q` is therefore

```
Q = [3 1 -4
     4 3  0];
```

Note that the second row of `Q` has been padded with a trailing zero. The block diagram has two external inputs `uc` and `u1` (inputs 1 and 2 of `sys`), and two external outputs `y1` and `y2` (outputs 2 and 3 of `sys`). Accordingly, set `inputs` and `outputs` as follows.

```
inputs = [1 2];
outputs = [2 3];
```

You can obtain a state-space model for the overall interconnection by typing

```
sysc = connect(sys,Q,inputs,outputs)

a =
                x1          x2          x3          x4
   x1           -5           0           0           0
   x2      0.84223    0.076636      5.6007     0.47644
   x3      -2.9012      -33.029      45.164     -1.6411
   x4      0.65708      -11.996       16.06     -1.6283

b =
                uc          u1
   x1            4           0
   x2            0   -0.076001
   x3            0    -1.5011
   x4            0   -0.57391

c =
                x1          x2          x3          x4
   y1      -0.22148     -5.6818      5.6568    -0.12529
   y2       0.46463     -8.4826      11.356     0.26283

d =
                uc          u1
   y1            0   -0.66204
   y2            0   -0.40582

Continuous-time system.
```

Note that the inputs and outputs are as desired.

See Also

`append`	Append LTI systems
`feedback`	Feedback connection
`minreal`	Minimal state-space realization
`parallel`	Parallel connection
`series`	Series connection

References

[1] Edwards, J.W., “A Fortran Program for the Analysis of Linear Continuous and Sampled-Data Systems,” *NASA Report TM X56038*, Dryden Research Center, 1976.

Purpose

Output and state covariance of a system driven by white noise

Syntax

```
[P,Q] = covar(sys,W)
```

Description

`covar` calculates the stationary covariance of the output y of an LTI model `sys` driven by Gaussian white noise inputs w. This function handles both continuous- and discrete-time cases.

`P = covar(sys,W)` returns the steady-state output response covariance

$$P = E(yy^T)$$

given the noise intensity

$$E(w(t)w(\tau)^T) = W\,\delta(t-\tau) \qquad \text{(continuous time)}$$

$$E(w[k]w[l]^T) = W\,\delta_{kl} \qquad \text{(discrete time)}$$

`[P,Q] = covar(sys,W)` also returns the steady-state state covariance

$$Q = E(xx^T)$$

when `sys` is a state-space model (otherwise `Q` is set to `[]`).

When applied to an `N`-dimensional LTI array `sys`, `covar` returns multi-dimensional arrays P, Q such that

`P(:,:,i1,...iN)` and `Q(:,:,i1,...iN)` are the covariance matrices for the model `sys(:,:,i1,...iN)`.

Example

Compute the output response covariance of the discrete SISO system

$$H(z) = \frac{2z+1}{z^2+0.2z+0.5}, \qquad T_s = 0.1$$

due to Gaussian white noise of intensity `W = 5`. Type

```
sys = tf([2 1],[1 0.2 0.5],0.1);
p = covar(sys,5)
```

and MATLAB returns

```
p =
    30.3167
```

You can compare this output of covar to simulation results.

```
randn('seed',0)
w = sqrt(5)*randn(1,1000);  % 1000 samples

% Simulate response to w with LSIM:
y = lsim(sys,w);

% Compute covariance of y values
psim = sum(y .* y)/length(w);
```

This yields

```
psim =
    32.6269
```

The two covariance values p and psim do not agree perfectly due to the finite simulation horizon.

Algorithm

Transfer functions and zero-pole-gain models are first converted to state space with ss.

For continuous-time state-space models

$$\dot{x} = Ax + Bw$$
$$y = Cx + Dw$$

Q is obtained by solving the Lyapunov equation

$$AQ + QA^T + BWB^T = 0$$

The output response covariance P is finite only when $D = 0$ and then $P = CQC^T$.

In discrete time, the state covariance solves the discrete Lyapunov equation

$$AQA^T - Q + BWB^T = 0$$

and P is given by $P = CQC' + DWD'$

Note that P is well defined for nonzero D in the discrete case.

Limitations

The state and output covariances are defined for *stable* systems only. For continuous systems, the output response covariance P is finite only when the D matrix is zero (strictly proper system).

See Also

`dlyap`	Solver for discrete-time Lyapunov equations
`lyap`	Solver for continuous-time Lyapunov equations

References

[1] Bryson, A.E. and Y.C. Ho, *Applied Optimal Control*, Hemisphere Publishing, 1975, pp. 458-459.

Purpose

Form the controllability matrix

Syntax

```
Co = ctrb(A,B)
Co = ctrb(sys)
```

Description

`ctrb` computes the controllability matrix for state-space systems. For an *n*-by-*n* matrix `A` and an *n*-by-*m* matrix `B`, `ctrb(A,B)` returns the controllability matrix

$$Co = \left[B \; AB \; A^2B \; \ldots \; A^{n-1}B\right] \qquad \textbf{(11-1)}$$

where Co has n rows and nm columns.

`Co = ctrb(sys)` calculates the controllability matrix of the state-space LTI object `sys`. This syntax is equivalent to executing

```
Co = ctrb(sys.A,sys.B)
```

The system is controllable if `Co` has full rank n.

Example

Check if the system with the following data

```
A =
     1      1
     4     –2

B =
     1     –1
     1     –1
```

is controllable. Type

```
Co=ctrb(A,B);

% Number of uncontrollable states
unco=length(A)–rank(Co)
```

and MATLAB returns

```
unco =
     1
```

Limitations

The calculation of `Co` may be ill-conditioned with respect to inversion. An indication of this can be seen from this simple example.

$$A = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ \delta \end{bmatrix}$$

This pair is controllable if $\delta \neq 0$ but if $\delta < \sqrt{eps}$, where *eps* is the relative machine precision. `ctrb(A,B)` returns

$$\begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \delta & \delta \end{bmatrix}$$

which is not full rank. For cases like these, it is better to determine the controllability of a system using `ctrbf`.

See Also

`ctrbf`	Compute the controllability staircase form
`obsv`	Compute the observability matrix

Purpose

Compute the controllability staircase form

Syntax

```
[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C)
[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C,tol)
```

Description

If the controllability matrix of (A, B) has rank $r \le n$, where n is the size of A, then there exists a similarity transformation such that

$$\overline{A} = TAT^T, \qquad \overline{B} = TB, \qquad \overline{C} = CT^T$$

where T is unitary, and the transformed system has a *staircase* form, in which the uncontrollable modes, if there are any, are in the upper left corner.

$$\overline{A} = \begin{bmatrix} A_{uc} & 0 \\ A_{21} & A_c \end{bmatrix}, \qquad \overline{B} = \begin{bmatrix} 0 \\ B_c \end{bmatrix}, \qquad \overline{C} = \begin{bmatrix} C_{nc} & C_c \end{bmatrix}$$

where (A_c, B_c) is controllable, all eigenvalues of A_{uc} are uncontrollable, and

$$C_c(sI - A_c)^{-1}B_c = C(sI - A)^{-1}B.$$

`[Abar,Bbar,Cbar,T,k] = ctrbf(A,B,C)` decomposes the state-space system represented by `A`, `B`, and `C` into the controllability staircase form, `Abar`, `Bbar`, and `Cbar`, described above. `T` is the similarity transformation matrix and `k` is a vector of length *n*, where *n* is the order of the system represented by `A`. Each entry of `k` represents the number of controllable states factored out during each step of the transformation matrix calculation. The number of nonzero elements in `k` indicates how many iterations were necessary to calculate `T`, and `sum(k)` is the number of states in A_c, the controllable portion of `Abar`.

`ctrbf(A,B,C,tol)` uses the tolerance `tol` when calculating the controllable/uncontrollable subspaces. When the tolerance is not specified, it defaults to `10*n*norm(A,1)*eps`.

Example

Compute the controllability staircase form for

```
A =
     1      1
     4     −2

B =
     1     −1
     1     −1

C =
     1      0
     0      1
```

and locate the uncontrollable mode.

```
[Abar,Bbar,Cbar,T,k]=ctrbf(A,B,C)

Abar =
   −3.0000         0
   −3.0000    2.0000

Bbar =
    0.0000    0.0000
    1.4142   −1.4142

Cbar =
   −0.7071    0.7071
    0.7071    0.7071

T =
   −0.7071    0.7071
    0.7071    0.7071
k =
     1     0
```

The decomposed system `Abar` shows an uncontrollable mode located at –3 and a controllable mode located at 2.

See also the function `minreal`, which uses `ctrbf` to find the minimal realization of a system.

Algorithm

`ctrbf` is an M-file that implements the Staircase Algorithm of [1].

See Also

`ctrb`	Form the controllability matrix
`minreal`	Minimum realization and pole-zero cancellation

References

[1] Rosenbrock, M.M., *State-Space and Multivariable Theory*, John Wiley, 1970.

d2c

Purpose

Convert discrete-time LTI models to continuous time

Syntax

```
sysc = d2c(sysd)
sysc = d2c(sysd,method)
```

Description

`d2c` converts LTI models from discrete to continuous time using one of the following conversion methods:

`'zoh'`	Zero-order hold on the inputs. The control inputs are assumed piecewise constant over the sampling period.
`'tustin'`	Bilinear (Tustin) approximation to the derivative.
`'prewarp'`	Tustin approximation with frequency prewarping.
`'matched'`	Matched pole-zero method of [1] (for SISO systems only).

The string *method* specifies the conversion method. If *method* is omitted then zero-order hold (`'zoh'`) is assumed. See "Continuous/Discrete Conversions of LTI Models" in Chapter 3 of this manual and reference [1] for more details on the conversion methods.

Example

Consider the discrete-time model with transfer function

$$H(z) = \frac{z-1}{z^2 + z + 0.3}$$

and sample time $T_s = 0.1$ second. You can derive a continuous-time zero-order-hold equivalent model by typing

```
Hc = d2c(H)
```

Discretizing the resulting model `Hc` with the zero-order hold method (this is the default method) and sampling period $T_s = 0.1$ gives back the original discrete model $H(z)$. To see this, type

```
c2d(Hc,0.1)
```

To use the Tustin approximation instead of zero-order hold, type

```
Hc = d2c(H,'tustin')
```

As with zero-order hold, the inverse discretization operation

```
c2d(Hc,0.1,'tustin')
```

gives back the original $H(z)$.

Algorithm

The `'zoh'` conversion is performed in state space and relies on the matrix logarithm (see `logm` in *Using MATLAB*).

Limitations

The Tustin approximation is not defined for systems with poles at $z = -1$ and is ill-conditioned for systems with poles near $z = -1$.

The zero-order hold method cannot handle systems with poles at $z = 0$. In addition, the `'zoh'` conversion increases the model order for systems with negative real poles, [2]. This is necessary because the matrix logarithm maps real negative poles to complex poles. As a result, a discrete model with a single pole at $z = -0.5$ would be transformed to a continuous model with a single *complex* pole at $\log(-0.5) \approx -0.6931 + j\pi$. Such a model is not meaningful because of its complex time response.

To ensure that all complex poles of the continuous model come in conjugate pairs, `d2c` replaces negative real poles $z = -\alpha$ with a pair of complex conjugate poles near $-\alpha$. The conversion then yields a continuous model with higher order. For example, the discrete model with transfer function

$$H(z) = \frac{z + 0.2}{(z + 0.5)(z^2 + z + 0.4)}$$

and sample time 0.1 second is converted by typing

```
Ts = 0.1
H = zpk(-0.2,-0.5,1,Ts) * tf(1,[1 1 0.4],Ts)
Hc = d2c(H)
```

MATLAB responds with

```
Warning: System order was increased to handle real negative poles.

Zero/pole/gain:
  -33.6556 (s-6.273) (s^2 + 28.29s + 1041)
-----------------------------------------
(s^2 + 9.163s + 637.3) (s^2 + 13.86s + 1035)
```

Convert `Hc` back to discrete time by typing

```
c2d(Hc,Ts)
```

yielding

```
Zero/pole/gain:
     (z+0.5) (z+0.2)
-------------------------
(z+0.5)^2 (z^2 + z + 0.4)

Sampling time: 0.1
```

This discrete model coincides with $H(z)$ after canceling the pole/zero pair at $z = -0.5$.

See Also

`c2d`	Continuous- to discrete-time conversion
`d2d`	Resampling of discrete models
`logm`	Matrix logarithm

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

[2] Kollár, I., G.F. Franklin, and R. Pintelon, "On the Equivalence of z-domain and s-domain Models in System Identification," *Proceedings of the IEEE Instrumentation and Measurement Technology Conference*, Brussels, Belgium, June, 1996, Vol. 1, pp. 14-19.

Purpose

Resample discrete-time LTI models or add input delays

Syntax

```
sys1 = d2d(sys,Ts)
```

Description

`sys1 = d2d(sys,Ts)` resamples the discrete-time LTI model `sys` to produce an equivalent discrete-time model `sys1` with the new sample time `Ts` (in seconds). The resampling assumes zero-order hold on the inputs and is equivalent to consecutive `d2c` and `c2d` conversions.

```
sys1 = c2d(d2c(sys),Ts)
```

Example

Consider the zero-pole-gain model

$$H(z) = \frac{z-0.7}{z-0.5}$$

with sample time 0.1 second. You can resample this model at 0.05 second by typing

```
H = zpk(0.7,0.5,1,0.1)
H2 = d2d(H,0.05)
```

```
Zero/pole/gain:
(z–0.8243)
----------
(z–0.7071)

Sampling time: 0.05
```

Note that the inverse resampling operation, performed by typing `d2d(H2,0.1)`, yields back the initial model $H(z)$.

```
Zero/pole/gain:
(z–0.7)
-------
(z–0.5)

Sampling time: 0.1
```

See Also

`c2d`	Continuous- to discrete-time conversion
`d2c`	Discrete- to continuous-time conversion

damp

Purpose

Compute damping factors and natural frequencies

Syntax

```
[Wn,Z] = damp(sys)
[Wn,Z,P] = damp(sys)
```

Description

`damp` calculates the damping factor and natural frequencies of the poles of an LTI model `sys`. When invoked without lefthand arguments, a table of the eigenvalues in increasing frequency, along with their damping factors and natural frequencies, is displayed on the screen.

`[Wn,Z] = damp(sys)` returns column vectors `Wn` and `Z` containing the natural frequencies ω_n and damping factors ζ of the poles of `sys`. For discrete-time systems with poles z and sample time T_s, `damp` computes "equivalent" continuous-time poles s by solving

$$z = e^{sT_s}$$

The values `Wn` and `Z` are then relative to the continuous-time poles s. Both `Wn` and `Z` are empty if the sample time is unspecified.

`[Wn,Z,P] = damp(sys)` returns an additional vector `P` containing the (true) poles of `sys`. Note that `P` returns the same values as `pole(sys)` (up to reordering).

Example

Compute and display the eigenvalues, natural frequencies, and damping factors of the continuous transfer function

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

Type

```
H = tf([2 5 1],[1 2 3])

Transfer function:
2 s^2 + 5 s + 1
---------------
 s^2 + 2 s + 3
```

Type

```
damp(H)
```

and MATLAB returns

```
Eigenvalue            Damping     Freq. (rad/s)

-1.00e+000 + 1.41e+000i     5.77e-001       1.73e+000
-1.00e+000 - 1.41e+000i     5.77e-001       1.73e+000
```

See Also

`eig`	Calculate eigenvalues and eigenvectors
`esort,dsort`	Sort system poles
`pole`	Compute system poles
`pzmap`	Pole-zero map
`zero`	Compute (transmission) zeros

Purpose

Solve discrete-time algebraic Riccati equations (DARE)

Syntax

```
[X,L,G,rr] = dare(A,B,Q,R)
[X,L,G,rr] = dare(A,B,Q,R,S,E)

[X,L,G,report] = dare(A,B,Q,...,'report')
[X1,X2,L,report] = dare(A,B,Q,...,'implicit')
```

Description

`[X,L,G,rr] = dare(A,B,Q,R)` computes the unique solution X of the discrete-time algebraic Riccati equation

$$Ric(X) = A^TXA - X - A^TXB(B^TXB + R)^{-1}B^TXA + Q = 0$$

such that the "closed-loop" matrix

$$A_{cl} = A - B(B^TXB + R)^{-1}B^TXA$$

has all its eigenvalues inside the unit disk. The matrix X is symmetric and called the *stabilizing* solution of $Ric(X) = 0$. `[X,L,G,rr] = dare(A,B,Q,R)` also returns:

- The eigenvalues `L` of A_{cl}
- The gain matrix

  $$G = (B^TXB + R)^{-1}B^TXA$$

- The relative residual `rr` defined by

  $$rr = \frac{\|Ric(X)\|_F}{\|X\|_F}$$

`[X,L,G,rr] = dare(A,B,Q,R,S,E)` solves the more general DARE:

$$A^TXA - E^TXE - (A^TXB + S)(B^TXB + R)^{-1}(B^TXA + S^T) + Q = 0$$

The corresponding gain matrix and closed-loop eigenvalues are

$$G = (B^TXB + R)^{-1}(B^TXA + S^T)$$

and `L = eig(A-B*G,E)`.

Two additional syntaxes are provided to help develop applications such as H_∞-optimal control design.

`[X,L,G,report] = dare(A,B,Q,...,'report')` turns off the error messages when the solution X fails to exist and returns a failure report instead. The value of `report` is:

- `-1` when the associated symplectic pencil has eigenvalues on or very near the unit circle (failure)
- `-2` when there is no finite solution, that is, $X = X_2 X_1^{-1}$ with X_1 singular (failure)
- The relative residual rr defined above when the solution exists (success)

Alternatively, `[X1,X2,L,report] = dare(A,B,Q,...,'implicit')` also turns off error messages but now returns X in implicit form as

$$X = X_2 X_1^{-1}$$

Note that this syntax returns `report = 0` when successful.

Algorithm

`dare` implements the algorithms described in [1]. It uses the QZ algorithm to deflate the extended symplectic pencil and compute its stable invariant subspace.

Limitations

The (A, B) pair must be stabilizable (that is, all eigenvalues of A outside the unit disk must be controllable). In addition, the associated symplectic pencil must have no eigenvalue on the unit circle. Sufficient conditions for this to hold are (Q, A) detectable when $S = 0$ and $R > 0$, or

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0$$

See Also

`care`	Solve continuous-time Riccati equations
`dlyap`	Solve discrete-time Lyapunov equations

References

[1] Arnold, W.F., III and A.J. Laub, "Generalized Eigenproblem Algorithms and Software for Algebraic Riccati Equations," *Proc. IEEE*, 72 (1984), pp. 1746–1754.

Purpose

Compute low frequency (DC) gain of LTI system

Syntax

```
k = dcgain(sys)
```

Description

`k = dcgain(sys)` computes the DC gain `k` of the LTI model `sys`.

Continuous Time

The continuous-time DC gain is the transfer function value at the frequency $s = 0$. For state-space models with matrices (A, B, C, D), this value is

$$K = D - CA^{-1}B$$

Discrete Time

The discrete-time DC gain is the transfer function value at $z = 1$. For state-space models with matrices (A, B, C, D), this value is

$$K = D + C(I - A)^{-1}B$$

Remark

The DC gain is infinite for systems with integrators.

Example

To compute the DC gain of the MIMO transfer function

$$H(s) = \begin{bmatrix} 1 & \dfrac{s-1}{s^2+s+3} \\ \dfrac{1}{s+1} & \dfrac{s+2}{s-3} \end{bmatrix}$$

type

```
H = [1 tf([1 -1],[1 1 3]) ; tf(1,[1 1]) tf([1 2],[1 -3])]
dcgain(H)

ans =
    1.0000   -0.3333
    1.0000   -0.6667
```

See Also

`evalfr`	Evaluates frequency response at single frequency
`norm`	LTI system norms

delay2z

Purpose

Replace delays of discrete-time TF, SS, or ZPK models by poles at z=0, or replace delays of FRD models by a phase shift

Syntax

```
sys = delay2z(sys)
```

Description

`sys = delay2z(sys)` maps all time delays to poles at z=0 for discrete-time TF, ZPK, or SS models `sys`. Specifically, a delay of `k` sampling periods is replaced by `(1/z)^k` in the transfer function corresponding to the model.

For FRD models, `delay2z` absorbs all time delays into the frequency response data, and is applicable to both continuous- and discrete-time FRDs.

Example

```
z=tf('z',-1);
sys=(-.4*z -.1)/(z^2 + 1.05*z + .08)

Transfer function:

-0.4 z - 0.1
-------------------
z^2 + 1.05 z + 0.08

Sampling time: unspecified

sys.Inputd = 1;
sys = delay2z(sys)

Transfer function:

     -0.4 z - 0.1
-----------------------
z^3 + 1.05 z^2 + 0.08 z

Sampling time: unspecified
```

See Also

`hasdelay`	True for LTI models with delays
`pade`	Pade approximation of time delays
`totaldelay`	Combine delays for an LTI model

Purpose

Design linear-quadratic (LQ) state-feedback regulator for discrete-time plant

Syntax

```
[K,S,e] = dlqr(a,b,Q,R)
[K,S,e] = dlqr(a,b,Q,R,N)
```

Description

`[K,S,e] = dlqr(a,b,Q,R,N)` calculates the optimal gain matrix `K` such that the state-feedback law

$$u[n] = -Kx[n]$$

minimizes the quadratic cost function

$$J(u) = \sum_{n=1}^{\infty} (x[n]^T Q x[n] + u[n]^T R u[n] + 2x[n]^T N u[n])$$

for the discrete-time state-space mode

$$lx[n+1] = Ax[n] + Bu[n]$$

The default value `N=0` is assumed when `N` is omitted.

In addition to the state-feedback gain `K`, `dlqr` returns the solution `S` of the associated discrete-time Riccati equation

$$A^T SA - S - (A^T SB + N)(B^T XB + R)^{-1}(B^T SA + N^T) + Q = 0$$

and the closed-loop eigenvalues `e = eig(a-b*K)`. Note that K is derived from S by

$$K = (B^T XB + R)^{-1}(B^T SA + N^T)$$

Limitations

The problem data must satisfy:

- The pair (A, B) is stabilizable.
- $R > 0$ and $Q - NR^{-1}N^T \geq 0$.
- $(Q - NR^{-1}N^T, A - BR^{-1}N^T)$ has no unobservable mode on the unit circle.

See Also

`dare`	Solve discrete Riccati equations
`lqgreg`	LQG regulator

`lqr`	State-feedback LQ regulator for continuous plant
`lqrd`	Discrete LQ regulator for continuous plant
`lqry`	State-feedback LQ regulator with output weighting

Purpose

Solve discrete-time Lyapunov equations

Syntax

```
X = dlyap(A,Q)
```

Description

`dlyap` solves the discrete-time Lyapunov equation

$$A^T X A - X + Q = 0$$

where A and Q are n-by-n matrices.

The solution X is symmetric when Q is symmetric, and positive definite when Q is positive definite and A has all its eigenvalues inside the unit disk.

Diagnostics

The discrete-time Lyapunov equation has a (unique) solution if the eigenvalues $\alpha_1, \alpha_2, ..., \alpha_n$ of A satisfy $\alpha_i \alpha_j \neq 1$ for all (i, j).

If this condition is violated, `dlyap` produces the error message

```
Solution does not exist or is not unique.
```

See Also

`covar`	Covariance of system response to white noise
`lyap`	Solve continuous Lyapunov equations

drmodel, drss

Purpose

Generate stable random discrete test models

Syntax

```
sys = drss(n)
sys = drss(n,p)
sys = drss(n,p,m)
sys = drss(n,p,m,s1,...sn)

[num,den] = drmodel(n)
[A,B,C,D] = drmodel(n)
[A,B,C,D] = drmodel(n,p,m)
```

Description

`sys = drss(n)` produces a random *n*-th order stable model with one input and one output, and returns the model in the state-space object `sys`.

`drss(n,p)` produces a random *n*-th order stable model with one input and `p` outputs.

`drss(n,m,p)` generates a random *n*-th order stable model with `m` inputs and `p` outputs.

`drss(n,p,m,s1,...sn)` generates a *s1*–by–*sn* array of random *n*-th order stable model with `m` inputs and `p` outputs.

In all cases, the discrete-time state-space model or array returned by `drss` has an unspecified sampling time. To generate transfer function or zero-pole-gain systems, convert `sys` using `tf` or `zpk`.

`drmodel(n)` produces a random *n*-th order stable model and returns either the transfer function numerator `num` and denominator `den` or the state-space matrices `A`, `B`, `C`, and `D`, based on the number of output arguments. The resulting model always has one input and one output.

`[A,B,C,D] = drmodel(n,m,p)` produces a random *n*-th order stable state-space model with `m` inputs and `p` outputs.

Example

Generate a random discrete LTI system with three states, two inputs, and two outputs.

```
sys = drss(3,2,2)

a =
                    x1          x2          x3
           x1      0.38630    −0.21458    −0.09914
           x2     −0.23390    −0.15220    −0.06572
           x3     −0.03412     0.11394    −0.22618

b =
                    u1          u2
           x1      0.98833     0.51551
           x2            0     0.33395
           x3      0.42350     0.43291

c =
                    x1          x2          x3
           y1      0.22595     0.76037           0
           y2            0           0           0

d =
                    u1          u2
           y1            0     0.68085
           y2      0.78333     0.46110

Sampling time: unspecified
Discrete-time system.
```

See Also

`rmodel`, `rss`	Generate stable random continuous test models
`tf`	Convert LTI systems to transfer functions form
`zpk`	Convert LTI systems to zero-pole-gain form

dsort

Purpose

Sort discrete-time poles by magnitude

Syntax

```
s = dsort(p)
[s,ndx] = dsort(p)
```

Description

`dsort` sorts the discrete-time poles contained in the vector `p` in descending order by magnitude. Unstable poles appear first.

When called with one lefthand argument, `dsort` returns the sorted poles in `s`.

`[s,ndx] = dsort(p)` also returns the vector `ndx` containing the indices used in the sort.

Example

Sort the following discrete poles.

```
p =
  -0.2410 + 0.5573i
  -0.2410 - 0.5573i
   0.1503
  -0.0972
  -0.2590

s = dsort(p)

s =
  -0.2410 + 0.5573i
  -0.2410 - 0.5573i
  -0.2590
   0.1503
  -0.0972
```

Limitations

The poles in the vector `p` must appear in complex conjugate pairs.

See Also

`eig`	Calculate eigenvalues and eigenvectors
`esort`, `sort`	Sort system poles
`pole`	Compute system poles
`pzmap`	Pole-zero map
`zero`	Compute (transmission) zeros

Purpose

Specify descriptor state-space models

Syntax

```
sys = dss(a,b,c,d,e)
sys = dss(a,b,c,d,e,Ts)
sys = dss(a,b,c,d,e,ltisys)

sys = dss(a,b,c,d,e,'Property1',Value1,...,'PropertyN',ValueN)
sys = dss(a,b,c,d,e,Ts,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

`sys = dss(a,b,c,d,e)` creates the continuous-time descriptor state-space model

$$E\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

The E matrix must be nonsingular. The output `sys` is an SS model storing the model data (see "LTI Objects" on page 2-3). Note that `ss` produces the same type of object. If the matrix $D = 0$, do can simply set `d` to the scalar `0` (zero).

`sys = dss(a,b,c,d,e,Ts)` creates the discrete-time descriptor model

$$Ex[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sample time `Ts` (in seconds).

`sys = dss(a,b,c,d,e,ltisys)` creates a descriptor model with generic LTI properties inherited from the LTI model `ltisys` (including the sample time). See "LTI Properties" on page 2-26 for an overview of generic LTI properties.

Any of the previous syntaxes can be followed by property name/property value pairs

```
'Property',Value
```

Each pair specifies a particular LTI property of the model, for example, the input names or some notes on the model history. See `set` and the example below for details.

Example

The command

```
sys = dss(1,2,3,4,5,'td',0.1,'inputname','voltage',...
                                'notes','Just an example')
```

creates the model

$$5\dot{x} = x + 2u$$
$$y = 3x + 4u$$

with a 0.1 second input delay. The input is labeled `'voltage'`, and a note is attached to tell you that this is just an example.

See Also

`dssdata`	Retrieve A, B, C, D, E matrices of descriptor model
`get`	Get properties of LTI models
`set`	Set properties of LTI models
`ss`	Specify (regular) state-space models

Purpose

Quick access to descriptor state-space data

Syntax

```
[a,b,c,d,e] = dssdata(sys)
[a,b,c,d,e,Ts] = dssdata(sys)
```

Description

`[a,b,c,d,e] = dssdata(sys)` extracts the descriptor matrix data (A, B, C, D, E) from the state-space model `sys`. If `sys` is a transfer function or zero-pole-gain model, it is first converted to state space. Note that `dssdata` is then equivalent to `ssdata` because it always returns $E = I$.

`[a,b,c,d,e,Ts] = dssdata(sys)` also returns the sample time `Ts` in addition to `a`, `b`, `c`, `d`, and `e`.

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.notes
```

See Also

`dss`	Specify descriptor state-space models
`get`	Get properties of LTI models
`ssdata`	Quick access to state-space data
`tfdata`	Quick access to transfer function data
`zpkdata`	Quick access to zero-pole-gain data

esort

Purpose

Sort continuous-time poles by real part

Syntax

```
s = esort(p)
[s,ndx] = esort(p)
```

Description

`esort` sorts the continuous-time poles contained in the vector `p` by real part. Unstable eigenvalues appear first and the remaining poles are ordered by decreasing real parts.

When called with one left-hand argument, `s = esort(p)` returns the sorted eigenvalues in `s`.

`[s,ndx] = esort(p)` returns the additional argument `ndx`, a vector containing the indices used in the sort.

Example

Sort the following continuous eigenvalues.

```
p
p =
  -0.2410+ 0.5573i
  -0.2410- 0.5573i
   0.1503
  -0.0972
  -0.2590

esort(p)

ans =
   0.1503
  -0.0972
  -0.2410+ 0.5573i
  -0.2410- 0.5573i
  -0.2590
```

Limitations

The eigenvalues in the vector `p` must appear in complex conjugate pairs.

See Also

`dsort`, `sort`	Sort system poles
`eig`	Calculate eigenvalues and eigenvectors
`pole`	Compute system poles
`pzmap`	Pole-zero map
`zero`	Compute (transmission) zeros

Purpose

Form state estimator given estimator gain

Syntax

```
est = estim(sys,L)
est = estim(sys,L,sensors,known)
```

Description

`est = estim(sys,L)` produces a state/output estimator `est` given the plant state-space model `sys` and the estimator gain `L`. All inputs w of `sys` are assumed stochastic (process and/or measurement noise), and all outputs y are measured. The estimator `est` is returned in state-space form (SS object). For a continuous-time plant `sys` with equations

$$\dot{x} = Ax + Bw$$
$$y = Cx + Dw$$

`estim` generates plant output and state estimates $\hat{y}$ and $\hat{x}$ as given by the following model.

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x})$$
$$\begin{bmatrix}\hat{y}\\ \hat{x}\end{bmatrix} = \begin{bmatrix}C\\ I\end{bmatrix}\hat{x}$$

The discrete-time estimator has similar equations.

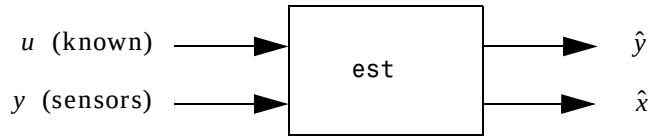
`est = estim(sys,L,sensors,known)` handles more general plants `sys` with both known inputs u and stochastic inputs w, and both measured outputs y and nonmeasured outputs z.

$$\dot{x} = Ax + B_1 w + B_2 u$$
$$\begin{bmatrix}z\\ y\end{bmatrix} = \begin{bmatrix}C_1\\ C_2\end{bmatrix}x + \begin{bmatrix}D_{11}\\ D_{21}\end{bmatrix}w + \begin{bmatrix}D_{12}\\ D_{22}\end{bmatrix}u$$

The index vectors `sensors` and `known` specify which outputs y are measured and which inputs u are known. The resulting estimator `est` uses both u and y to produce the output and state estimates.

$$\dot{\hat{x}} = A\hat{x} + B_2 u + L(y - C_2\hat{x} - D_{22}u)$$

$$\begin{bmatrix}\hat{y}\\ \hat{x}\end{bmatrix} = \begin{bmatrix}C_2\\ I\end{bmatrix}\hat{x} + \begin{bmatrix}D_{22}\\ 0\end{bmatrix}u$$



`estim` handles both continuous- and discrete-time cases. You can use the functions `place` (pole placement) or `kalman` (Kalman filtering) to design an adequate estimator gain L. Note that the estimator poles (eigenvalues of $A - LC$) should be faster than the plant dynamics (eigenvalues of A) to ensure accurate estimation.

Example

Consider a state-space model `sys` with seven outputs and four inputs. Suppose you designed a Kalman gain matrix L using outputs 4, 7, and 1 of the plant as sensor measurements, and inputs 1,4, and 3 of the plant as known (deterministic) inputs. You can then form the Kalman estimator by

```
sensors = [4,7,1];
known = [1,4,3];
est = estim(sys,L,sensors,known)
```

See the function `kalman` for direct Kalman estimator design.

See Also

`kalman`	Design Kalman estimator
`place`	Pole placement
`reg`	Form regulator given state-feedback and estimator gains

evalfr

Purpose

Evaluate frequency response at a single (complex) frequency

Syntax

```
frsp = evalfr(sys,f)
```

Description

`frsp = evalfr(sys,f)` evaluates the transfer function of the TF, SS, or ZPK model `sys` at the complex number `f`. For state-space models with data (A, B, C, D), the result is

$$H(f) = D + C(fI - A)^{-1}B$$

`evalfr` is a simplified version of `freqresp` meant for quick evaluation of the response at a single point. Use `freqresp` to compute the frequency response over a set of frequencies.

Example

To evaluate the discrete-time transfer function

$$H(z) = \frac{z-1}{z^2+z+1}$$

at $z = 1 + j$, type

```
H = tf([1 -1],[1 1 1],-1)
z = 1+j
evalfr(H,z)

ans =
  2.3077e-01 +  1.5385e-01i
```

Limitations

The response is not finite when `f` is a pole of `sys`.

See Also

`bode`	Bode frequency response
`freqresp`	Frequency response over a set of frequencies
`sigma`	Singular value response

Purpose

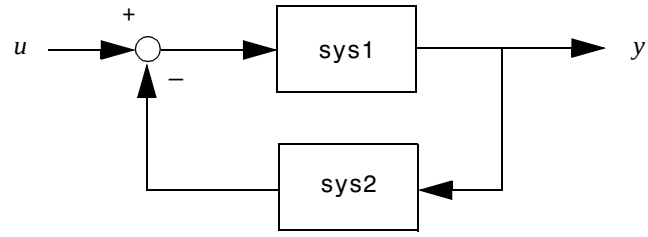
Feedback connection of two LTI models

Syntax

```
sys = feedback(sys1,sys2)
sys = feedback(sys1,sys2,sign)
sys = feedback(sys1,sys2,feedin,feedout,sign)
```

Description

`sys = feedback(sys1,sys2)` returns an LTI model `sys` for the negative feedback interconnection.



The closed-loop model `sys` has u as input vector and y as output vector. The LTI models `sys1` and `sys2` must be both continuous or both discrete with identical sample times. Precedence rules are used to determine the resulting model type (see “Precedence Rules” on page 2-5).

To apply positive feedback, use the syntax

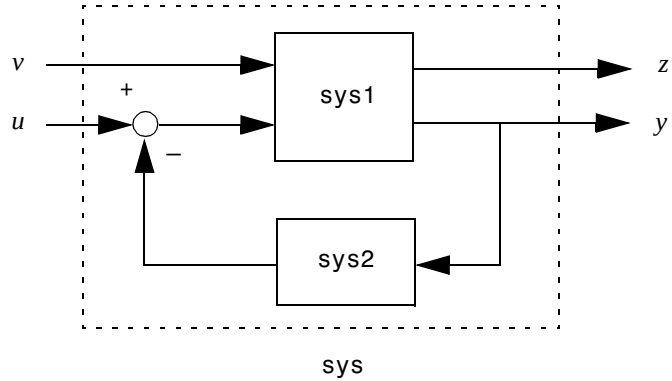
```
sys = feedback(sys1,sys2,+1)
```

By default, `feedback(sys1,sys2)` assumes negative feedback and is equivalent to `feedback(sys1,sys2,–1)`.

Finally,

```
sys = feedback(sys1,sys2,feedin,feedout)
```

computes a closed-loop model sys for the more general feedback loop.



The vector feedin contains indices into the input vector of sys1 and specifies which inputs *u* are involved in the feedback loop. Similarly, feedout specifies which outputs *y* of sys1 are used for feedback. The resulting LTI model sys has the same inputs and outputs as sys1 (with their order preserved). As before, negative feedback is applied by default and you must use

```
sys = feedback(sys1,sys2,feedin,feedout,+1)
```

to apply positive feedback.

For more complicated feedback structures, use append and connect.

Remark

You can specify static gains as regular matrices, for example,

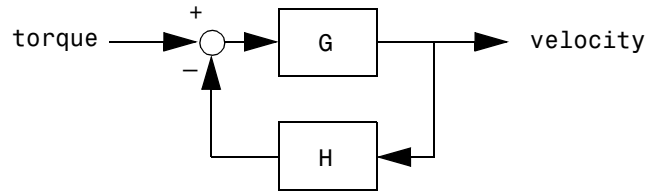
```
sys = feedback(sys1,2)
```

However, at least one of the two arguments sys1 and sys2 should be an LTI object. For feedback loops involving two static gains k1 and k2, use the syntax

```
sys = feedback(tf(k1),k2)
```

Examples

Example 1



To connect the plant

$$G(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

with the controller

$$H(s) = \frac{5(s + 2)}{s + 10}$$

using negative feedback, type

```
G = tf([2 5 1],[1 2 3],'inputname','torque',...
                       'outputname','velocity');
H = zpk(-2,-10,5)
Cloop = feedback(G,H)
```

and MATLAB returns

```
Zero/pole/gain from input "torque" to output "velocity":
0.18182 (s+10) (s+2.281) (s+0.2192)
---------------------------------
 (s+3.419) (s^2 + 1.763s + 1.064)
```

The result is a zero-pole-gain model as expected from the precedence rules. Note that `Cloop` inherited the input and output names from `G`.

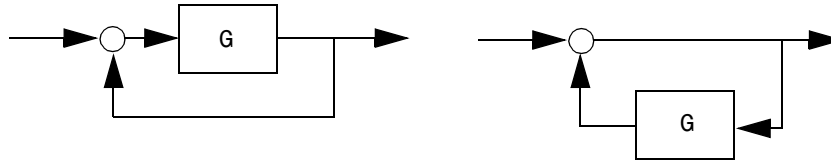
Example 2

Consider a state-space plant `P` with five inputs and four outputs and a state-space feedback controller `K` with three inputs and two outputs. To connect outputs 1, 3, and 4 of the plant to the controller inputs, and the controller outputs to inputs 4 and 2 of the plant, use

```
feedin = [4 2];
feedout = [1 3 4];
Cloop = feedback(P,K,feedin,feedout)
```

Example 3

You can form the following negative-feedback loops



by

```
Cloop = feedback(G,1)    % left diagram
Cloop = feedback(1,G)    % right diagram
```

Limitations

The feedback connection should be free of algebraic loop. If D_1 and D_2 are the feedthrough matrices of `sys1` and `sys2`, this condition is equivalent to:

- $I + D_1 D_2$ nonsingular when using negative feedback
- $I - D_1 D_2$ nonsingular when using positive feedback.

See Also

`star`	Star product of LTI systems (LFT connection)
`series`	Series connection
`parallel`	Parallel connection
`connect`	Derive state-space model for block diagram interconnection

Purpose

Specify discrete transfer functions in DSP format

Syntax

```
sys = filt(num,den)
sys = filt(num,den,Ts)
sys = filt(M)

sys = filt(num,den,'Property1',Value1,...,'PropertyN',ValueN)
sys = filt(num,den,Ts,'Property1',Value1,...,'PropertyN',ValueN)
```

Description

In digital signal processing (DSP), it is customary to write transfer functions as rational expressions in z^{-1} and to order the numerator and denominator terms in *ascending* powers of z^{-1}, for example,

$$H(z^{-1}) = \frac{2 + z^{-1}}{1 + 0.4z^{-1} + 2z^{-2}}$$

The function `filt` is provided to facilitate the specification of transfer functions in DSP format.

`sys = filt(num,den)` creates a discrete-time transfer function `sys` with numerator(s) `num` and denominator(s) `den`. The sample time is left unspecified (`sys.Ts = -1`) and the output `sys` is a TF object.

`sys = filt(num,den,Ts)` further specifies the sample time `Ts` (in seconds).

`sys = filt(M)` specifies a static filter with gain matrix `M`.

Any of the previous syntaxes can be followed by property name/property value pairs of the form

```
'Property',Value
```

Each pair specifies a particular LTI property of the model, for example, the input names or the transfer function variable. See “LTI Properties” on page 2-26 and the `set` entry for additional information on LTI properties and admissible property values.

Arguments

For SISO transfer functions, `num` and `den` are row vectors containing the numerator and denominator coefficients ordered in ascending powers of z^{-1}. For example, `den = [1 0.4 2]` represents the polynomial $1 + 0.4z^{-1} + 2z^{-2}$.

MIMO transfer functions are regarded as arrays of SISO transfer functions (one per I/O channel), each of which is characterized by its numerator and denominator. The input arguments `num` and `den` are then cell arrays of row vectors such that:

- `num` and `den` have as many rows as outputs and as many columns as inputs.
- Their (i,j) entries `num{i,j}` and `den{i,j}` specify the numerator and denominator of the transfer function from input `j` to output `i`.

If all SISO entries have the same denominator, you can also set `den` to the row vector representation of this common denominator. See also "MIMO Transfer Function Models" on page 2-10 for alternative ways to specify MIMO transfer functions.

Remark

`filt` behaves as `tf` with the `Variable` property set to `'z^-1'` or `'q'`. See `tf` entry below for details.

Example

Typing the commands

```
num = {1 , [1 0.3]}
den = {[1 1 2] ,[5 2]}
H = filt(num,den,'inputname',{'channel1' 'channel2'})
```

creates the two-input digital filter

$$H(z^{-1}) = \begin{bmatrix} \dfrac{1}{1 + z^{-1} + 2z^{-2}} & \dfrac{1 + 0.3z^{-1}}{5 + 2z^{-1}} \end{bmatrix}$$

with unspecified sample time and input names `'channel1'` and `'channel2'`.

See Also

`tf`	Create transfer functions
`zpk`	Create zero-pole-gain models
`ss`	Create state-space models

Purpose

Create a frequency response data (FRD) object or convert another model type to an FRD model

Syntax

```
sys = frd(response,frequency)
sys = frd(response,frequency,Ts)
sys = frd
sys = frd(response,frequency,ltisys)

sysfrd = frd(sys,frequency)
sysfrd = frd(sys,frequency,'Units',units)
```

Description

`sys = frd(response,frequency)` creates an FRD model `sys` from the frequency response data stored in the multidimensional array `response`. The vector `frequency` represents the underlying frequencies for the frequency response data. See Table 11-14, "Data Format for the Argument response in FRD Models," on page 80.

`sys = frd(response,frequency,Ts)` creates a discrete-time FRD model `sys` with scalar sample time `Ts`. Set `Ts = –1` to create a discrete-time FRD model without specifying the sample time.

`sys = frd` creates an empty FRD model.

The input argument list for any of these syntaxes can be followed by property name/property value pairs of the form

```
'PropertyName',PropertyValue
```

You can use these extra arguments to set the various properties of FRD models (see the `set` command, or "LTI Properties" on page 2-26 and "Model-Specific Properties" on page 2-28). These properties include `'Units'`. The default units for FRD models are in `'rad/s'`.

To force an FRD model `sys` to inherit all of its generic LTI properties from any existing LTI model `refsys`, use the syntax `sys = frd(response,frequency,ltisys)`.

`sysfrd = frd(sys,frequency)` converts a TF, SS, or ZPK model to an FRD model. The frequency response is computed at the frequencies provided by the vector `frequency`.

`sysfrd = frd(sys,frequency,'Units',units)` converts an FRD model from a TF, SS, or ZPK model while specifying the units for `frequency` to be `units` (`'rad/s'` or `'Hz'`).

Arguments

When you specify a SISO or MIMO FRD model, or an array of FRD models, the input argument `frequency` is always a vector of length `Nf`, where `Nf` is the number of frequency data points in the FRD. The specification of the input argument `response` is summarized in the following table.

Table 11-14: Data Format for the Argument response in FRD Models

Model Form	Response Data Format
SISO model	Vector of length `Nf` for which `response(i)` is the frequency response at the frequency `frequency(i)`
MIMO model with `Ny` outputs and `Nu` inputs	`Ny`-by-`Nu`-by-`Nf` multidimensional array for which `response(i,j,k)` specifies the frequency response from input `j` to output `i` at frequency `frequency(k)`
`S1`-by-`...`-by-`Sn` array of models with `Ny` outputs and `Nu` inputs	Multidimensional array of size `[Ny Nu S1 ... Sn]` for which `response(i,j,k,:)` specifies the array of frequency response data from input `j` to output `i` at frequency `frequency(k)`

Remarks

See "Frequency Response Data (FRD) Models" on page 2-17 for more information on single FRD models, and "Building LTI Arrays Using tf, zpk, ss, and frd" on page 4-17 for information on arrays of FRD models.

Example

Type the commands

```
freq = logspace(1,2);
resp = .05*(freq).*exp(i*2*freq);
sys = frd(resp,freq)
```

to create a SISO FRD model.

See Also

`chgunits`	Change units for an FRD model
`frdata`	Quick access to data for an FRD model
`set`	Set the properties for an LTI model
`ss`	Create state-space models

`tf`	Create transfer functions
`zpk`	Create zero-pole-gain models

frdata

Purpose

Quick access to data for a frequency response data object

Syntax

```
[response,freq] = frdata(sys)
[response,freq,Ts] = frdata(sys)
[response,freq] = frdata(sys,'v')
```

Description

`[response,freq] = frdata(sys)` returns the response data and frequency samples of the FRD model `sys`. For an FRD model with `Ny` outputs and `Nu` inputs at `Nf` frequencies:

- `response` is an `Ny`-by-`Nu`-by-`Nf` multidimensional array where the `(i,j)` entry specifies the response from input `j` to output `i`.
- `freq` is a column vector of length `Nf` that contains the frequency samples of the FRD model.

See Table 11-14, “Data Format for the Argument response in FRD Models,” on page 80 for more information on the data format for FRD response data.

For SISO FRD models, the syntax

```
[response,freq] = frdata(sys,'v')
```

forces `frdata` to return the response data and frequencies directly as column vectors rather than as cell arrays (see example below).

`[response,freq,Ts] = frdata(sys)` also returns the sample time `Ts`.

Other properties of `sys` can be accessed with `get` or by direct structure-like referencing (e.g., `sys.Units`).

Arguments

The input argument `sys` to `frdata` must be an FRD model.

Example

Typing the commands

```
freq = logspace(1,2,2);
resp = .05*(freq).*exp(i*2*freq);
sys = frd(resp,freq);
[resp,freq] = frdata(sys,'v')
```

returns the FRD model data

```
resp =
   0.2040 + 0.4565i
   2.4359 - 4.3665i

freq =
    10
   100
```

See Also

`frd`	Create or convert to FRD models
`get`	Get the properties for an LTI model
`set`	Set model properties

Purpose

Compute frequency response over grid of frequencies

Syntax

```
H = freqresp(sys,w)
```

Description

`H = freqresp(sys,w)` computes the frequency response of the LTI model `sys` at the real frequency points specified by the vector `w`. The frequencies must be in radians/sec. For single LTI Models, `freqresp(sys,w)` returns a 3-D array `H` with the frequency as the last dimension (see "Arguments" below). For LTI arrays of size `[Ny Nu S1 ... Sn]`, `freqresp(sys,w)` returns a `[Ny–by–Nu–by–S1–by–...–by–Sn]` length (w) array.

In continuous time, the response at a frequency ω is the transfer function value at $s = j\omega$. For state-space models, this value is given by

$$H(j\omega) = D + C(j\omega I - A)^{-1}B$$

In discrete time, the real frequencies `w(1)`,...,`w(N)` are mapped to points on the unit circle using the transformation $z = e^{j\omega T_s}$

where T_s is the sample time. The transfer function is then evaluated at the resulting z values. The default $T_s = 1$ is used for models with unspecified sample time.

Remark

If `sys` is an FRD model, `freqresp(sys,w)`, `w` can only include frequencies in `sys.frequency`.

Arguments

The output argument `H` is a 3-D array with dimensions

(number of outputs) × (number of inputs) × (length of `w`)

For SISO systems, `H(1,1,k)` gives the scalar response at the frequency `w(k)`. For MIMO systems, the frequency response at `w(k)` is `H(:,:,k)`, a matrix with as many rows as outputs and as many columns as inputs.

Example

Compute the frequency response of

$$P(s) = \begin{bmatrix} 0 & \frac{1}{s+1} \\ \frac{s-1}{s+2} & 1 \end{bmatrix}$$

at the frequencies $\omega = 1, 10, 100$. Type

```
w = [1 10 100]
H = freqresp(P,w)
```

```
H(:,:,1) =

         0            0.5000– 0.5000i
  –0.2000+ 0.6000i    1.0000


H(:,:,2) =

         0            0.0099– 0.0990i
    0.9423+ 0.2885i   1.0000


H(:,:,3) =

         0            0.0001– 0.0100i
    0.9994+ 0.0300i   1.0000
```

The three displayed matrices are the values of $P(j\omega)$ for

$\omega = 1, \quad \omega = 10, \quad \omega = 100$

The third index in the 3-D array `H` is relative to the frequency vector `w`, so you can extract the frequency response at $\omega = 10$ rad/sec by

```
H(:,:,w==10)
```

```
ans =
         0            0.0099– 0.0990i
    0.9423+ 0.2885i   1.0000
```

Algorithm

For transfer functions or zero-pole-gain models, `freqresp` evaluates the numerator(s) and denominator(s) at the specified frequency points. For continuous-time state-space models (A, B, C, D), the frequency response is

$$D + C(j\omega - A)^{-1}B, \qquad \omega = \omega_1, ..., \omega_N$$

When numerically safe, A is diagonalized for fast evaluation of this expression at the frequencies $\omega_1, ..., \omega_N$. Otherwise, A is reduced to upper Hessenberg form and the linear equation $(j\omega - A)X = B$ is solved at each frequency point, taking advantage of the Hessenberg structure. The reduction to Hessenberg form provides a good compromise between efficiency and reliability. See [1] for more details on this technique.

Diagnostics

If the system has a pole on the $j\omega$ axis (or unit circle in the discrete-time case) and `w` happens to contain this frequency point, the gain is infinite, $j\omega I - A$ is singular, and `freqresp` produces the following warning message.

```
Singularity in freq. response due to jw-axis or unit circle pole.
```

See Also

`evalfr`	Response at single complex frequency
`bode`	Bode plot
`nyquist`	Nyquist plot
`nichols`	Nichols plot
`sigma`	Singular value plot
`ltiview`	LTI system viewer

References

[1] Laub, A.J., "Efficient Multivariable Frequency Response Computations," *IEEE Transactions on Automatic Control*, AC-26 (1981), pp. 407–408.

Purpose

Generate test input signals for `lsim`

Syntax

```
[u,t] = gensig(type,tau)
[u,t] = gensig(type,tau,Tf,Ts)
```

Description

`[u,t] = gensig(type,tau)` generates a scalar signal `u` of class `type` and with period `tau` (in seconds). The following types of signals are available.

`type = 'sin'`	Sine wave.
`type = 'square'`	Square wave.
`type = 'pulse'`	Periodic pulse.

`gensig` returns a vector `t` of time samples and the vector `u` of signal values at these samples. All generated signals have unit amplitude.

`[u,t] = gensig(type,tau,Tf,Ts)` also specifies the time duration `Tf` of the signal and the spacing `Ts` between the time samples `t`.

You can feed the outputs `u` and `t` directly to `lsim` and simulate the response of a single-input linear system to the specified signal. Since `t` is uniquely determined by `Tf` and `Ts`, you can also generate inputs for multi-input systems by repeated calls to `gensig`.

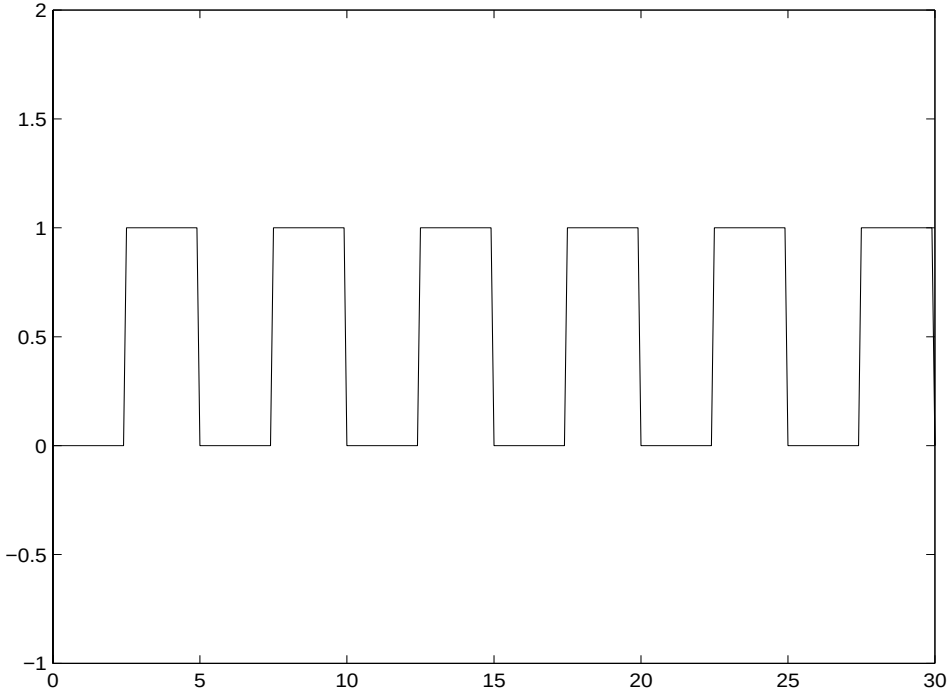
Example

Generate a square wave with period 5 seconds, duration 30 seconds, and sampling every 0.1 seconds.

```
[u,t] = gensig('square',5,30,0.1)
```

Plot the resulting signal.

```
plot(t,u)
axis([0 30 –1 2])
```



See Also

`lsim` Simulate response to arbitrary inputs

Purpose

Access/query LTI property values

Syntax

```
Value = get(sys,'PropertyName')
get(sys)
```

Description

`Value = get(sys,'PropertyName')` returns the current value of the property *PropertyName* of the LTI model sys. The string '*PropertyName*' can be the full property name (for example, 'UserData') or any unambiguous case-insensitive abbreviation (for example, 'user'). You can specify any generic LTI property, or any property specific to the model sys (see "LTI Properties" on page 2-26 for details on generic and model-specific LTI properties).

Without left-hand argument,

```
get(sys)
```

displays all properties of sys and their values.

Example

Consider the discrete-time SISO transfer function defined by

```
h = tf(1,[1 2],0.1,'inputname','voltage','user','hello');
```

You can display all LTI properties of h with

```
get(h)
            num: {[0 1]}
            den: {[1 2]}
       Variable: 'z'
             Ts: 0.1
     InputDelay: 0
    OutputDelay: 0
  ioDelayMatrix: 0
      InputName: {'voltage'}
     OutputName: {''}
     InputGroup: {0x2 cell}
    OutputGroup: {0x2 cell}
          Notes: {}
       UserData: 'hello'
```

or query only about the numerator and sample time values by

```
get(h,'num')

ans =
    [1x2 double]
```

and

```
get(h,'ts')

ans =
    0.1000
```

Because the numerator data (`num` property) is always stored as a cell array, the first command evaluates to a cell array containing the row vector `[0 1]`.

Remark

An alternative to the syntax

```
Value = get(sys,'PropertyName')
```

is the structure-like referencing

```
Value = sys.PropertyName
```

For example,

```
sys.Ts
sys.a
sys.user
```

return the values of the sample time, *A* matrix, and `UserData` property of the (state-space) model `sys`.

See Also

`frdata`	Quick access to frequency response data
`set`	Set/modify LTI properties
`ssdata`	Quick access to state-space data
`tfdata`	Quick access to transfer function data
`zpkdata`	Quick access to zero-pole-gain data

Purpose

Compute controllability and observability gramians

Syntax

```
Wc = gram(sys,'c')
Wo = gram(sys,'o')
```

Description

`gram` calculates controllability and observability gramians. You can use gramians to study the controllability and observability properties of state-space models and for model reduction [1,2]. They have better numerical properties than the controllability and observability matrices formed by `ctrb` and `obsv`.

Given the continuous-time state-space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

the controllability gramian is defined by

$$W_c = \int_0^\infty e^{A\tau} B B^T e^{A^T\tau} d\tau$$

and the observability gramian by

$$W_o = \int_0^\infty e^{A^T\tau} C^T C e^{A\tau} d\tau$$

The discrete-time counterparts are

$$W_c = \sum_{k=0}^{\infty} A^k B B^T (A^T)^k, \qquad W_o = \sum_{k=0}^{\infty} (A^T)^k C^T C A^k$$

The controllability gramian is positive definite if and only if (A, B) is controllable. Similarly, the observability gramian is positive definite if and only if (C, A) is observable.

Use the commands

```
Wc = gram(sys,'c')   % controllability gramian
Wo = gram(sys,'o')   % observability gramian
```

to compute the gramians of a continuous or discrete system. The LTI model sys must be in state-space form.

Algorithm

The controllability gramian W_c is obtained by solving the continuous-time Lyapunov equation

$$AW_c + W_cA^T + BB^T = 0$$

or its discrete-time counterpart

$$AW_cA^T - W_c + BB^T = 0$$

Similarly, the observability gramian W_o solves the Lyapunov equation

$$A^TW_o + W_oA + C^TC = 0$$

in continuous time, and the Lyapunov equation

$$A^TW_oA - W_o + C^TC = 0$$

in discrete time.

Limitations

The A matrix must be stable (all eigenvalues have negative real part in continuous time, and magnitude strictly less than one in discrete time).

See Also

`balreal`	Gramian-based balancing of state-space realizations
`ctrb`	Controllability matrix
`lyap`, `dlyap`	Lyapunov equation solvers
`obsv`	Observability matrix

References

[1] Kailath, T., *Linear Systems*, Prentice-Hall, 1980.

Purpose

Test if an LTI model has time delays

Syntax

```
hasdelay(sys)
```

Description

`hasdelay(sys)` returns 1 (true) if the LTI model `sys` has input delays, output delays, or I/O delays, and 0 (false) otherwise.

See Also

`delay2z`	Changes transfer functions of discrete-time LTI models with delays to rational functions or absorbs FRD delays into the frequency response phase information
`totaldelay`	Combines delays for an LTI model

impulse

Purpose

Compute the impulse response of LTI models

Syntax

```
impulse(sys)
impulse(sys,t)

impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,t)
impulse(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[y,t,x] = impulse(sys)
```

Description

`impulse` calculates the unit impulse response of a linear system. The impulse response is the response to a Dirac input $\delta(t)$ for continuous-time systems and to a unit pulse at $t = 0$ for discrete-time systems. Zero initial state is assumed in the state-space case. When invoked without left-hand arguments, this function plots the impulse response on the screen.

`impulse(sys)` plots the impulse response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. The impulse response of multi-input systems is the collection of impulse responses for each input channel. The duration of simulation is determined automatically to display the transient behavior of the response.

`impulse(sys,t)` sets the simulation horizon explicitly. You can specify either a final time `t = Tfinal` (in seconds), or a vector of evenly spaced time samples of the form

```
t = 0:dt:Tfinal
```

For discrete systems, the spacing `dt` should match the sample period. For continuous systems, `dt` becomes the sample time of the discretized simulation model (see "Algorithm"), so make sure to choose `dt` small enough to capture transient phenomena.

To plot the impulse responses of several LTI models `sys1`,..., `sysN` on a single figure, use

```
impulse(sys1,sys2,...,sysN)
impulse(sys1,sys2,...,sysN,t)
```

As with bode or plot, you can specify a particular color, linestyle, and/or marker for each system, for example,

```
impulse(sys1,'y:',sys2,'g--')
```

See "Plotting and Comparing Multiple Systems" on page 5-13 and the bode entry in this chapter for more details.

When invoked with lefthand arguments,

```
[y,t] = impulse(sys)
[y,t,x] = impulse(sys)       % for state-space models only
y = impulse(sys,t)
```

return the output response y, the time vector t used for simulation, and the state trajectories x (for state-space models only). No plot is drawn on the screen. For single-input systems, y has as many rows as time samples (length of t), and as many columns as outputs. In the multi-input case, the impulse responses of each input channel are stacked up along the third dimension of y. The dimensions of y are then

(length of t) × (number of outputs) × (number of inputs)

and y(:,:,j) gives the response to an impulse disturbance entering the jth input channel. Similarly, the dimensions of x are

(length of t) × (number of states) × (number of inputs)

Example

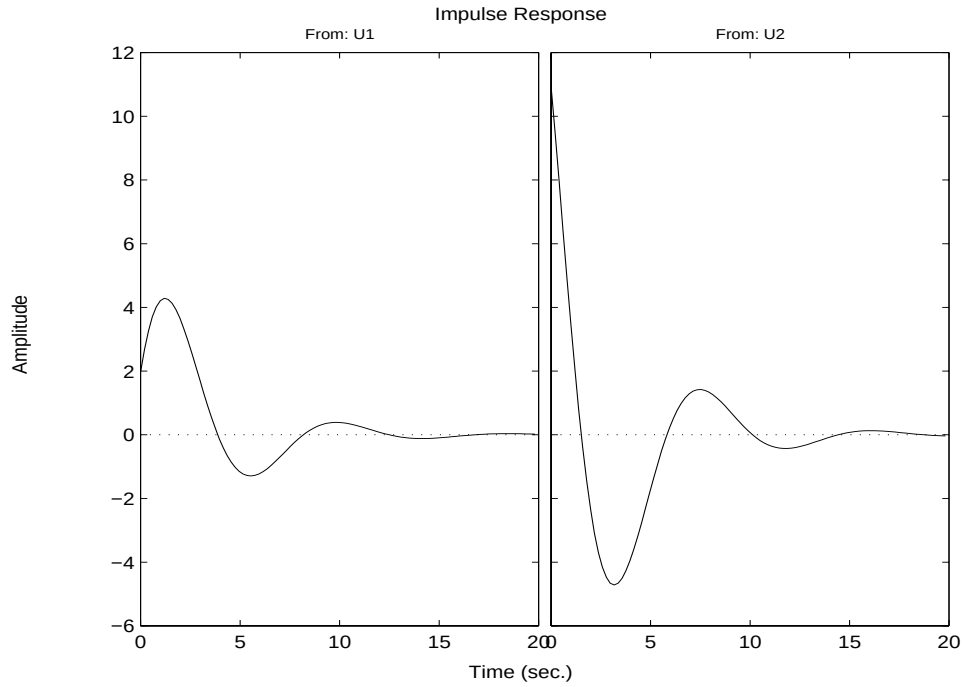
To plot the impulse response of the second-order state-space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5572 & -0.7814 \\ 0.7814 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

use the following commands.

```
a = [–0.5572   –0.7814;0.7814  0];
b = [1 –1;0 2];
c = [1.9691  6.4493];
sys = ss(a,b,c,0);
impulse(sys)
```



The left plot shows the impulse response of the first input channel, and the right plot shows the impulse response of the second input channel.

You can store the impulse response data in MATLAB arrays by

```
[y,t] = impulse(sys)
```

Because this system has two inputs, y is a 3-D array with dimensions

```
size(y)

ans =
   101     1     2
```

(the first dimension is the length of t). The impulse response of the first input channel is then accessed by

```
y(:,:,1)
```

Algorithm

Continuous-time models are first converted to state space. The impulse response of a single-input state-space model

$$\dot{x} = Ax + bu$$
$$y = Cx$$

is equivalent to the following unforced response with initial state b.

$$\dot{x} = Ax, \qquad x(0) = b$$
$$y = Cx$$

To simulate this response, the system is discretized using zero-order hold on the inputs. The sampling period is chosen automatically based on the system dynamics, except when a time vector `t = 0:dt:Tf` is supplied (`dt` is then used as sampling period).

Limitations

The impulse response of a continuous system with nonzero D matrix is infinite at $t = 0$. `impulse` ignores this discontinuity and returns the lower continuity value Cb at $t = 0$.

See Also

`ltiview`	LTI system viewer
`step`	Step response
`initial`	Free response to initial condition
`lsim`	Simulate response to arbitrary inputs

Purpose

Compute the initial condition response of state-space models

Syntax

```
initial(sys,x0)
initial(sys,x0,t)

initial(sys1,sys2,...,sysN,x0)
initial(sys1,sys2,...,sysN,x0,t)
initial(sys1,'PlotStyle1',...,sysN,'PlotStyleN',x0)

[y,t,x] = initial(sys,x0)
```

Description

`initial` calculates the unforced response of a state-space model with an initial condition on the states.

$$\dot{x} = Ax, \qquad x(0) = x_0$$
$$y = Cx$$

This function is applicable to either continuous- or discrete-time models. When invoked without lefthand arguments, `initial` plots the initial condition response on the screen.

`initial(sys,x0)` plots the response of `sys` to an initial condition `x0` on the states. `sys` can be any *state-space* model (continuous or discrete, SISO or MIMO, with or without inputs). The duration of simulation is determined automatically to reflect adequately the response transients.

`initial(sys,x0,t)` explicitly sets the simulation horizon. You can specify either a final time `t = Tfinal` (in seconds), or a vector of evenly spaced time samples of the form

```
t = 0:dt:Tfinal
```

For discrete systems, the spacing `dt` should match the sample period. For continuous systems, `dt` becomes the sample time of the discretized simulation model (see `impulse`), so make sure to choose `dt` small enough to capture transient phenomena.

To plot the initial condition responses of several LTI models on a single figure, use

```
initial(sys1,sys2,...,sysN,x0)
initial(sys1,sys2,...,sysN,x0,t)
```

(see `impulse` for details).

When invoked with lefthand arguments,

```
[y,t,x] = initial(sys,x0)
[y,t,x] = initial(sys,x0,t)
```

return the output response `y`, the time vector `t` used for simulation, and the state trajectories `x`. No plot is drawn on the screen. The array `y` has as many rows as time samples (length of `t`) and as many columns as outputs. Similarly, `x` has `length(t)` rows and as many columns as states.

Example

Plot the response of the state-space model

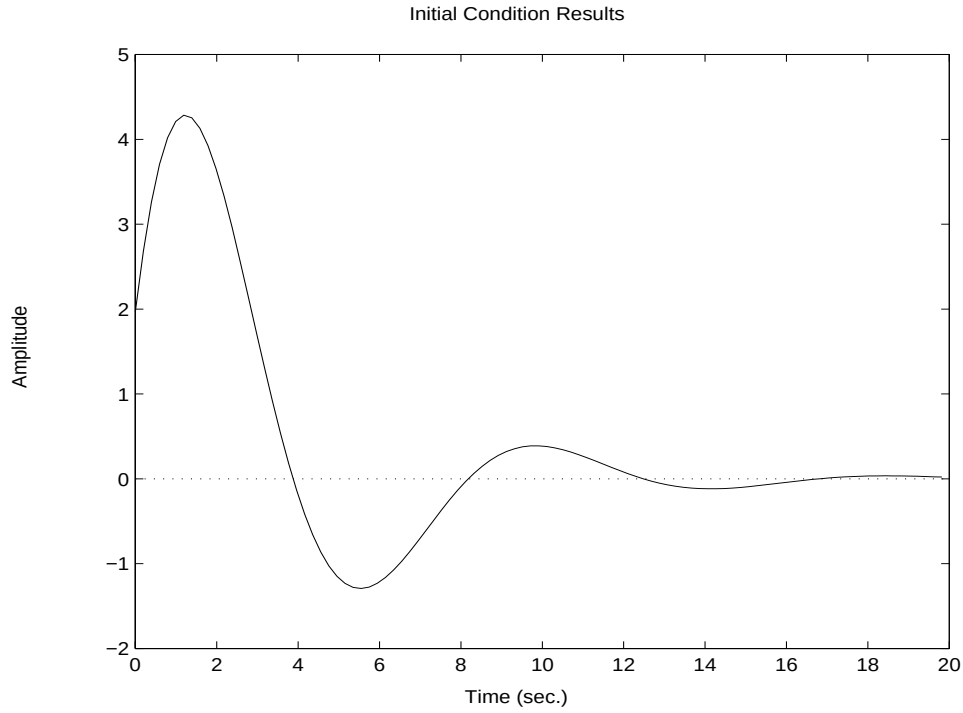
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5572 & -0.7814 \\ 0.7814 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

to the initial condition

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

```
a = [-0.5572   -0.7814;0.7814   0];
c = [1.9691   6.4493];
x0 = [1 ; 0]

sys = ss(a,[],c,[]);
initial(sys,x0)
```



See Also

`impulse`	Impulse response
`lsim`	Simulate response to arbitrary inputs
`ltiview`	LTI system viewer
`step`	Step response

Purpose

Invert LTI systems

Syntax

```
isys = inv(sys)
```

Description

inv inverts the input/output relation

$$y = G(s)u$$

to produce the LTI system with the transfer matrix $H(s) = G(s)^{-1}$.

$$u = H(s)y$$

This operation is defined only for square systems (same number of inputs and outputs) with an invertible feedthrough matrix D. inv handles both continuous- and discrete-time systems.

Example

Consider

$$H(s) = \begin{bmatrix} 1 & \frac{1}{s+1} \\ 0 & 1 \end{bmatrix}$$

At the MATLAB prompt, type

```
H = [1 tf(1,[1 1]);0 1]
Hi = inv(H)
```

to invert it. MATLAB returns

```
Transfer function from input 1 to output...
 #1:  1

 #2:  0

Transfer function from input 2 to output...
       -1
 #1:  -----
      s + 1

 #2:  1
```

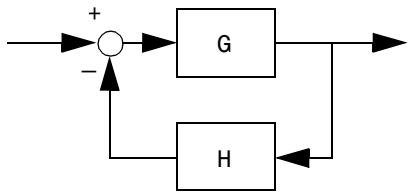
You can verify that

```
H * Hi
```

is the identity transfer function (static gain `I`).

Limitations

Do not use `inv` to model feedback connections such as



While it seems reasonable to evaluate the corresponding closed-loop transfer function $(I + GH)^{-1}G$ as

```
inv(1+g*h) * g
```

this typically leads to nonminimal closed-loop models. For example,

```
g = zpk([],1,1)
h = tf([2 1],[1 0])
cloop = inv(1+g*h) * g
```

yields a third-order closed-loop model with an unstable pole-zero cancellation at `s = 1`.

```
cloop

Zero/pole/gain:
      s (s-1)
-------------------
(s-1) (s^2 + s + 1)
```

Use `feedback` or `star` to avoid such pitfalls.

```
cloop = feedback(g,h)

Zero/pole/gain:
      s
-------------
(s^2 + s + 1)
```

isct, isdt

Purpose

Determine whether an LTI model is continuous or discrete

Syntax

```
boo = isct(sys)
boo = isdt(sys)
```

Description

`boo = isct(sys)` returns 1 (true) if the LTI model `sys` is continuous and 0 (false) otherwise. `sys` is continuous if its sample time is zero, that is, `sys.Ts=0`.

`boo = isdt(sys)` returns 1 (true) if `sys` is discrete and 0 (false) otherwise. Discrete-time LTI models have a nonzero sample time, except for empty models and static gains, which are regarded as either continuous or discrete as long as their sample time is not explicitly set to a nonzero value. Thus both

```
isct(tf(10))
isdt(tf(10))
```

are true. However, if you explicitly label a gain as discrete, for example, by typing

```
g = tf(10,'ts',0.01)
```

`isct(g)` now returns false and only `isdt(g)` is true.

See Also

`isa`	Determine LTI model type
`isempty`	True for empty LTI models
`isproper`	True for proper LTI models

Purpose

Test if an LTI model is empty

Syntax

```
boo = isempty(sys)
```

Description

`isempty(sys)` returns `1` (true) if the LTI model `sys` has no input or no output, and `0` (false) otherwise.

Example

Both commands

```
isempty(tf)   % tf by itself returns an empty transfer function
isempty(ss(1,2,[],[]))
```

return `1` (true) while

```
isempty(ss(1,2,3,4))
```

returns `0` (false).

See Also

`issiso`	True for SISO systems
`size`	I/O dimensions and array dimensions of LTI models

isproper

Purpose

Test if an LTI model is proper

Syntax

```
boo = isproper(sys)
```

Description

`isproper(sys)` returns 1 (true) if the LTI model `sys` is proper and 0 (false) otherwise.

State-space models are always proper. SISO transfer functions or zero-pole-gain models are proper if the degree of their numerator is less than or equal to the degree of their denominator. MIMO transfer functions are proper if all their SISO entries are proper.

Example

The following commands

```
isproper(tf([1 0],1))        % transfer function s
isproper(tf([1 0],[1 1]))    % transfer function s/(s+1)
```

return false and true, respectively.

Purpose

Test if an LTI model is single-input/single-output (SISO)

Syntax

```
boo = issiso(sys)
```

Description

`issiso(sys)` returns `1` (true) if the LTI model `sys` is SISO and `0` (false) otherwise.

See Also

`isempty`	True for empty LTI models
`size`	I/O dimensions and array dimensions of LTI models

Purpose

Design continuous- or discrete-time Kalman estimator

Syntax

```
[kest,L,P] = kalman(sys,Qn,Rn,Nn)
[kest,L,P,M,Z] = kalman(sys,Qn,Rn,Nn)   % discrete time only
[kest,L,P] = kalman(sys,Qn,Rn,Nn,sensors,known)
```

Description

`kalman` designs a Kalman state estimator given a state-space model of the plant and the process and measurement noise covariance data. The Kalman estimator is the optimal solution to the following continuous or discrete estimation problems.

Continuous-Time Estimation

Given the continuous plant

$$\dot{x} = Ax + Bu + Gw \qquad \text{(state equation)}$$
$$y_v = Cx + Du + Hw + v \qquad \text{(measurement equation)}$$

with known inputs u and process and measurement white noise w, v satisfying

$$E(w) = E(v) = 0, \quad E(ww^T) = Q, \quad E(vv^T) = R, \quad E(wv^T) = N$$

construct a state estimate $\hat{x}(t)$ that minimizes the steady-state error covariance

$$P = \lim_{t \to \infty} E(\{x - \hat{x}\}\{x - \hat{x}\}^T)$$

The optimal solution is the Kalman filter with equations

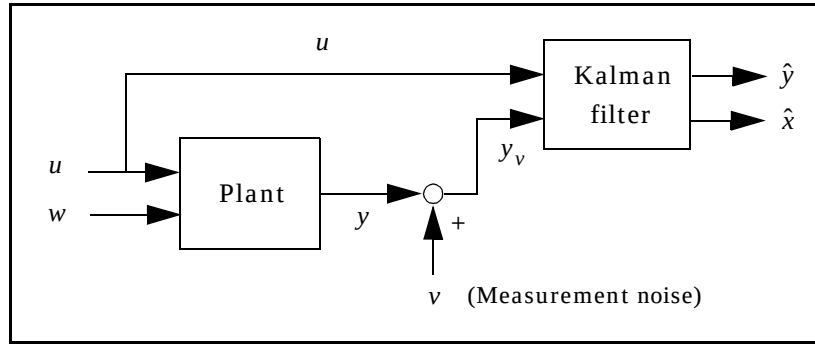
$$\dot{\hat{x}} = A\hat{x} + Bu + L(y_v - C\hat{x} - Du)$$

$$\begin{bmatrix} \hat{y} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} \hat{x} + \begin{bmatrix} D \\ 0 \end{bmatrix} u$$

where the filter gain L is determined by solving an algebraic Riccati equation. This estimator uses the known inputs u and the measurements y_v to generate

the output and state estimates $\hat{y}$ and $\hat{x}$. Note that $\hat{y}$ estimates the true plant output

$$y = Cx + Du + Hw$$



Kalman estimator

Discrete-Time Estimation

Given the discrete plant

$$x[n+1] = Ax[n] + Bu[n] + Gw[n]$$
$$y_v[n] = Cx[n] + Du[n] + Hw[n] + v[n]$$

and the noise covariance data

$$E(w[n]w[n]^T) = Q, \quad E(v[n]v[n]^T) = R, \quad E(w[n]v[n]^T) = N$$

the Kalman estimator has equations

$$\hat{x}[n+1|n] = A\hat{x}[n|n-1] + Bu[n] + L(y_v[n] - C\hat{x}[n|n-1] - Du[n])$$

$$\begin{bmatrix} \hat{y}[n|n] \\ \hat{x}[n|n] \end{bmatrix} = \begin{bmatrix} C(I-MC) \\ I-MC \end{bmatrix} \hat{x}[n|n-1] + \begin{bmatrix} (I-CM)D & CM \\ -MD & M \end{bmatrix} \begin{bmatrix} u[n] \\ y_v[n] \end{bmatrix}$$

and generates optimal "current" output and state estimates $\hat{y}[n|n]$ and $\hat{x}[n|n]$ using all available measurements including $y_v[n]$. The gain matrices L and M are derived by solving a discrete Riccati equation. The *innovation gain* M is used to update the prediction $\hat{x}[n|n-1]$ using the new measurement $y_v[n]$.

$$\hat{x}[n|n] = \hat{x}[n|n-1] + M\Big(\underbrace{y_v[n] - C\hat{x}[n|n-1] - Du[n]}_{\text{innovation}}\Big)$$

Usage

`[kest,L,P] = kalman(sys,Qn,Rn,Nn)` returns a state-space model `kest` of the Kalman estimator given the plant model `sys` and the noise covariance data `Qn`, `Rn`, `Nn` (matrices Q, R, N above). `sys` must be a state-space model with matrices

$$A,\ \begin{bmatrix} B & G \end{bmatrix},\ C,\ \begin{bmatrix} D & H \end{bmatrix}$$

The resulting estimator `kest` has $[u\ ;\ y_v]$ as inputs and $[\hat{y}\ ;\ \hat{x}]$ (or their discrete-time counterparts) as outputs. You can omit the last input argument `Nn` when $N = 0$.

The function `kalman` handles both continuous and discrete problems and produces a continuous estimator when `sys` is continuous, and a discrete estimator otherwise. In continuous time, `kalman` also returns the Kalman gain `L` and the steady-state error covariance matrix `P`. Note that `P` is the solution of the associated Riccati equation. In discrete time, the syntax

```
[kest,L,P,M,Z] = kalman(sys,Qn,Rn,Nn)
```

returns the filter gain L and innovations gain M, as well as the steady-state error covariances

$$P = \lim_{n\to\infty} E(e[n|n-1]e[n|n-1]^T), \qquad e[n|n-1] = x[n] - x[n|n-1]$$

$$Z = \lim_{n\to\infty} E(e[n|n]e[n|n]^T), \qquad e[n|n] = x[n] - x[n|n]$$

Finally, use the syntaxes

```
[kest,L,P] = kalman(sys,Qn,Rn,Nn,sensors,known)
[kest,L,P,M,Z] = kalman(sys,Qn,Rn,Nn,sensors,known)
```

for more general plants sys where the known inputs u and stochastic inputs w are mixed together, and not all outputs are measured. The index vectors sensors and known then specify which outputs y of sys are measured and which inputs u are known. All other inputs are assumed stochastic.

Example

See examples on "Control Design Tools" on page 1-20, "LQG Design for the x-Axis" on page 9-34, and "Kalman Filtering" on page 9-50.

Limitations

The plant and noise data must satisfy:

- (C, A) detectable
- $\overline{R} > 0$ and $\overline{Q} - \overline{N}\overline{R}^{-1}\overline{N}^T \geq 0$
- $(A - \overline{N}\overline{R}^{-1}C, \overline{Q} - \overline{N}\overline{R}^{-1}\overline{N}^T)$ has no uncontrollable mode on the imaginary axis (or unit circle in discrete time)

with the notation

$$\overline{Q} = GQG^T$$
$$\overline{R} = R + HN + N^T H^T + HQH^T$$
$$\overline{N} = G(QH^T + N)$$

See Also

care	Solve continuous-time Riccati equations
dare	Solve discrete-time Riccati equations
estim	Form estimator given estimator gain
kalmd	Discrete Kalman estimator for continuous plant
lqgreg	Assemble LQG regulator
lqr	Design state-feedback LQ regulator

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

kalmd

Purpose

Design discrete Kalman estimator for continuous plant

Syntax

```
[kest,L,P,M,Z] = kalmd(sys,Qn,Rn,Ts)
```

Description

`kalmd` designs a discrete-time Kalman estimator that has response characteristics similar to a continuous-time estimator designed with `kalman`. This command is useful to derive a discrete estimator for digital implementation after a satisfactory continuous estimator has been designed.

`[kest,L,P,M,Z] = kalmd(sys,Qn,Rn,Ts)` produces a discrete Kalman estimator `kest` with sample time `Ts` for the continuous-time plant

$$\dot{x} = Ax + Bu + Gw \qquad \text{(state equation)}$$
$$y_v = Cx + Du + v \qquad \text{(measurement equation)}$$

with process noise w and measurement noise v satisfying

$$E(w) = E(v) = 0\,, \quad E(ww^T) = Q_n\,, \quad E(vv^T) = R_n\,, \quad E(wv^T) = 0$$

The estimator `kest` is derived as follows. The continuous plant `sys` is first discretized using zero-order hold with sample time `Ts` (see `c2d` entry), and the continuous noise covariance matrices Q_n and R_n are replaced by their discrete equivalents

$$Q_d = \int_0^{T_s} e^{A\tau} G Q G^T e^{A^T\tau} d\tau$$
$$R_d = R / T_s$$

The integral is computed using the matrix exponential formulas in [2]. A discrete-time estimator is then designed for the discretized plant and noise. See `kalman` for details on discrete-time Kalman estimation.

`kalmd` also returns the estimator gains `L` and `M`, and the discrete error covariance matrices `P` and `Z` (see `kalman` for details).

Limitations

The discretized problem data should satisfy the requirements for `kalman`.

See Also

`kalman` Design Kalman estimator

`lqgreg`	Assemble LQG regulator
`lqrd`	Discrete LQ-optimal gain for continuous plant

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1990.

[2] Van Loan, C.F., "Computing Integrals Involving the Matrix Exponential," *IEEE Trans. Automatic Control*, AC-15, October 1970.

Purpose

Redheffer star product (linear fractional transformation) of two LTI models

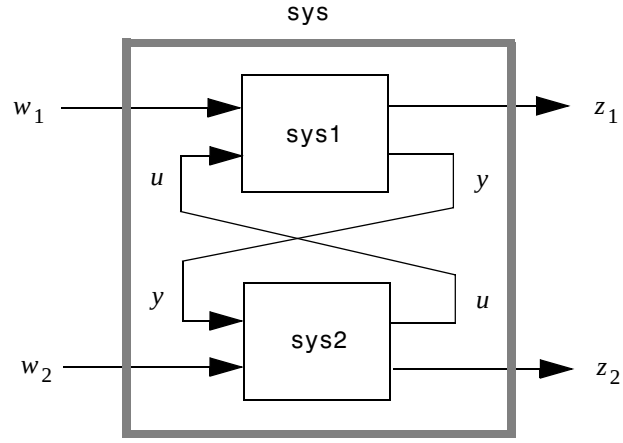
Syntax

```
sys = lft(sys1,sys2)
sys = lft(sys1,sys2,nu,ny)
```

Description

`lft` forms the star product or linear fractional transformation (LFT) of two LTI models or LTI arrays. Such interconnections are widely used in robust control techniques.

`sys = lft(sys1,sys2,nu,ny)` forms the star product `sys` of the two LTI models (or LTI arrays) `sys1` and `sys2`. The star product amounts to the following feedback connection for single LTI models (or for each model in an LTI array).



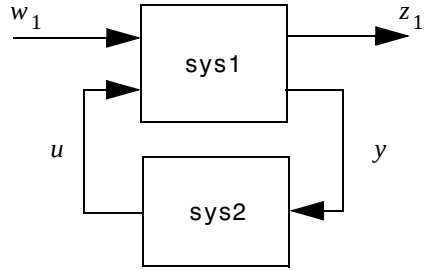
This feedback loop connects the first `nu` outputs of `sys2` to the last `nu` inputs of `sys1` (signals u), and the last `ny` outputs of `sys1` to the first `ny` inputs of `sys2` (signals y). The resulting system `sys` maps the input vector $[w_1 ; w_2]$ to the output vector $[z_1 ; z_2]$.

The abbreviated syntax

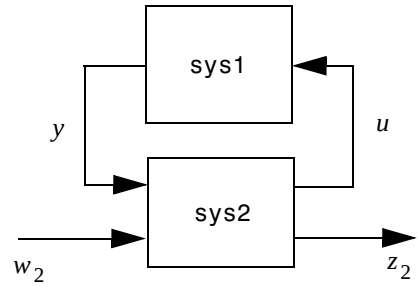
```
sys = lft(sys1,sys2)
```

produces:

- The lower LFT of sys1 and sys2 if sys2 has fewer inputs and outputs than sys1. This amounts to deleting w_2 and z_2 in the above diagram.
- The upper LFT of sys1 and sys2 if sys1 has fewer inputs and outputs than sys2. This amounts to deleting w_1 and z_1 in the above diagram.



Lower LFT connection



Upper LFT connection

Algorithm

The closed-loop model is derived by elementary state-space manipulations.

Limitations

There should be no algebraic loop in the feedback connection.

See Also

`connect`	Derive state-space model for block diagram interconnection
`feedback`	Feedback connection

Purpose

Form LQG regulator given state-feedback gain and Kalman estimator

Syntax

```
rlqg = lqgreg(kest,k)
rlqg = lqgreg(kest,k,'current')   % discrete-time only

rlqg = lqgreg(kest,k,controls)
```

Description

`lqgreg` forms the LQG regulator by connecting the Kalman estimator designed with `kalman` and the optimal state-feedback gain designed with `lqr`, `dlqr`, or `lqry`. The LQG regulator minimizes some quadratic cost function that trades off regulation performance and control effort. This regulator is dynamic and relies on noisy output measurements to generate the regulating commands (see "LQG Regulator" on page 7-10 for details).

In continuous time, the LQG regulator generates the commands

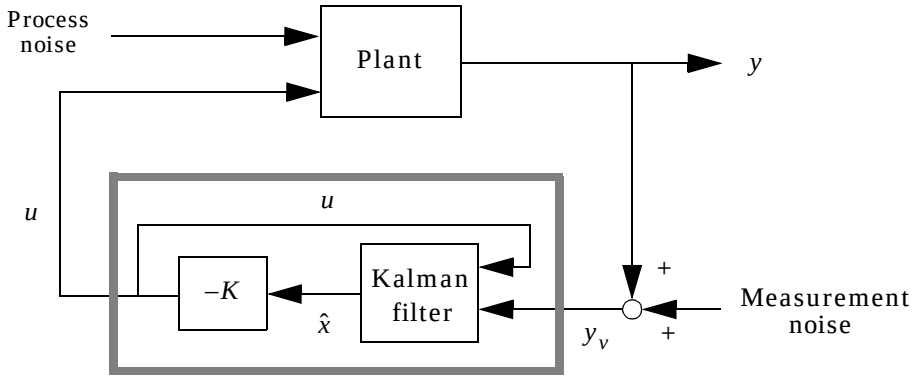
$$u = -K\hat{x}$$

where $\hat{x}$ is the Kalman state estimate. The regulator state-space equations are

$$\dot{\hat{x}} = \left[A - LC - (B - LD)K\right]\hat{x} + Ly_v$$

$$u = -K\hat{x}$$

where y_v is the vector of plant output measurements (see `kalman` for background and notation). The diagram below shows this dynamic regulator in relation to the plant.



LQG regulator

In discrete time, you can form the LQG regulator using either the prediction $\hat{x}[n|n-1]$ of $x[n]$ based on measurements up to $y_v[n-1]$, or the current state estimate $\hat{x}[n|n]$ based on all available measurements including $y_v[n]$. While the regulator

$$u[n] = -K\hat{x}[n|n-1]$$

is always well-defined, the *current regulator*

$$u[n] = -K\hat{x}[n|n]$$

is causal only when $I - KMD$ is invertible (see `kalman` for the notation). In addition, practical implementations of the current regulator should allow for the processing time required to compute $u[n]$ once the measurements $y_v[n]$ become available (this amounts to a time delay in the feedback loop).

Usage

`rlqg = lqgreg(kest,k)` returns the LQG regulator `rlqg` (a state-space model) given the Kalman estimator `kest` and the state-feedback gain matrix `k`. The same function handles both continuous- and discrete-time cases. Use consistent tools to design `kest` and `k`:

- Continuous regulator for continuous plant: use `lqr` or `lqry` and `kalman`.
- Discrete regulator for discrete plant: use `dlqr` or `lqry` and `kalman`.
- Discrete regulator for continuous plant: use `lqrd` and `kalmd`.

In discrete time, `lqgreg` produces the regulator

$$u[n] = -K\hat{x}[n|n-1]$$

by default (see "Description"). To form the "current" LQG regulator instead, use

$$u[n] = -K\hat{x}[n|n]$$

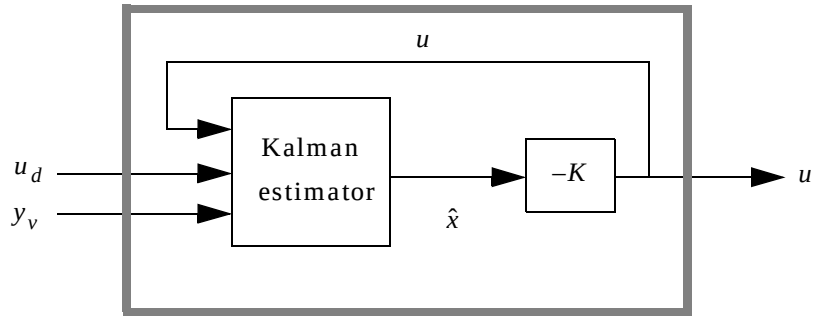
the syntax

```
rlqg = lqgreg(kest,k,'current')
```

This syntax is meaningful only for discrete-time problems.

`rlqg = lqgreg(kest,k,controls)` handles estimators that have access to additional known plant inputs u_d. The index vector `controls` then specifies which estimator inputs are the controls u, and the resulting LQG regulator `rlqg` has u_d and y_v as inputs (see figure below).

Note: Always use *positive* feedback to connect the LQG regulator to the plant.



LQG regulator

Example

See the examples “Control Design Tools” on page 1-20 and “LQG Regulation” on page 9-31.

See Also

`kalman`	Kalman estimator design
`kalmd`	Discrete Kalman estimator for continuous plant
`lqr`, `dlqr`	State-feedback LQ regulator
`lqrd`	Discrete LQ regulator for continuous plant
`lqry`	LQ regulator with output weighting
`reg`	Form regulator given state-feedback and estimator gains

Purpose

Design linear-quadratic (LQ) state-feedback regulator for continuous plant

Syntax

```
[K,S,e] = lqr(A,B,Q,R)
[K,S,e] = lqr(A,B,Q,R,N)
```

Description

`[K,S,e] = lqr(A,B,Q,R,N)` calculates the optimal gain matrix `K` such that the state-feedback law $u = -Kx$

minimizes the quadratic cost function

$$J(u) = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u)\,dt$$

for the continuous-time state-space model $\dot{x} = Ax + Bu$

The default value `N=0` is assumed when `N` is omitted.

In addition to the state-feedback gain `K`, `lqr` returns the solution `S` of the associated Riccati equation

$$A^T S + SA - (SB + N)R^{-1}(B^T S + N^T) + Q = 0$$

and the closed-loop eigenvalues `e = eig(A-B*K)`. Note that K is derived from S by

$$K = R^{-1}(B^T S + N^T)$$

Limitations

The problem data must satisfy:

- The pair (A, B) is stabilizable.
- $R > 0$ and $Q - NR^{-1}N^T \geq 0$.
- $(Q - NR^{-1}N^T, A - BR^{-1}N^T)$ has no unobservable mode on the imaginary axis.

See Also

`care`	Solve continuous Riccati equations
`dlqr`	State-feedback LQ regulator for discrete plant
`lqgreg`	Form LQG regulator
`lqrd`	Discrete LQ regulator for continuous plant
`lqry`	State-feedback LQ regulator with output weighting

Purpose

Design discrete LQ regulator for continuous plant

Syntax

```
[Kd,S,e] = lqrd(A,B,Q,R,Ts)
[Kd,S,e] = lqrd(A,B,Q,R,N,Ts)
```

Description

`lqrd` designs a discrete full-state-feedback regulator that has response characteristics similar to a continuous state-feedback regulator designed using `lqr`. This command is useful to design a gain matrix for digital implementation after a satisfactory continuous state-feedback gain has been designed.

`[Kd,S,e] = lqrd(A,B,Q,R,Ts)` calculates the discrete state-feedback law

$$u[n] = -K_d x[n]$$

that minimizes a discrete cost function equivalent to the continuous cost function

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt$$

The matrices `A` and `B` specify the continuous plant dynamics

$$\dot{x} = Ax + Bu$$

and `Ts` specifies the sample time of the discrete regulator. Also returned are the solution `S` of the discrete Riccati equation for the discretized problem and the discrete closed-loop eigenvalues `e = eig(Ad–Bd*Kd)`.

`[Kd,S,e] = lqrd(A,B,Q,R,N,Ts)` solves the more general problem with a cross-coupling term in the cost function.

$$J = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u)\, dt$$

Algorithm

The equivalent discrete gain matrix `Kd` is determined by discretizing the continuous plant and weighting matrices using the sample time `Ts` and the zero-order hold approximation.

With the notation

$$\Phi(\tau) = e^{A\tau}, \qquad A_d = \Phi(T_s)$$

$$\Gamma(\tau) = \int_0^\tau e^{A\eta} B\, d\eta, \qquad B_d = \Gamma(T_s)$$

the discretized plant has equations

$$x[n+1] = A_d x[n] + B_d u[n]$$

and the weighting matrices for the equivalent discrete cost function are

$$\begin{bmatrix} Q_d & N_d \\ N_d^T & R_d \end{bmatrix} = \int_0^{T_s} \begin{bmatrix} \Phi^T(\tau) & 0 \\ \Gamma^T(\tau) & I \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} \Phi(\tau) & \Gamma(\tau) \\ 0 & I \end{bmatrix} d\tau$$

The integrals are computed using matrix exponential formulas due to Van Loan (see [2]). The plant is discretized using `c2d` and the gain matrix is computed from the discretized data using `dlqr`.

Limitations

The discretized problem data should meet the requirements for `dlqr`.

See Also

`c2d`	Discretization of LTI model
`dlqr`	State-feedback LQ regulator for discrete plant
`kalmd`	Discrete Kalman estimator for continuous plant
`lqr`	State-feedback LQ regulator for continuous plant

References

[1] Franklin, G.F., J.D. Powell, and M.L. Workman, *Digital Control of Dynamic Systems*, Second Edition, Addison-Wesley, 1980, pp. 439–440

[2] Van Loan, C.F., "Computing Integrals Involving the Matrix Exponential," *IEEE Trans. Automatic Control*, AC-15, October 1970.

Purpose

Linear-quadratic (LQ) state-feedback regulator with output weighting

Syntax

```
[K,S,e] = lqry(sys,Q,R)
[K,S,e] = lqry(sys,Q,R,N)
```

Description

Given the plant

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

or its discrete-time counterpart, `lqry` designs a state-feedback control

$$u = -Kx$$

that minimizes the quadratic cost function with output weighting

$$J(u) = \int_0^\infty (y^T Q y + u^T R u + 2y^T N u)\,dt$$

(or its discrete-time counterpart). The function `lqry` is equivalent to `lqr` or `dlqr` with weighting matrices:

$$\begin{bmatrix} \bar{Q} & \bar{N} \\ \bar{N}^T & \bar{R} \end{bmatrix} = \begin{bmatrix} C^T & 0 \\ D^T & I \end{bmatrix} \begin{bmatrix} Q & N \\ N^T & R \end{bmatrix} \begin{bmatrix} C & D \\ 0 & I \end{bmatrix}$$

`[K,S,e] = lqry(sys,Q,R,N)` returns the optimal gain matrix `K`, the Riccati solution `S`, and the closed-loop eigenvalues `e = eig(A-B*K)`. The state-space model `sys` specifies the continuous- or discrete-time plant data (A, B, C, D). The default value `N=0` is assumed when `N` is omitted.

Example

See “LQG Design for the x-Axis” on page 9-34 for an example.

Limitations

The data $A, B, \bar{Q}, \bar{R}, \bar{N}$ must satisfy the requirements for `lqr` or `dlqr`.

See Also

`lqr`	State-feedback LQ regulator for continuous plant
`dlqr`	State-feedback LQ regulator for discrete plant
`kalman`	Kalman estimator design
`lqgreg`	Form LQG regulator

lsim

Purpose

Simulate LTI model response to arbitrary inputs

Syntax

```
lsim(sys,u,t)
lsim(sys,u,t,x0)

lsim(sys1,sys2,...,sysN,u,t)
lsim(sys1,sys2,...,sysN,u,t,x0)
lsim(sys1,'PlotStyle1',...,sysN,'PlotStyleN',u,t)

[y,t,x] = lsim(sys,u,t,x0)
```

Description

`lsim` simulates the (time) response of continuous or discrete linear systems to arbitrary inputs. When invoked without left-hand arguments, `lsim` plots the response on the screen.

`lsim(sys,u,t)` produces a plot of the zero-initial condition time response of the LTI model `sys` to the input time history `t,u`. The vector `t` specifies the time samples for the simulation and consists of regularly spaced time samples

```
t = T0:dt:Tfinal
```

The matrix `u` must have as many rows as time samples (`length(t)`) and as many columns as system inputs. Each row `u(i,:)` specifies the input value(s) at the time sample `t(i)`.

The LTI model `sys` can be continuous or discrete, SISO or MIMO. In discrete time, `u` must be sampled at the same rate as the system (`t` is then redundant and can be omitted or set to the empty matrix). In continuous time, the time sampling `dt=t(2)-t(1)` is used to discretize the continuous model. Automatic resampling is performed if `dt` is too large (undersampling) and may give rise to hidden oscillations (see "Algorithm").

`lsim(sys,u,t,x0)` specifies a nonzero initial condition `x0` for the system states. This initial condition is applied at `t(1)`. This syntax applies only to state-space models.

Finally,

```
lsim(sys1,sys2,...,sysN,u,t)
```

simulates the responses of several LTI models to the same input history t,u and plots these responses on a single figure. As with bode or plot, you can specify a particular color, linestyle, and/or marker for each system, for example,

```
lsim(sys1,'y:',sys2,'g--',u,t,x0)
```

The multisystem behavior is similar to that of bode or step.

When invoked with left-hand arguments,

```
[y,t] = lsim(sys,u,t)
[y,t,x] = lsim(sys,u,t)       % for state-space models only
[y,t,x] = lsim(sys,u,t,x0)     % with initial state
```

return the output response y, the time vector t used for simulation, and the state trajectories x (for state-space models only). No plot is drawn on the screen. The matrix y has as many rows as time samples (length(t)) and as many columns as system outputs. The same holds for x with "outputs" replaced by states. Note that the output t may differ from the specified time vector when the input data is undersampled (see "Algorithm").

Example

Simulate and plot the response of the system

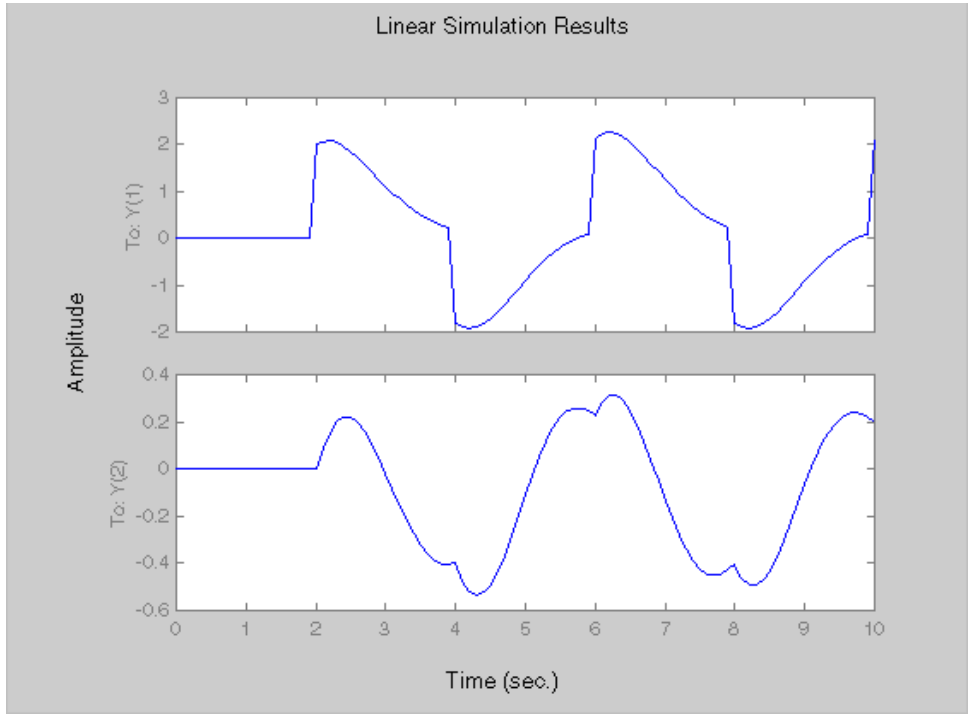
$$H(s) = \begin{bmatrix} \dfrac{2s^2 + 5s + 1}{s^2 + 2s + 3} \\ \dfrac{s - 1}{s^2 + s + 5} \end{bmatrix}$$

to a square wave with period of four seconds. First generate the square wave with gensig. Sample every 0.1 second during 10 seconds:

```
[u,t] = gensig('square',4,10,0.1);
```

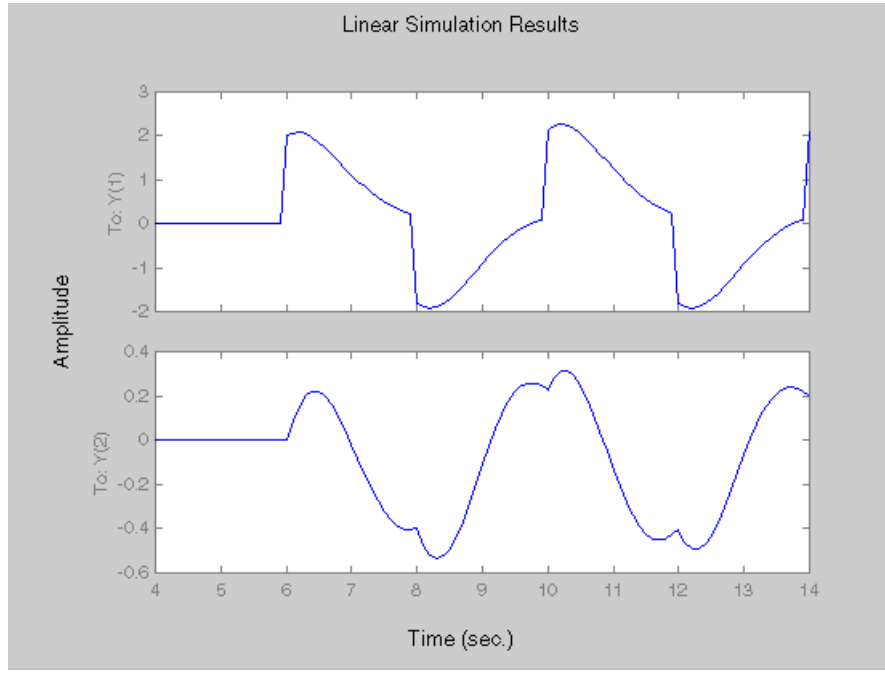
Then simulate with `lsim`.

```
H = [tf([2 5 1],[1 2 3]) ; tf([1 -1],[1 1 5])];
lsim(H,u,t)
```



Note that if you begin the simulation with a nonzero value for t(1), then the zero-initial condition response is shifted in time, as shown below.

```
lsim(H,u,t+4)
```



Algorithm

Discrete-time systems are simulated with `ltitr` (state space) or `filter` (transfer function and zero-pole-gain).

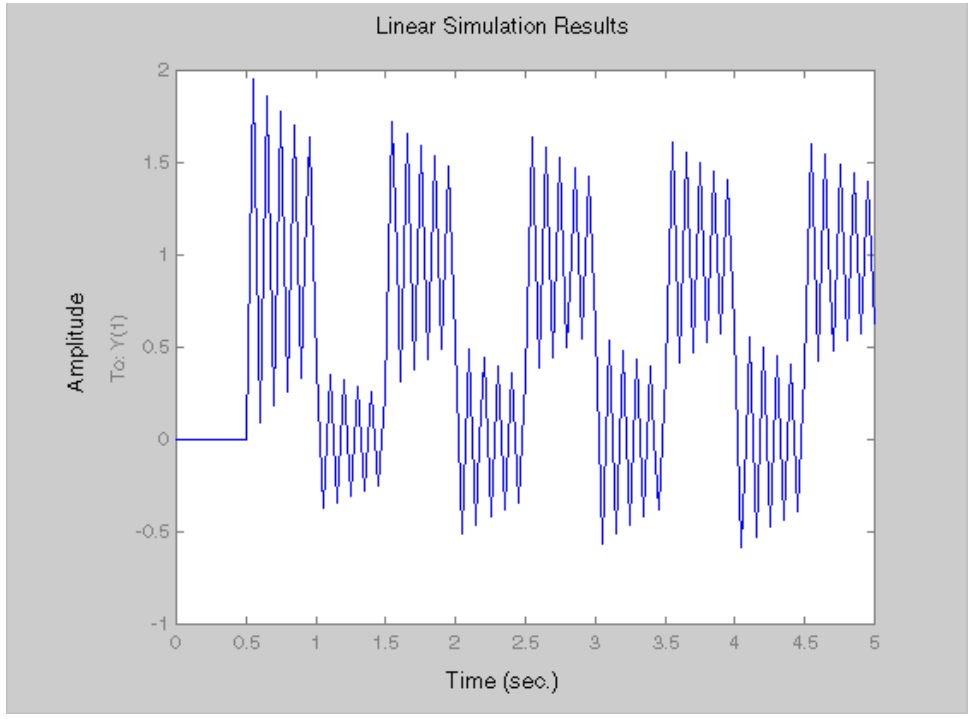
Continuous-time systems are discretized with `c2d` using either the `'zoh'` or `'foh'` method (`'foh'` is used for smooth input signals and `'zoh'` for discontinuous signals such as pulses or square waves). By default, the sampling period is set to the spacing `dt` between the user-supplied time samples `t`. However, if `dt` is not small enough to capture intersample behavior, `lsim` selects a smaller sampling period and resamples the input data using linear interpolation for smooth signals and zero-order hold for square signals. The time vector returned by `lsim` is then different from the specified `t` vector.

To illustrate why resampling is sometimes necessary, consider the second-order model

$$H(s) = \frac{\omega^2}{s^2 + 2s + \omega^2}, \qquad \omega = 62.83$$

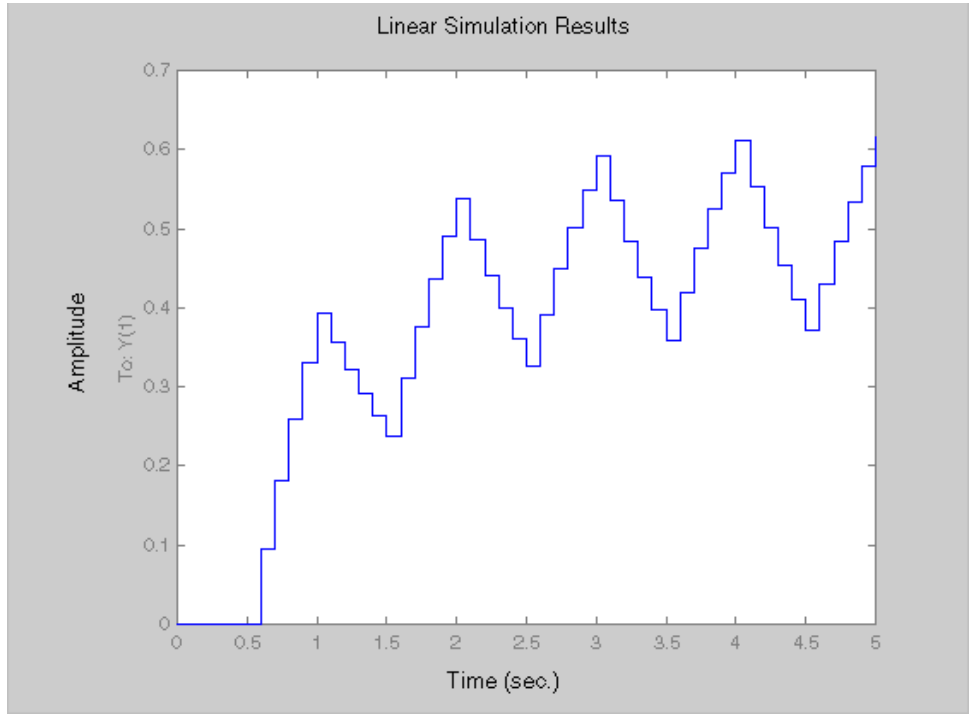
To simulate its response to a square wave with period 1 second, you can proceed as follows:

```
w2 = 62.83^2
h = tf(w2,[1 2 w2]);

t = 0:0.1:5;            % vector of time samples
u = (rem(t,1)>=0.5);    % square wave values
lsim(h,u,t)
```



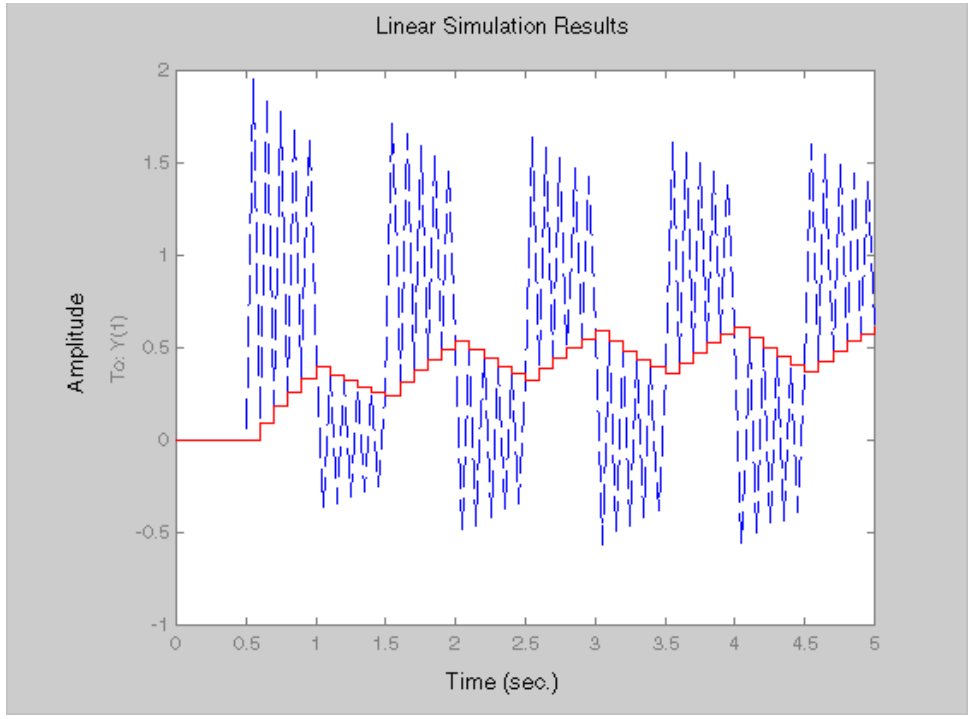
The response exhibits strong oscillations. Less obvious from this plot is the fact that `lsim` has resampled the input to reveal the oscillatory behavior. To see this, discretize $H(s)$ using the sampling period 0.1 second (spacing in your `t` vector) and simulate the response of the discretized model:

```
hd = c2d(h,0.1);
lsim(hd,u,t)
```



The two responses look quite different. To clarify this discrepancy, superimpose the two plots by

```
lsim(h,'b--',hd,'r-',u,t)
```



The cause is now obvious: `hd` is undersampled and its response (solid line) masks the intersample oscillations of the continuous model $H(s)$.

By comparing the suggested sampling `dt=t(2)-t(1)` against the system dynamics, `lsim` detects such undersampling and resamples the input to produce accurate continuous-time simulations.

See Also

`gensig`	Generate test input signals for `lsim`
`impulse`	Impulse response
`initial`	Free response to initial condition
`ltiview`	LTI system viewer
`step`	Step response

Purpose

Initialize an LTI Viewer for LTI system response analysis

Syntax

```
ltiview
ltiview(plottype,sys)
ltiview(plottype,sys,extras)

ltiview(plottype,sys1,sys2,...sysN)
ltiview(plottype,sys1,sys2,...sysN,extras)
ltiview(plottype,sys1,PlotStyle1,sys2,PlotStyle2,...)
```

Description

`ltiview` when invoked without input arguments, initializes a new LTI Viewer for LTI system response analysis.

Only frequency-domain analysis functions can be applied to FRDs.

`ltiview(plottype,sys)` initializes an LTI Viewer containing the LTI response type indicated by *`plottype`* for the LTI model `sys`. The string *`plottype`* can be any one of the following:

```
'step'
'impulse'
'initial'
'lsim'
'pzmap'
'bode'
'nyquist'
'nichols'
'sigma'
```

or, *`plottype`* can be a cell vector containing up to six of these plot types.

For example,

```
ltiview({'step';'nyquist'},sys)
```

displays the plots of both of these response types for a given system `sys`.

`ltiview(plottype,sys,extras)` allows the additional input arguments supported by the various LTI model response functions to be passed to the ltiview command.

`extras` is one or more input arguments as specified by the function named in *`plottype`*. These arguments may be required or optional, depending on the type of LTI response. For example, if *`plottype`* is `'step'` then `extras` may be the desired final time, `Tfinal`, as shown below.

```
ltiview('step',sys,Tfinal)
```

However, if *`plottype`* is `'initial'`, the `extras` arguments must contain the initial conditions `x0` and may contain other arguments, such as `Tfinal`.

```
ltiview('initial',sys,x0,Tfinal)
```

See the individual references pages of each possible *`plottype`* commands for a list of appropriate arguments for `extras`.

Finally,

```
ltiview(plottype,sys1,sys2,...sysN)
ltiview(plottype,sys1,sys2,...sysN,extras)
ltiview(plottype,sys1,PlotStyle1,sys2,PlotStyle2,...)
```

initializes an LTI Viewer containing the responses of multiple LTI models, using the plot styles in `PlotStyle`, when applicable. See the individual reference pages of the LTI response functions for more information on specifying plot styles.

Example

See Chapter 6, “The LTI Viewer.”

See Also

`bode`	Bode response
`impulse`	Impulse response
`initial`	Response to initial condition
`lsim`	Simulate LTI model response to arbitrary inputs
`nichols`	Nichols response
`nyquist`	Nyquist response
`pzmap`	Pole/zero map
`sigma`	Singular value response
`step`	Step response

Purpose

Solve continuous-time Lyapunov equations

Syntax

```
X = lyap(A,Q)
X = lyap(A,B,C)
```

Description

`lyap` solves the special and general forms of the Lyapunov matrix equation. Lyapunov equations arise in several areas of control, including stability theory and the study of the RMS behavior of systems.

`X = lyap(A,Q)` solves the Lyapunov equation

$$AX + XA^T + Q = 0$$

where A and Q are square matrices of identical sizes. The solution `X` is a symmetric matrix if Q is.

`X = lyap(A,B,C)` solves the generalized Lyapunov equation (also called Sylvester equation).

$$AX + XB + C = 0$$

The matrices A, B, C must have compatible dimensions but need not be square.

Algorithm

`lyap` transforms the A and B matrices to complex Schur form, computes the solution of the resulting triangular system, and transforms this solution back [1].

Limitations

The continuous Lyapunov equation has a (unique) solution if the eigenvalues $\alpha_1, \alpha_2, ..., \alpha_n$ of A and $\beta_1, \beta_2, ..., \beta_n$ of B satisfy

$$\alpha_i + \beta_j \neq 0 \qquad \text{for all pairs } (i, j)$$

If this condition is violated, `lyap` produces the error message

```
Solution does not exist or is not unique.
```

See Also

`covar`	Covariance of system response to white noise
`dlyap`	Solve discrete Lyapunov equations

References

[1] Bartels, R.H. and G.W. Stewart, “Solution of the Matrix Equation AX + XB = C,” *Comm. of the ACM*, Vol. 15, No. 9, 1972.

[2] Bryson, A.E. and Y.C. Ho, *Applied Optimal Control*, Hemisphere Publishing, 1975. pp. 328–338.

Purpose

Compute gain and phase margins and associated crossover frequencies

Syntax

```
[Gm,Pm,Wcg,Wcp] = margin(sys)
[Gm,Pm,Wcg,Wcp] = margin(mag,phase,w)
margin(sys)
```

Description

`margin` calculates the gain margin, phase margin, and associated crossover frequencies of SISO open-loop models. The gain and phase margins indicate the relative stability of the control system when the loop is closed. When invoked without left-hand arguments, `margin` produces a Bode plot and displays the margins on this plot.

The gain margin is the amount of gain increase required to make the loop gain unity at the frequency where the phase angle is –180°. In other words, the gain margin is $1/g$ if g is the gain at the –180° phase frequency. Similarly, the phase margin is the difference between the phase of the response and –180° when the loop gain is 1.0. The frequency at which the magnitude is 1.0 is called the unity-gain frequency or crossover frequency. It is generally found that gain margins of three or more combined with phase margins between 30 and 60 degrees result in reasonable trade-offs between bandwidth and stability.

`[Gm,Pm,Wcg,Wcp] = margin(sys)` computes the gain margin `Gm`, the phase margin `Pm`, and the corresponding crossover frequencies `Wcg` and `Wcp`, given the SISO open-loop model `sys`. This function handles both continuous- and discrete-time cases. When faced with several crossover frequencies, `margin` returns the smallest gain and phase margins.

`[Gm,Pm,Wcg,Wcp] = margin(mag,phase,w)` derives the gain and phase margins from the Bode frequency response data (magnitude, phase, and frequency vector). Interpolation is performed between the frequency points to estimate the margin values. This approach is generally less accurate.

When invoked without left-hand argument,

```
margin(sys)
```

plots the open-loop Bode response with the gain and phase margins marked by vertical lines.

Example

You can compute the gain and phase margins of the open-loop discrete-time transfer function. Type

```
hd = tf([0.04798 0.0464],[1 −1.81 0.9048],0.1)
```

MATLAB responds with

```
Transfer function:
 0.04798 z + 0.0464
---------------------
z^2 − 1.81 z + 0.9048

Sampling time: 0.1
```

Type

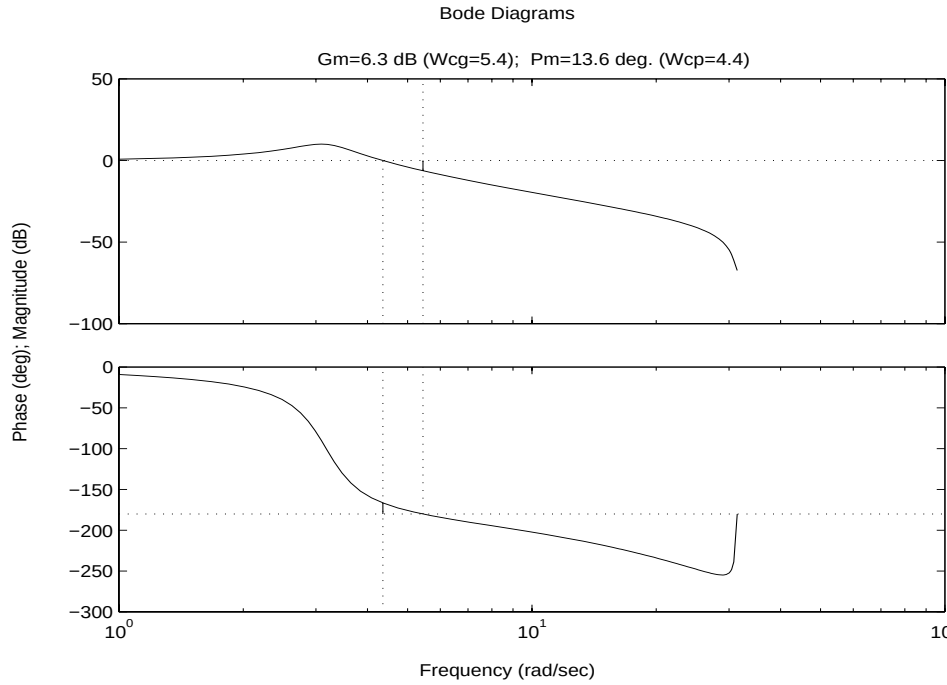
```
[Gm,Pm,Wcg,Wcp] = margin(hd);
[Gm,Pm,Wcg,Wcp]
```

and MATLAB returns

```
ans =
    2.0517   13.5712    5.4374    4.3544
```

You can also display these margins graphically.

```
margin(hd)
```



Algorithm

The phase margin is computed using H_∞ theory, and the gain margin by solving $H(j\omega) = \overline{H(j\omega)}$ for the frequency ω.

See Also

`bode`	Bode frequency response
`ltiview`	LTI system viewer

minreal

Purpose

Minimal realization or pole-zero cancellation

Syntax

```
sysr = minreal(sys)
sysr = minreal(sys,tol)
```

Description

`sysr = minreal(sys)` eliminates uncontrollable or unobservable state in state-space models, or cancels pole-zero pairs in transfer functions or zero-pole-gain models. The output `sysr` has minimal order and the same response characteristics as the original model `sys`.

`sysr = minreal(sys,tol)` specifies the tolerance used for state elimination or pole-zero cancellation. The default value is `tol = sqrt(eps)` and increasing this tolerance forces additional cancellations.

Example

The commands

```
g = zpk([],1,1)
h = tf([2 1],[1 0])
cloop = inv(1+g*h) * g
```

produce the nonminimal zero-pole-gain model by typing `cloop`.

```
Zero/pole/gain:
      s (s-1)
-------------------
(s-1) (s^2 + s + 1)
```

To cancel the pole-zero pair at $s = 1$, type

```
cloop = minreal(cloop)
```

and MATLAB returns

```
Zero/pole/gain:
      s
-------------
(s^2 + s + 1)
```

Algorithm

Pole-zero cancellation is a straightforward search through the poles and zeros looking for matches that are within tolerance. Transfer functions are first converted to zero-pole-gain form.

See Also

`balreal`	Gramian-based input/output balancing
`modred`	Model order reduction
`sminreal`	Structured model reduction

modred

Purpose

Model order reduction

Syntax

```
rsys = modred(sys,elim)
rsys = modred(sys,elim,'mdc')
rsys = modred(sys,elim,'del')
```

Description

`modred` reduces the order of a continuous or discrete state-space model `sys`. This function is usually used in conjunction with `balreal`. Two order reduction techniques are available:

- `rsys = modred(sys,elim)` or `rsys = modred(sys,elim,'mdc')` produces a reduced-order model `rsys` with matching DC gain (or equivalently, matching steady state in the step response). The index vector `elim` specifies the states to be eliminated. The resulting model `rsys` has `length(elim)` fewer states. This technique consists of setting the derivative of the eliminated states to zero and solving for the remaining states.
- `rsys = modred(sys,elim,'del')` simply deletes the states specified by `elim`. While this method does not guarantee matching DC gains, it tends to produce better approximations in the frequency domain (see example below).

If the state-space model `sys` has been balanced with `balreal` and the gramians have m small diagonal entries, you can reduce the model order by eliminating the last m states with `modred`.

Example

Consider the continuous fourth-order model

$$h(s) = \frac{s^3 + 11s^2 + 36s + 26}{s^4 + 14.6s^3 + 74.96s^2 + 153.7s + 99.65}$$

To reduce its order, first compute a balanced state-space realization with `balreal` by typing

```
h = tf([1 11 36 26],[1 14.6 74.96 153.7 99.65])
[hb,g] = balreal(h)
g'
```

MATLAB returns

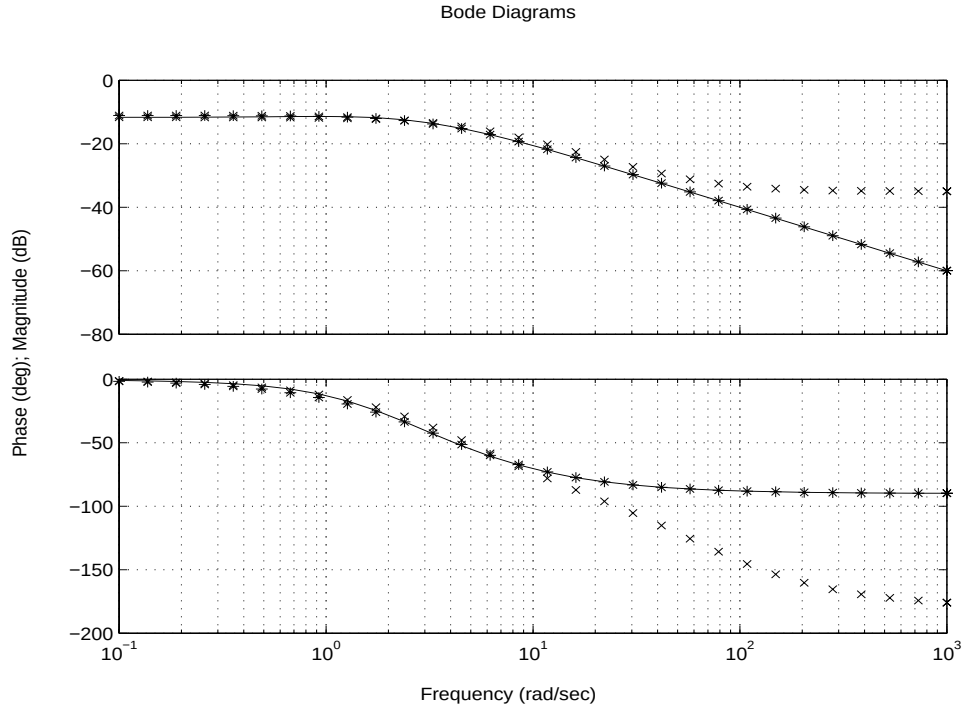
```
ans =
   1.3938e-01    9.5482e-03    6.2712e-04    7.3245e-06
```

The last three diagonal entries of the balanced gramians are small, so eliminate the last three states with `modred` using both matched DC gain and direct deletion methods.

```
hmdc = modred(hb,2:4,'mdc')
hdel = modred(hb,2:4,'del')
```

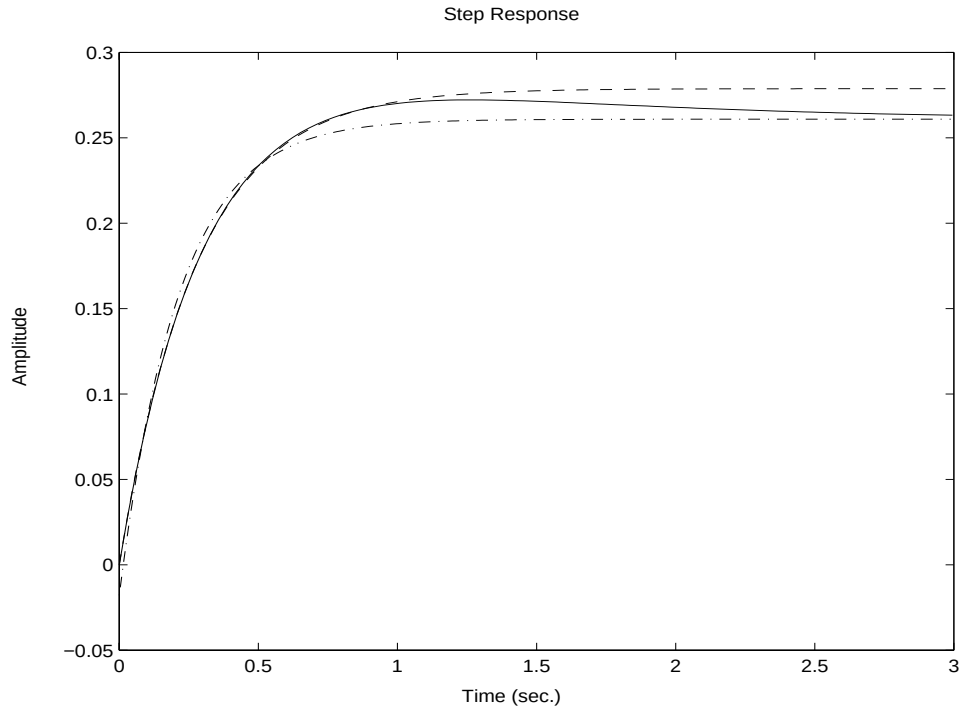
Both `hmdc` and `hdel` are first-order models. Compare their Bode responses against that of the original model $h(s)$.

```
bode(h,'-',hmdc,'x',hdel,'*')
```



The reduced-order model `hdel` is clearly a better frequency-domain approximation of $h(s)$. Now compare the step responses.

```
step(h,'-',hmdc,'-.',hdel,'--')
```



While `hdel` accurately reflects the transient behavior, only `hmdc` gives the true steady-state response.

Algorithm

The algorithm for the matched DC gain method is as follows. For continuous-time models

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

the state vector is partitioned into x_1, to be kept, and x_2, to be eliminated.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix}u$$

$$y = \begin{bmatrix}C_1 & C_2\end{bmatrix}x + Du$$

Next, the derivative of x_2 is set to zero and the resulting equation is solved for x_1. The reduced-order model is given by

$$\dot{x}_1 = [A_{11} - A_{12}A_{22}^{-1}A_{21}]x_1 + [B_1 - A_{12}A_{22}^{-1}B_2]u$$

$$y = [C_1 - C_2A_{22}^{-1}A_{21}]x + [D - C_2A_{22}^{-1}B_2]u$$

The discrete-time case is treated similarly by setting

$$x_2[n+1] = x_2[n]$$

Limitations

With the matched DC gain method, A_{22} must be invertible in continuous time, and $I - A_{22}$ must be invertible in discrete time.

See Also

`balreal`	Input/output balancing of state-space models
`minreal`	Minimal state-space realizations

ndims

Purpose

Provide the number of the dimensions of an LTI model or LTI array

Syntax

```
n = ndims(sys)
```

Description

`n = ndims(sys)` is the number of dimensions of an LTI model or an array of LTI models `sys`. A single LTI model has two dimensions (one for outputs, and one for inputs). An LTI array has $2+p$ dimensions, where $p \geq 2$ is the number of array dimensions. For example, a 2-by-3-by-4 array of models has 2+3=5 dimensions.

```
ndims(sys) = length(size(sys))
```

Example

```
sys = rss(3,1,1,3);
ndims(sys)

ans =
     4
```

`ndims` returns 4 for this 3-by-1 array of SISO models.

See Also

`size`	Returns a vector containing the lengths of the dimensions of an LTI array or model

Purpose

Superimpose a Nichols chart on a Nichols plot

Syntax

```
ngrid
```

Description

`ngrid` superimposes Nichols chart grid lines over the Nichols frequency response of a SISO LTI system. The range of the Nichols grid lines is set to encompass the entire Nichols frequency response.

The chart relates the complex number $H/(1+H)$ to H, where H is any complex number. For SISO systems, when H is a point on the open-loop frequency response, then

$$\frac{H}{1+H}$$

is the corresponding value of the closed-loop frequency response assuming unit negative feedback.

If the current axis is empty, `ngrid` generates a new Nichols chart grid in the region –40 dB to 40 dB in magnitude and –360 degrees to 0 degrees in phase. If the current axis does not contain a SISO Nichols frequency response, `ngrid` returns a warning.

Example

Plot the Nichols response with Nichols grid lines for the system.

$$H(s) = \frac{-4s^4 + 48s^3 - 18s^2 + 250s + 600}{s^4 + 30s^3 + 282s^2 + 525s + 60}$$

Type

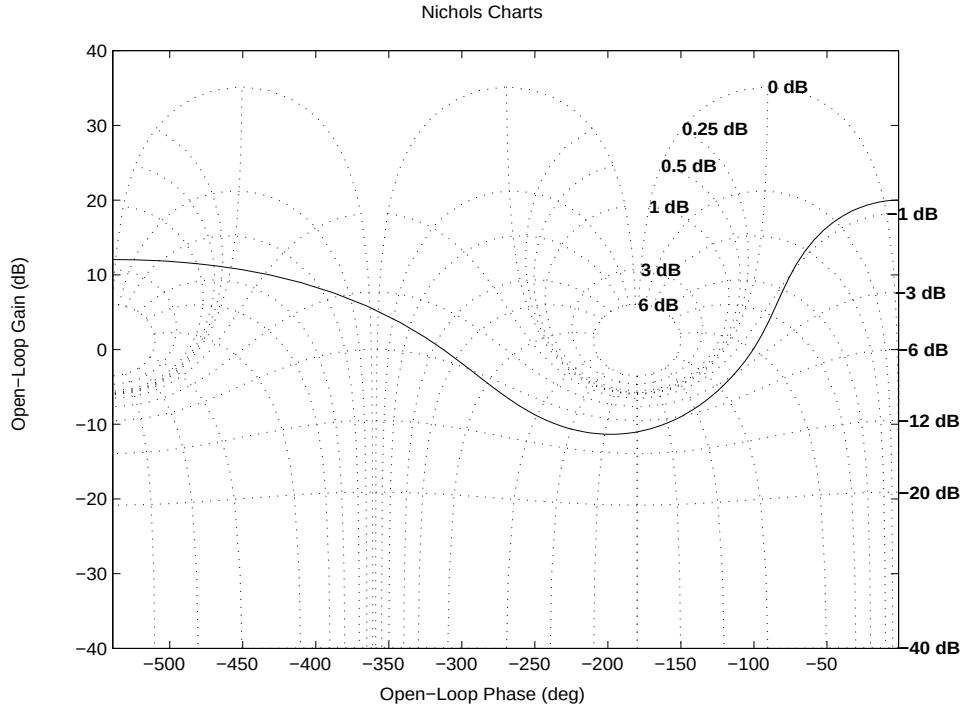
```
H = tf([-4 48 -18 250 600],[1 30 282 525 60])
```

MATLAB returns

```
Transfer function:
- 4 s^4 + 48 s^3 - 18 s^2 + 250 s + 600
---------------------------------------
  s^4 + 30 s^3 + 282 s^2 + 525 s + 60
```

Type

```
nichols(H)
ngrid
```



See Also `nichols` Nichols plots

Purpose

Compute Nichols frequency response of LTI models

Syntax

```
nichols(sys)
nichols(sys,w)

nichols(sys1,sys2,...,sysN)
nichols(sys1,sys2,...,sysN,w)
nichols(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[mag,phase,w] = nichols(sys)
[mag,phase] = nichols(sys,w)
```

Description

`nichols` computes the frequency response of an LTI model and plots it in the Nichols coordinates. Nichols plots are useful to analyze open- and closed-loop properties of SISO systems, but offer little insight into MIMO control loops. Use `ngrid` to superimpose a Nichols chart on an existing SISO Nichols plot.

`nichols(sys)` produces a Nichols plot of the LTI model `sys`. This model can be continuous or discrete, SISO or MIMO. In the MIMO case, `nichols` produces an array of Nichols plots, each plot showing the response of one particular I/O channel. The frequency range and gridding are determined automatically based on the system poles and zeros.

`nichols(sys,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set `w = {wmin,wmax}`. To use particular frequency points, set `w` to the vector of desired frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. Frequencies should be specified in radians/sec.

`nichols(sys1,sys2,...,sysN)` or `nichols(sys1,sys2,...,sysN,w)` superimposes the Nichols plots of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but may otherwise be a mix of continuous- and discrete-time systems. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

```
nichols(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
```

See `bode` for an example.

When invoked with left-hand arguments,

```
[mag,phase,w] = nichols(sys)
[mag,phase] = nichols(sys,w)
```

return the magnitude and phase (in degrees) of the frequency response at the frequencies `w` (in rad/sec). The outputs `mag` and `phase` are 3-D arrays similar to those produced by `bode` (see `bode` on page 11-19). They have dimensions

(number of outputs) × (number of inputs) × (length of `w`)

Remark

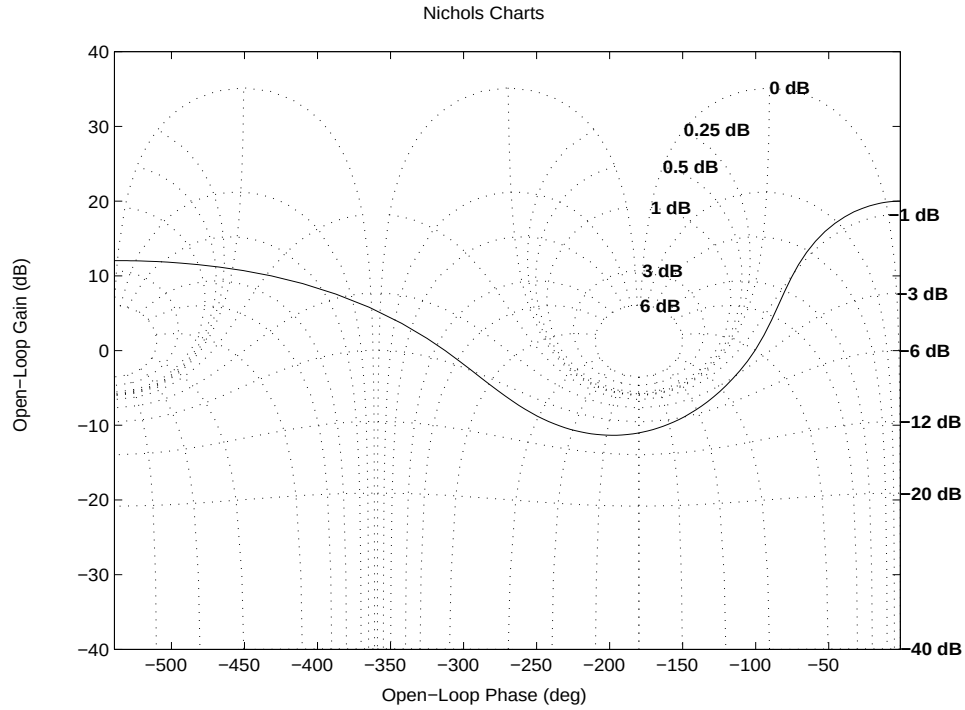
If `sys` is an FRD model, `nichols(sys,w)`, `w` can only include frequencies in `sys.frequency`.

Example

Plot the Nichols response of the system

$$H(s) = \frac{-4s^4 + 48s^3 - 18s^2 + 250s + 600}{s^4 + 30s^3 + 282s^2 + 525s + 60}$$

```
num = [-4 48 -18 250 600];
den = [1 30 282 525 60];
H = tf(num,den)

nichols(H); ngrid
```



Algorithm

See bode.

See Also

bode	Bode plot
evalfr	Response at single complex frequency
freqresp	Frequency response computation
ltiview	LTI system viewer
ngrid	Grid on Nichols plot
nyquist	Nyquist plot
sigma	Singular value plot

Purpose

Compute LTI model norms

Syntax

```
norm(sys)
norm(sys,2)

norm(sys,inf)
norm(sys,inf,tol)
[ninf,fpeak] = norm(sys)
```

Description

`norm` computes the H_2 or L_∞ norm of a continuous- or discrete-time LTI model.

H_2 Norm

The H_2 norm of a stable continuous system with transfer function $H(s)$, is the root-mean-square of its impulse response, or equivalently

$$\|H\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Trace}(H(j\omega)^H H(j\omega))\, d\omega}$$

This norm measures the steady-state covariance (or power) of the output response $y = Hw$ to unit white noise inputs w.

$$\|H\|_2^2 = \lim_{t\to\infty} E\{y(t)^T y(t)\}\ , \qquad E(w(t)w(\tau)^T) = \delta(t-\tau)I$$

Infinity Norm

The infinity norm is the peak gain of the frequency response, that is,

$$\|H(s)\|_\infty = \max_\omega |H(j\omega)| \qquad \text{(SISO case)}$$

$$\|H(s)\|_\infty = \max_\omega \sigma_{\max}(H(j\omega)) \qquad \text{(MIMO case)}$$

where $\sigma_{\max}(.)$ denotes the largest singular value of a matrix.

The discrete-time counterpart is

$$\|H(z)\|_\infty = \max_{\theta \in [0,\pi]} \sigma_{\max}(H(e^{j\theta}))$$

Usage

`norm(sys)` or `norm(sys,2)` both return the H_2 norm of the TF, SS, or ZPK model `sys`. This norm is infinite in the following cases:

- `sys` is unstable.
- `sys` is continuous and has a nonzero feedthrough (that is, nonzero gain at the frequency $\omega = \infty$).

Note that `norm(sys)` produces the same result as

```
sqrt(trace(covar(sys,1)))
```

`norm(sys,inf)` computes the infinity norm of any type of LTI model `sys`. This norm is infinite if `sys` has poles on the imaginary axis in continuous time, or on the unit circle in discrete time.

`norm(sys,inf,tol)` sets the desired relative accuracy on the computed infinity norm (the default value is `tol=1e-2`).

`[ninf,fpeak] = norm(sys,inf)` also returns the frequency `fpeak` where the gain achieves its peak value.

Example

Consider the discrete-time transfer function

$$H(z) = \frac{z^3 - 2.841z^2 + 2.875z - 1.004}{z^3 - 2.417z^2 + 2.003z - 0.5488}$$

with sample time 0.1 second. Compute its H_2 norm by typing

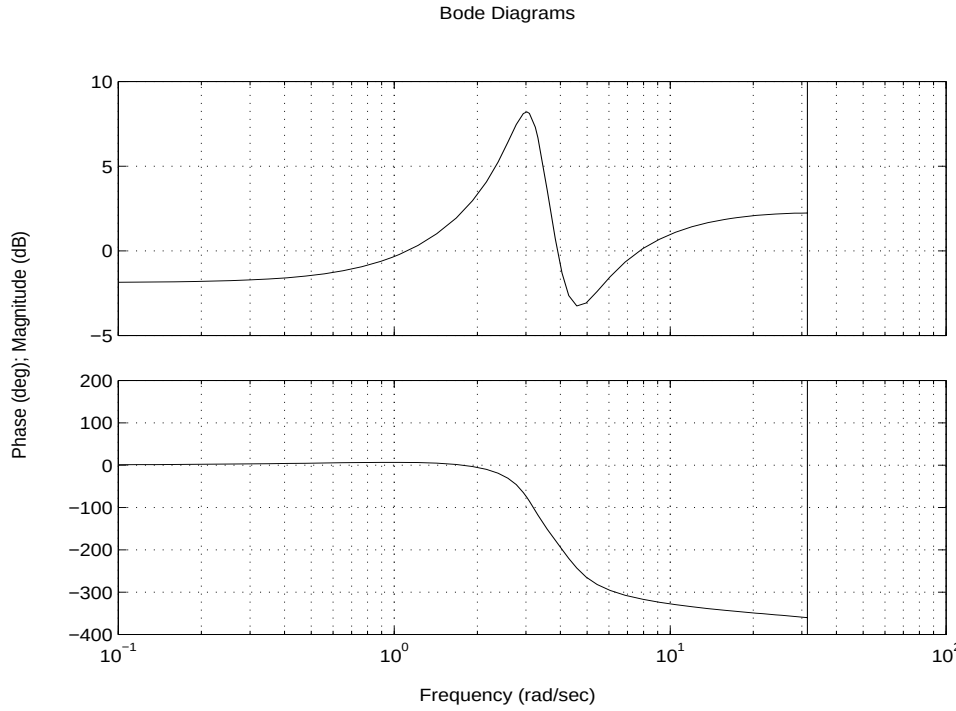
```
H = tf([1 -2.841 2.875 -1.004],[1 -2.417 2.003 -0.5488],0.1)
norm(H)

ans =
    1.2438
```

Compute its infinity norm by typing

```
[ninf,fpeak] = norm(H,inf)

ninf =
    2.5488

fpeak =
    3.0844
```

These values are confirmed by the Bode plot of $H(z)$.

```
bode(H)
```



The gain indeed peaks at approximately 3 rad/sec and its peak value in dB is found by typing

```
20*log10(ninf)
```

MATLAB returns

```
ans =
    8.1268
```

Algorithm

`norm` uses the same algorithm as `covar` for the H_2 norm, and the algorithm of [1] for the infinity norm. `sys` is first converted to state space.

See Also

`bode`	Bode plot
`freqresp`	Frequency response computation
`sigma`	Singular value plot

References

[1] Bruisma, N.A. and M. Steinbuch, "A Fast Algorithm to Compute the H_∞-Norm of a Transfer Function Matrix," *System Control Letters*, 14 (1990), pp. 287–293.

nyquist

Purpose

Compute Nyquist frequency response of LTI models

Syntax

```
nyquist(sys)
nyquist(sys,w)

nyquist(sys1,sys2,...,sysN)
nyquist(sys1,sys2,...,sysN,w)
nyquist(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[re,im,w] = nyquist(sys)
[re,im] = nyquist(sys,w)
```

Description

`nyquist` calculates the Nyquist frequency response of LTI models. When invoked without left-hand arguments, `nyquist` produces a Nyquist plot on the screen. Nyquist plots are used to analyze system properties including gain margin, phase margin, and stability.

`nyquist(sys)` plots the Nyquist response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. In the MIMO case, `nyquist` produces an array of Nyquist plots, each plot showing the response of one particular I/O channel. The frequency points are chosen automatically based on the system poles and zeros.

`nyquist(sys,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set `w = {wmin,wmax}`. To use particular frequency points, set `w` to the vector of desired frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. Frequencies should be specified in rad/sec.

`nyquist(sys1,sys2,...,sysN)` or `nyquist(sys1,sys2,...,sysN,w)` superimposes the Nyquist plots of several LTI models on a single figure. All systems must have the same number of inputs and outputs, but may otherwise be a mix of continuous- and discrete-time systems. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

```
nyquist(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
```

See `bode` for an example.

When invoked with left-hand arguments

```
[re,im,w] = nyquist(sys)
[re,im] = nyquist(sys,w)
```

return the real and imaginary parts of the frequency response at the frequencies `w` (in rad/sec). `re` and `im` are 3-D arrays with the frequency as last dimension (see "Arguments" below for details).

Remark

If `sys` is an FRD model, `nyquist(sys,w)`, `w` can only include frequencies in `sys.frequency`.

Arguments

The output arguments `re` and `im` are 3-D arrays with dimensions

(number of outputs) × (number of inputs) × (length of `w`)

For SISO systems, the scalars `re(1,1,k)` and `im(1,1,k)` are the real and imaginary parts of the response at the frequency ω_k = `w(k)`.

`re(1,1,k)` = $\mathrm{Re}(h(j\omega_k))$

`im(1,1,k)` = $\mathrm{Im}(h(j\omega_k))$

For MIMO systems with transfer function $H(s)$, `re(:,:,k)` and `im(:,:,k)` give the real and imaginary parts of $H(j\omega_k)$ (both arrays with as many rows as outputs and as many columns as inputs). Thus,

`re(i,j,k)` = $\mathrm{Re}(h_{ij}(j\omega_k))$

`im(i,j,k)` = $\mathrm{Im}(h_{ij}(j\omega_k))$

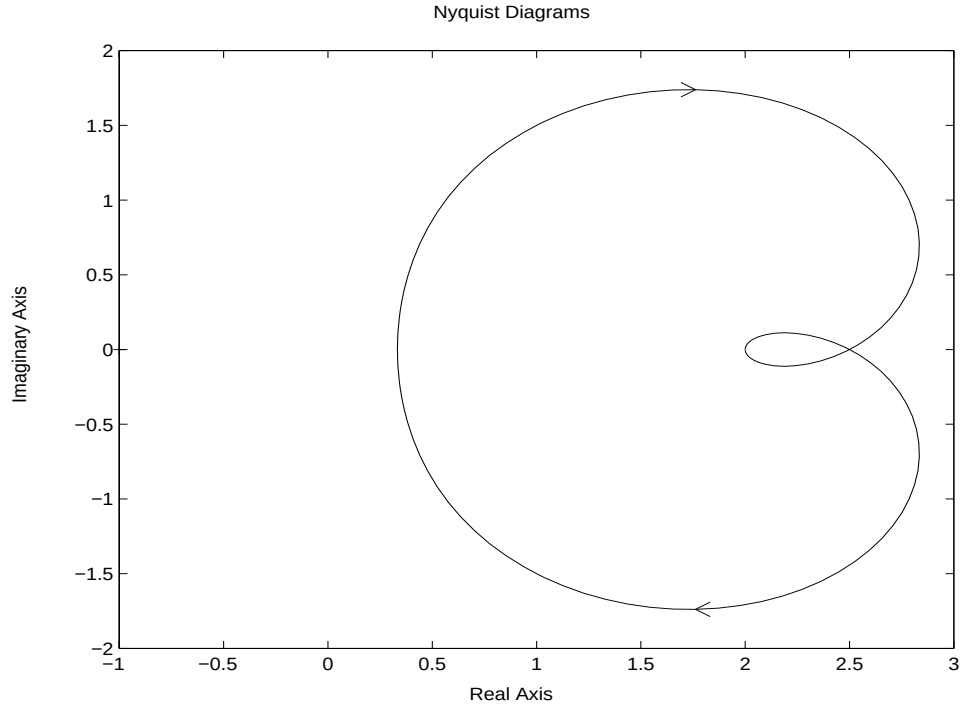
where h_{ij} is the transfer function from input j to output i.

Example

Plot the Nyquist response of the system

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

```
H = tf([2 5 1],[1 2 3])
nyquist(H)
```



See Also

`bode`	Bode plot
`evalfr`	Response at single complex frequency
`freqresp`	Frequency response computation
`ltiview`	LTI system viewer
`nichols`	Nichols plot
`sigma`	Singular value plot

Purpose

Form the observability matrix

Syntax

```
Ob = obsv(A,B)
Ob = obsv(sys)
```

Description

`obsv` computes the observability matrix for state-space systems. For an *n*-by-*n* matrix `A` and a *p*-by-*n* matrix `C`, `obsv(A,C)` returns the observability matrix

$$Ob = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

with *n* columns and *np* rows.

`Ob = obsv(sys)` calculates the observability matrix of the state-space model `sys`. This syntax is equivalent to executing

```
Ob = obsv(sys.A,sys.C)
```

The model is observable if `Ob` has full rank *n*.

Example

Determine if the pair

```
A =
     1     1
     4    -2

C =
     1     0
     0     1
```

is observable. Type

```
Ob = obsv(A,C);

% Number of unobservable states
unob = length(A)-rank(Ob)
```

MATLAB responds with

```
unob =
     0
```

See Also

`obsvf` Compute the observability staircase form

Purpose

Compute the observability staircase form

Syntax

```
[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C)
[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C,tol)
```

Description

If the observability matrix of `(A,C)` has rank $r \le n$, where n is the size of `A`, then there exists a similarity transformation such that

$$\overline{A} = TAT^T, \qquad \overline{B} = TB, \qquad \overline{C} = CT^T$$

where T is unitary and the transformed system has a *staircase* form with the unobservable modes, if any, in the upper left corner.

$$\overline{A} = \begin{bmatrix} A_{no} & A_{12} \\ 0 & A_o \end{bmatrix}, \qquad \overline{B} = \begin{bmatrix} B_{no} \\ B_o \end{bmatrix}, \qquad \overline{C} = \begin{bmatrix} 0 & C_o \end{bmatrix}$$

where (C_o, A_o) is observable, and the eigenvalues of A_{no} are the unobservable modes.

`[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C)` decomposes the state-space system with matrices `A`, `B`, and `C` into the observability staircase form `Abar`, `Bbar`, and `Cbar`, as described above. `T` is the similarity transformation matrix and `k` is a vector of length n, where n is the number of states in `A`. Each entry of `k` represents the number of observable states factored out during each step of the transformation matrix calculation [1]. The number of nonzero elements in `k` indicates how many iterations were necessary to calculate `T`, and `sum(k)` is the number of states in A_o, the observable portion of `Abar`.

`obsvf(A,B,C,tol)` uses the tolerance `tol` when calculating the observable/unobservable subspaces. When the tolerance is not specified, it defaults to `10*n*norm(a,1)*eps`.

Example

Form the observability staircase form of

```
A =
     1     1
     4    -2

B =
     1    -1
     1    -1

C =
     1     0
     0     1
```

by typing

```
[Abar,Bbar,Cbar,T,k] = obsvf(A,B,C)
```

```
Abar =
     1     1
     4    -2
Bbar =
     1     1
     1    -1
Cbar =
     1     0
     0     1
T =
     1     0
     0     1
k =
     2     0
```

Algorithm

`obsvf` is an M-file that implements the Staircase Algorithm of [1] by calling `ctrbf` and using duality.

See Also

`ctrbf`	Compute the controllability staircase form
`obsv`	Calculate the observability matrix

References

[1] Rosenbrock, M.M., *State-Space and Multivariable Theory*, John Wiley, 1970.

Purpose

Generate continuous second-order systems

Syntax

```
[A,B,C,D] = ord2(wn,z)
[num,den] = ord2(wn,z)
```

Description

`[A,B,C,D] = ord2(wn,z)` generates the state-space description `(A,B,C,D)` of the second-order system

$$h(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$$

given the natural frequency `wn` (ω_n) and damping factor `z` (ζ). Use `ss` to turn this description into a state-space object.

`[num,den] = ord2(wn,z)` returns the numerator and denominator of the second-order transfer function. Use `tf` to form the corresponding transfer function object.

Example

To generate an LTI model of the second-order transfer function with damping factor $\zeta = 0.4$ and natural frequency $\omega_n = 2.4$ rad/sec. , type

```
[num,den] = ord2(2.4,0.4)

num =
     1
den =
    1.0000    1.9200    5.7600

sys = tf(num,den)

Transfer function:
          1
-------------------
s^2 + 1.92 s + 5.76
```

See Also

`rmodel`, `rss`	Generate random stable continuous models
`ss`	Create a state-space LTI model
`tf`	Create a transfer function LTI model

pade

Purpose

Compute the Padé approximation of models with time delays

Syntax

```
[num,den] = pade(T,N)
pade(T,N)

sysx = pade(sys,N)
sysx = pade(sys,NI,NO,Nio)
```

Description

pade approximates time delays by rational LTI models. Such approximations are useful to model time delay effects such as transport and computation delays within the context of continuous-time systems. The Laplace transform of an time delay of T seconds is $\exp(-sT)$. This exponential transfer function is approximated by a rational transfer function using the Padé approximation formulas [1].

`[num,den] = pade(T,N)` returns the `N`th-order (diagonal) Padé approximation of the continuous-time I/O delay $\exp(-sT)$ in transfer function form. The row vectors `num` and `den` contain the numerator and denominator coefficients in descending powers of s. Both are `N`th-order polynomials.

When invoked without output arguments,

```
pade(T,N)
```

plots the step and phase responses of the `N`th-order Padé approximation and compares them with the exact responses of the model with I/O delay `T`. Note that the Padé approximation has unit gain at all frequencies.

`sysx = pade(sys,N)` produces a delay-free approximation `sysx` of the continuous delay system `sys`. All delays are replaced by their `N`th-order Padé approximation. See “Time Delays” on page 2-45 for details on LTI models with delays.

`sysx = pade(sys,NI,NO,Nio)` specifies independent approximation orders for each input, output, and I/O delay. These approximation orders are given by the arrays of integers `NI`, `NO`, and `Nio`, such that:

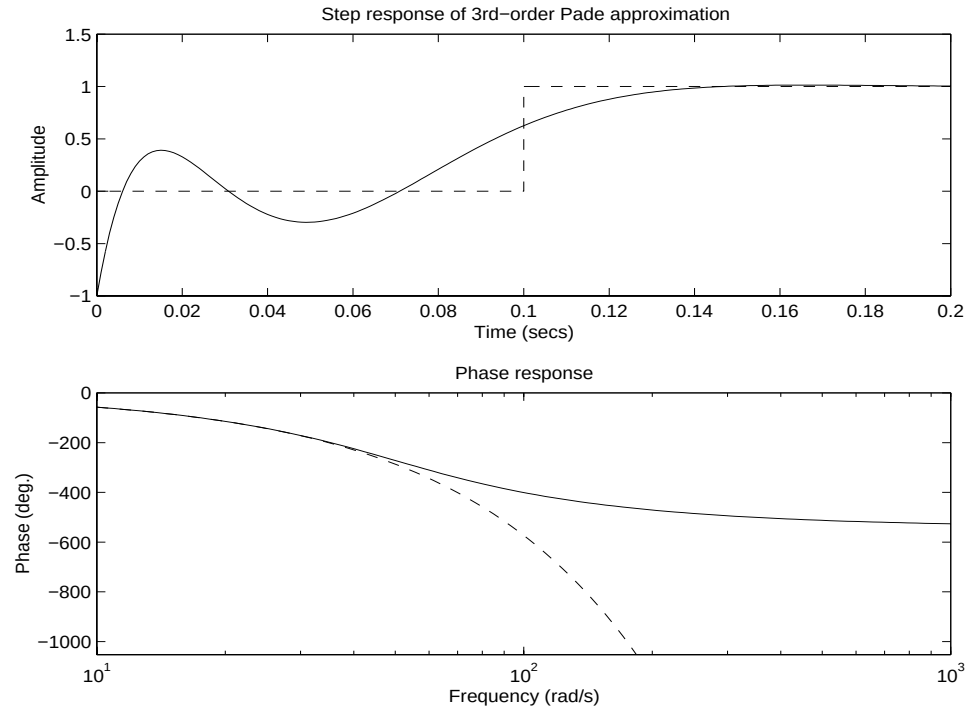
- `NI(j)` is the approximation order for the `j`-th input channel.
- `NO(i)` is the approximation order for the `i`-th output channel.
- `Nio(i,j)` is the approximation order for the I/O delay from input `j` to output `i`.

You can use scalar values to specify uniform approximation orders, and `[]` if there are no input, output, or I/O delays.

Example

Compute a third-order Padé approximation of a 0.1 second I/O delay and compare the time and frequency responses of the true delay and its approximation. To do this, type

```
pade(0.1,3)
```



Limitations

High-order Padé approximations produce transfer functions with clustered poles. Because such pole configurations tend to be very sensitive to perturbations, Padé approximations with order `N>10` should be avoided.

See Also

`c2d`	Discretization of continuous system
`delay2z`	Changes transfer functions of discrete-time LTI models with delays to rational functions or absorbs FRD delays into the frequency response phase information

References

[1] Golub, G. H. and C. F. Van Loan, *Matrix Computations*, Johns Hopkins University Press, Baltimore, 1989, pp. 557–558.

Purpose

Parallel connection of two LTI models

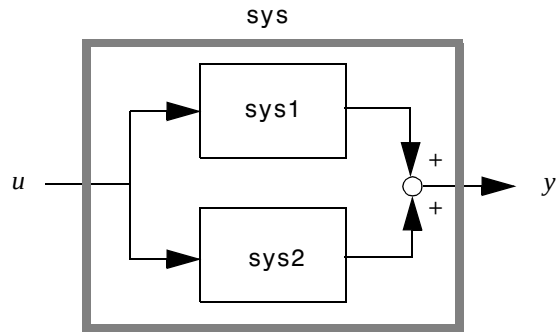
Syntax

```
sys = parallel(sys1,sys2)
sys = parallel(sys1,sys2,inp1,inp2,out1,out2)
```

Description

`parallel` connects two LTI models in parallel. This function accepts any type of LTI model. The two systems must be either both continuous or both discrete with identical sample time. Static gains are neutral and can be specified as regular matrices.

`sys = parallel(sys1,sys2)` forms the basic parallel connection shown below.

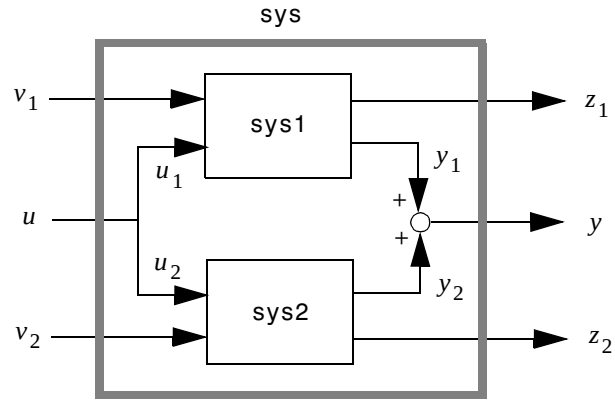


This command is equivalent to the direct addition

```
sys = sys1 + sys2
```

(See “Addition and Subtraction” on page 3-11 for details on LTI system addition.)

`sys = parallel(sys1,sys2,inp1,inp2,out1,out2)` forms the more general parallel connection.



The index vectors `inp1` and `inp2` specify which inputs u_1 of `sys1` and which inputs u_2 of `sys2` are connected. Similarly, the index vectors `out1` and `out2` specify which outputs y_1 of `sys1` and which outputs y_2 of `sys2` are summed. The resulting model `sys` has $[v_1 ; u ; v_2]$ as inputs and $[z_1 ; y ; z_2]$ as outputs.

Example

See “Steady-State Design” on page 9-51 for an example.

See Also

`append`	Append LTI systems
`feedback`	Feedback connection
`series`	Series connection

place

Purpose

Pole placement design

Syntax

```
K = place(A,B,p)
[K,prec,message] = place(A,B,p)
```

Description

Given the single- or multi-input system

$$\dot{x} = Ax + Bu$$

and a vector `p` of desired self-conjugate closed-loop pole locations, `place` computes a gain matrix `K` such that the state feedback $u = -Kx$ places the closed-loop poles at the locations `p`. In other words, the eigenvalues of $A - BK$ match the entries of `p` (up to the ordering).

`K = place(A,B,p)` computes a feedback gain matrix `K` that achieves the desired closed-loop pole locations `p`, assuming all the inputs of the plant are control inputs. The length of `p` must match the row size of `A`. `place` works for multi-input systems and is based on the algorithm from [1]. This algorithm uses the extra degrees of freedom to find a solution that minimizes the sensitivity of the closed-loop poles to perturbations in A or B.

`[K,prec,message] = place(A,B,p)` also returns `prec`, an estimate of how closely the eigenvalues of $A - BK$ match the specified locations `p` (`prec` measures the number of accurate decimal digits in the actual closed-loop poles). If some nonzero closed-loop pole is more than 10% off from the desired location, `message` contains a warning message.

You can also use `place` for estimator gain selection by transposing the `A` matrix and substituting `C'` for `B`.

```
l = place(A',C',p).'
```

Example

Consider a state-space system `(a,b,c,d)` with two inputs, three outputs, and three states. You can compute the feedback gain matrix needed to place the closed-loop poles at `p = [1.1 23 5.0]` by

```
p = [1 1.23 5.0];
K = place(a,b,p)
```

Algorithm

`place` uses the algorithm of [1] which, for multi-input systems, optimizes the choice of eigenvectors for a robust solution. We recommend `place` rather than `acker` even for single-input systems.

In high-order problems, some choices of pole locations result in very large gains. The sensitivity problems attached with large gains suggest caution in the use of pole placement techniques. See [2] for results from numerical testing.

See Also

`acker`	Pole placement using Ackermann's formula
`lqr`	State-feedback LQ regulator design
`rlocus`, `rlocfind`	Root locus design

References

[1] Kautsky, J. and N.K. Nichols, "Robust Pole Assignment in Linear State Feedback," *Int. J. Control*, 41 (1985), pp. 1129–1155.

[2] Laub, A.J. and M. Wette, *Algorithms and Software for Pole Assignment and Observers*, UCRL-15646 Rev. 1, EE Dept., Univ. of Calif., Santa Barbara, CA, Sept. 1984.

pole

Purpose

Compute the poles of an LTI system

Syntax

```
p = pole(sys)
```

Description

`pole` computes the poles `p` of the SISO or MIMO LTI model `sys`.

Algorithm

For state-space models, the poles are the eigenvalues of the A matrix, or the generalized eigenvalues of $A - \lambda E$ in the descriptor case.

For SISO transfer functions or zero-pole-gain models, the poles are simply the denominator roots (see `roots`).

For MIMO transfer functions (or zero-pole-gain models), the poles are computed as the union of the poles for each SISO entry. If some columns or rows have a common denominator, the roots of this denominator are counted only once.

Limitations

Multiple poles are numerically sensitive and cannot be computed to high accuracy. A pole λ with multiplicity m typically gives rise to a cluster of computed poles distributed on a circle with center λ and radius of order

$$\rho \approx \mathrm{eps}^{1/m}$$

See Also

`damp`	Damping and natural frequency of system poles
`esort`, `dsort`	Sort system poles
`pzmap`	Pole-zero map
`zero`	Compute (transmission) zeros

Purpose

Compute the pole-zero map of an LTI model

Syntax

```
pzmap(sys)
[p,z] = pzmap(sys)
```

Description

`pzmap(sys)` plots the pole-zero map of the continuous- or discrete-time LTI model `sys`. For SISO systems, `pzmap` plots the transfer function poles and zeros. For MIMO systems, it plots the system poles and transmission zeros. The poles are plotted as `x`'s and the zeros are plotted as `o`'s.

When invoked without left-hand arguments,

```
[p,z] = pzmap(sys)
```

returns the system poles and (transmission) zeros in the column vectors `p` and `z`. No plot is drawn on the screen.

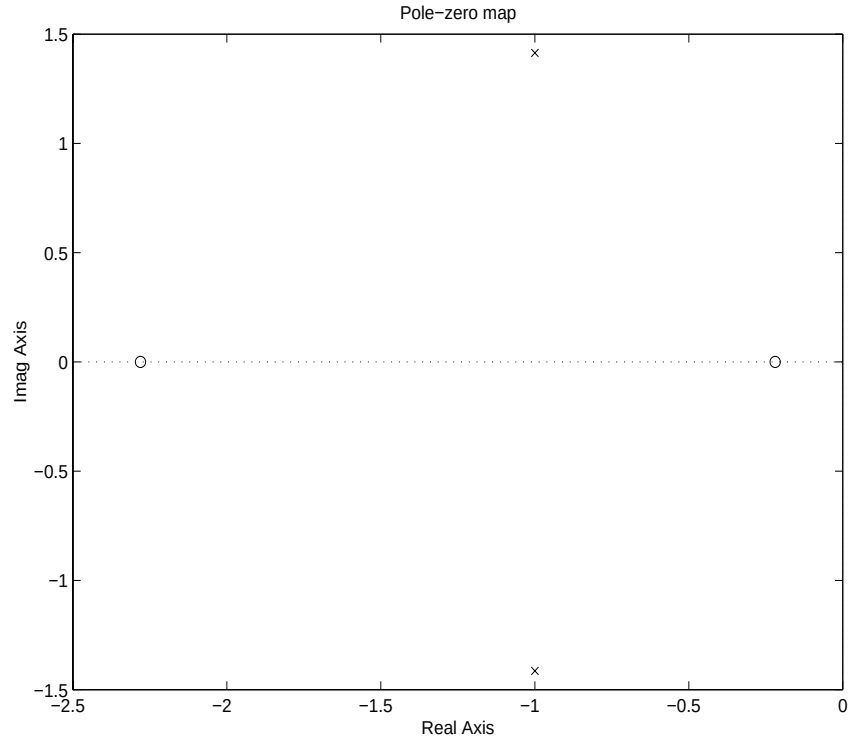
You can use the functions `sgrid` or `zgrid` to plot lines of constant damping ratio and natural frequency in the *s*- or *z*-plane.

Example

Plot the poles and zeros of the continuous-time system.

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

```
H = tf([2 5 1],[1 2 3])
pzmap(H)
```



Algorithm

`pzmap` uses a combination of `pole` and `zero`.

See Also

`damp`	Damping and natural frequency of system poles
`esort`, `dsort`	Sort system poles
`pole`	Compute system poles
`rlocus`	Root locus

`sgrid`, `zgrid`	Plot lines of constant damping and natural frequency
`zero`	Compute system (transmission) zeros

reg

Purpose

Form regulator given state-feedback and estimator gains

Syntax

```
rsys = reg(sys,K,L)
rsys = reg(sys,K,L,sensors,known,controls)
```

Description

`rsys = reg(sys,K,L)` forms a dynamic regulator or compensator `rsys` given a state-space model `sys` of the plant, a state-feedback gain matrix `K`, and an estimator gain matrix `L`. The gains `K` and `L` are typically designed using pole placement or LQG techniques. The function `reg` handles both continuous- and discrete-time cases.

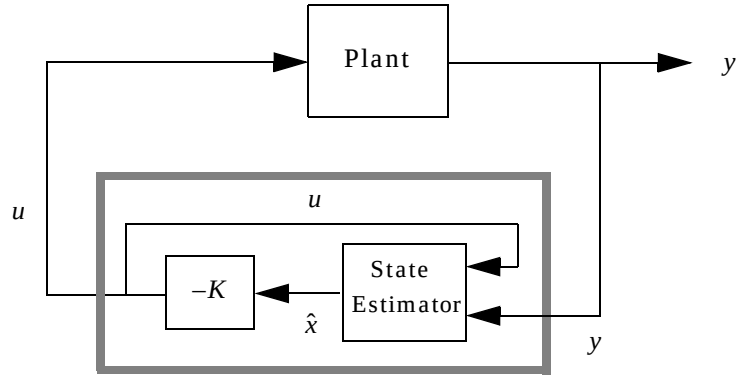
This syntax assumes that all inputs of `sys` are controls, and all outputs are measured. The regulator `rsys` is obtained by connecting the state-feedback law $u = -Kx$ and the state estimator with gain matrix `L` (see `estim`). For a plant with equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

this yields the regulator

$$\dot{\hat{x}} = \left[A - LC - (B - LD)K\right]\hat{x} + Ly$$
$$u = -K\hat{x}$$

This regulator should be connected to the plant using *positive* feedback.

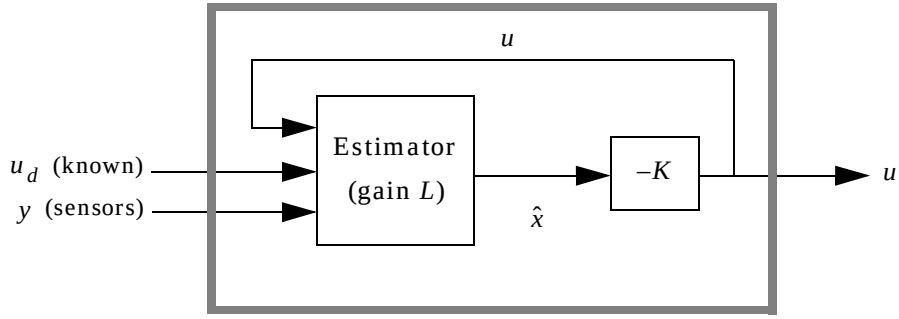


Regulator

`rsys = reg(sys,K,L,sensors,known,controls)` handles more general regulation problems where:

- The plant inputs consist of controls u, known inputs u_d, and stochastic inputs w.
- Only a subset y of the plant outputs is measured.

The index vectors `sensors`, `known`, and `controls` specify y, u_d, and u as subsets of the outputs and inputs of `sys`. The resulting regulator uses $[u_d ; y]$ as inputs to generate the commands u (see figure below).



Regulator `rsys`

Example

Given a continuous-time state-space model

```
sys = ss(A,B,C,D)
```

with seven outputs and four inputs, suppose you have designed:

- A state-feedback controller gain `K` using inputs 1, 2, and 4 of the plant as control inputs
- A state estimator with gain `L` using outputs 4, 7, and 1 of the plant as sensors, and input 3 of the plant as an additional known input

You can then connect the controller and estimator and form the complete regulation system by

```
controls = [1,2,4];
sensors = [4,7,1];
known = [3];
regulator = reg(sys,K,L,sensors,known,controls)
```

See Also

`estim`	Form state estimator given estimator gain
`kalman`	Kalman estimator design
`lqgreg`	Form LQG regulator
`lqr`, `dlqr`	State-feedback LQ regulator
`place`	Pole placement

Purpose

Change the shape of an LTI array

Syntax

```
sys = reshape(sys,s1,s2,...,sk)
sys = reshape(sys,[s1 s2 ... sk])
```

Description

`sys = reshape(sys,s1,s2,...,sk)` (or, equivalently, `sys = reshape(sys,[s1 s2 ... sk])`) reshapes the LTI array `sys` into an `s1`-by-`s2`-by...-`sk` array of LTI models. Equivalently, `sys = reshape(sys,[s1 s2 ... sk])` reshapes the LTI array `sys` into an `s1`-by-`s2`-by...-`sk` array of LTI models. With either syntax, there must be `s1*s2*...*sk` models in `sys` to begin with.

Example

```
sys = rss(4,1,1,2,3);
size(sys)

2x3 array of state-space models
Each model has 1 output, 1 input, and 4 states.

sys1 = reshape(sys,6);
size(sys1)

6x1 array of state-space models
Each model has 1 output, 1 input, and 4 states.
```

See Also

`ndims`	Provide the number of dimensions of an LTI array
`size`	Provide the lengths of each dimension of an LTI array

rlocfind

Purpose

Select feedback gain from root locus plot

Syntax

```
[k,poles] = rlocfind(sys)
[k,poles] = rlocfind(sys,p)
```

Description

`rlocfind` returns the feedback gain associated with a particular set of poles on the root locus. `rlocfind` works with both continuous- and discrete-time SISO systems.

`[k,poles] = rlocfind(sys)` is used for interactive gain selection from the root locus plot of the SISO system `sys` generated by `rlocus`. The function `rlocfind` puts up a crosshair cursor on the root locus plot that you use to select a particular pole location. The root locus gain associated with this point is returned in `k` and the column vector `poles` contains the closed-loop poles for this gain. To use this command, the root locus of the SISO open-loop model `sys` must be present in the current figure window.

`[k,poles] = rlocfind(sys,p)` takes a vector `p` of desired root locations and computes a root locus gain for each of these locations (that is, a gain for which one of the closed-loop roots is near the desired location). The `j`th entry of the vector `k` gives the computed gain for the pole location `p(j)`, and the `j`th column of the matrix `poles` lists the resulting closed-loop poles.

Example

Determine a feedback gain such that the closed-loop poles of the system

$$h(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

have damping ratio $\zeta = 0.707$.

```
h = tf([2 5 1],[1 2 3]);
rlocus(h)        % Plot the root locus

k = rlocfind(h)  % Select pole with ζ=.707 graphically
```

Algorithm

`[k,poles] = rlocfind(sys,p)` calculates the gain `k` from the following formula.

```
k = abs(d(p)/n(p))
```

where p is the complex point you supply as an input argument to `rlocfind` (or where you point to with the mouse), and `n` and `d` are respectively the numerator and denominator polynomials of the transfer function associated with `sys`. The poles associated with the gain `k` are then computed as

```
poles = roots(k*n(s) + d(s))
```

Limitations

`rlocfind` assumes that a root locus is in the current figure window when this function is called without second input argument.

See Also

`rlocus`	Plot root locus
`rltool`	Root Locus Design GUI

References

[1] Ogota, K., *Modern Control Engineering*, Prentice Hall, 1970.

Purpose

Evans root locus

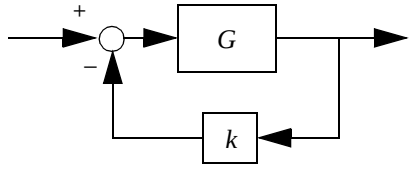
Syntax

```
rlocus(sys)
rlocus(sys,k)

[r,k] = rlocus(sys)
r = rlocus(sys,k)
```

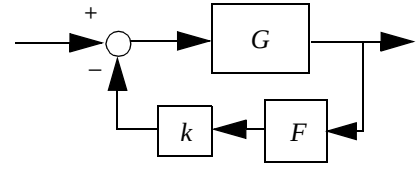
Description

`rlocus` computes the Evans root locus of a SISO open-loop model. The root locus gives the closed-loop pole trajectories as a function of the feedback gain k (assuming negative feedback). Root loci are used to study the effects of varying feedback gains on closed-loop pole locations. In turn, these locations provide indirect information on the time and frequency responses.

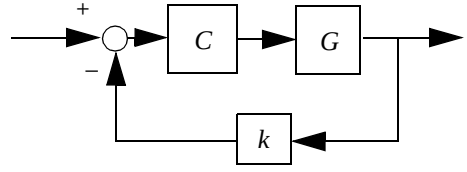
`rlocus(sys)` calculates and plots the root locus of the open-loop SISO model `sys`. This function can be applied to any of the following *negative* feedback loops by setting `sys` appropriately.



`sys = G`



`sys = F * G`



`sys = G * C`

If `sys` has transfer function

$$h(s) = \frac{n(s)}{d(s)}$$

the closed-loop poles are the roots of

$$d(s) + k\ n(s) = 0$$

`rlocus` adaptively selects a set of positive gains k to produce a smooth plot. Alternatively,

```
rlocus(sys,k)
```

uses the user-specified vector `k` of gains to plot the root locus.

When invoked with output arguments,

```
[r,k] = rlocus(sys)
r = rlocus(sys,k)
```

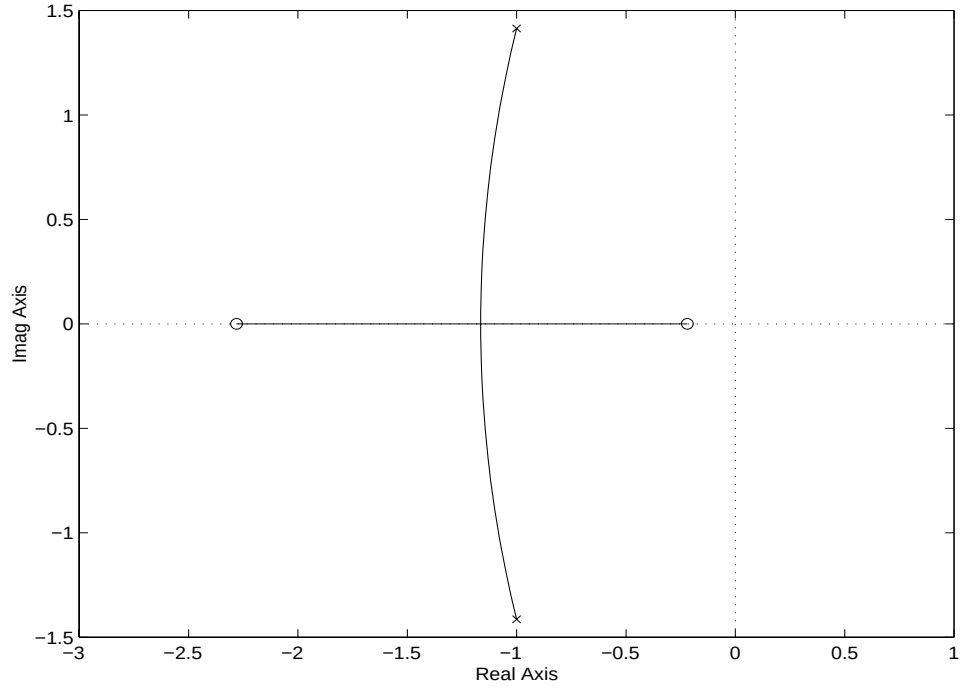
return the vector `k` of selected gains and the complex root locations `r` for these gains. The matrix `r` has `length(k)` columns and its `j`th column lists the closed-loop roots for the gain `k(j)`.

Example

Find and plot the root-locus of the following system.

$$h(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

```
h = tf([2 5 1],[1 2 3]);
rlocus(h)
```



For examples, see “Root Locus Design” on page 9-9 and “Hard-Disk Read/Write Head Controller” on page 9-20.

See Also

`rlocfind`	Select gain from root locus plot
`rltool`	Root Locus Design GUI
`pole`	System poles
`pzmap`	Pole-zero map

Purpose

Initialize the Root Locus Design GUI

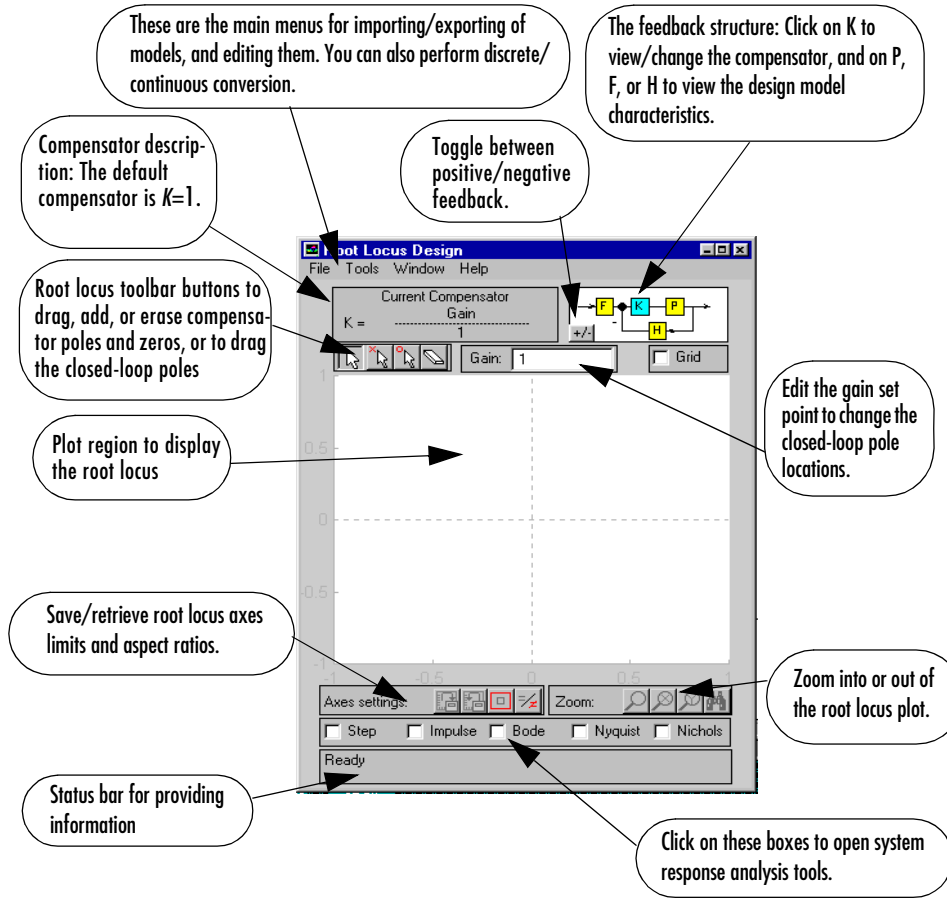
Syntax

```
rltool
rltool(sys)
rltool(sys,comp)
rltool(sys,comp,LocationFlag,FeedbackSign)
```

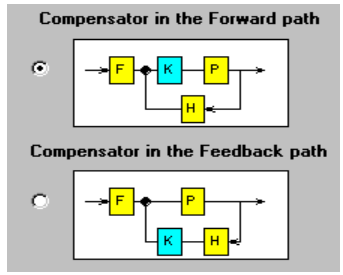
Description

When invoked without input arguments, `rltool` initializes a new Root Locus Design GUI for interactive compensator design. This GUI allows you to design a single-input/single-output (SISO) compensator using root locus techniques.

The Root Locus Design GUI looks like this.



This tool can be applied to SISO LTI systems whose feedback structure is in one of the following two configurations.



In either configuration, **F** is a pre-filter, **P** is the plant model, **H** is the sensor dynamics, and **K** is the compensator to be designed. In terms of the GUI design procedure, once you specify them, **F**, **P**, and **H** are *fixed* in the feedback structure. This triple, along with the feedback structure, is called the *design model*.

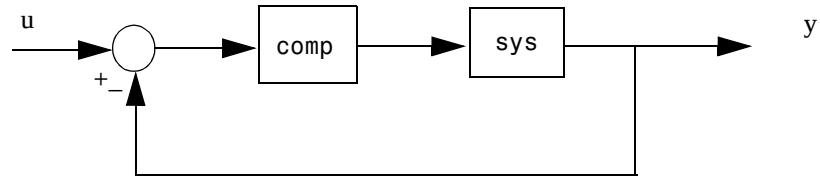
A design model can be constructed for the GUI by selecting the **Import Model** menu item from the **File** menu of the Root Locus Design GUI. Once you select the item, the **Import Design Model** window opens. You can then import SISO LTI models that have been created with `ss`, `tf`, or `zpk` in your workspace or on your disk (or SISO LTI blocks contained in open or saved Simulink models) into **F**, **P**, and **H**. Otherwise, you can specify your design model by defining **F**, **P**, and **H** manually with LTI models created using `ss`, `tf`, or `zpk` in the text boxes provided on the **Import Design Model** window.

`If sys` is any SISO LTI object (created with `ss`, `tf`, or `zpk`) that exists in the MATLAB workspace, `rltool(sys)` initializes a Root Locus Design GUI, by setting the plant model **P** to `sys`.

`rltool(sys,comp)` also initializes a Root Locus Design GUI for the plant model `sys`. In addition, the root locus compensator is initialized to `comp`, where `comp` is any SISO LTI object that exists in the MATLAB workspace.

When either the plant, or both the plant and the compensator are provided as arguments to `rltool`, the root locus of the closed-loop poles and their locations for the current compensator gain are drawn on the Root Locus Design GUI. The closed-loop model is generated by placing the compensator (`comp`) and plant

model (sys) in the forward loop of a negative unity feedback system, as shown in the diagram below.



In this case, **F** and **H** are taken to be 1, while **P** is sys. If you want to include **F** and **H** in the design model after loading `rltool(sys)` or `rltool(sys,comp)`, select the **Import Model** menu item from the **File** menu of the Root Locus Design GUI to load **F** and **H**.

`rltool(sys,comp,LocationFlag,FeedbackSign)` allows you to override the default compensator location and feedback sign. `LocationFlag` can be either 1 or 2:

- `LocationFlag` = 1: Places the compensator in the forward path (this is the default)
- `LocationFlag` = 2: Places the compensator in the feedback path
- `FeedbackSign` can be either 1 or -1:
  - `FeedbackSign` = –1 for negative feedback (this is the default)
  - `FeedbackSign` = 1 for positive feedback

See Also

`rlocus`	Plot root locus
`rlocfind`	Select gain from the root locus plot

Purpose

Generate stable random continuous test models

Syntax

```
sys = rss(n)
sys = rss(n,p)
sys = rss(n,p,m)
sys = rss(n,p,m,s1,...,sn)

[num,den] = rmodel(n)
[A,B,C,D] = rmodel(n)
[A,B,C,D] = rmodel(n,p,m)
```

Description

`rss(n)` produces a stable random `n`-th order model with one input and one output and returns the model in the state-space object `sys`.

`rss(n,p)` produces a random `n`th order stable model with one input and `p` outputs, and `rss(n,m,p)` produces a random `n`-th order stable model with `m` inputs and `p` outputs. The output `sys` is always a state-space model.

`rss(n,p,m,s1,...,sn)` produces an `s1`-by-`...`-by-`sn` array of random n-th order stable state-space models with `m` inputs and `p` outputs.

Use `tf`, `frd`, or `zpk` to convert the state-space object `sys` to transfer function, frequency response, or zero-pole-gain form.

`rmodel(n)` produces a random `n`-th order stable model and returns either the transfer function numerator `num` and denominator `den` or the state-space matrices `A`, `B`, `C`, and `D`, depending on the number of output arguments. The resulting model always has one input and one output.

`[A,B,C,D] = rmodel(n,m,p)` produces a stable random `n`th order state-space model with `m` inputs and `p` outputs.

Example

Obtain a stable random continuous LTI model with three states, two inputs, and two outputs by typing

```
sys = rss(3,2,2)

a =
                    x1          x2          x3
          x1    −0.54175     0.09729     0.08304
          x2     0.09729    −0.89491     0.58707
          x3     0.08304     0.58707    −1.95271

b =
                    u1          u2
          x1    −0.88844    −2.41459
          x2           0    −0.69435
          x3    −0.07162    −1.39139

c =
                    x1          x2          x3
          y1     0.32965     0.14718           0
          y2     0.59854    −0.10144     0.02805


d =
                    u1          u2
          y1    −0.87631    −0.32758
          y2           0           0

Continuous-time system.
```

See Also

`drmodel`, `drss`	Generate stable random discrete test models
`frd`	Convert LTI systems to frequency response form
`tf`	Convert LTI systems to transfer function form
`zpk`	Convert LTI systems to zero-pole-gain form

Purpose

Series connection of two LTI models

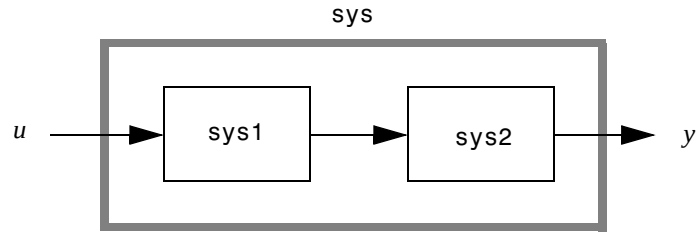
Syntax

```
sys = series(sys1,sys2)
sys = series(sys1,sys2,outputs1,inputs2)
```

Description

`series` connects two LTI models in series. This function accepts any type of LTI model. The two systems must be either both continuous or both discrete with identical sample time. Static gains are neutral and can be specified as regular matrices.

`sys = series(sys1,sys2)` forms the basic series connection shown below.

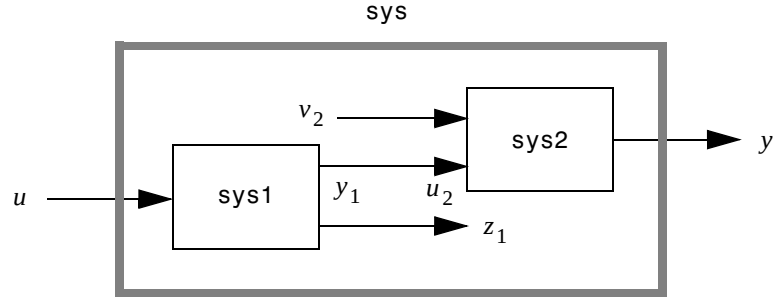


This command is equivalent to the direct multiplication

```
sys = sys2 * sys1
```

See “Multiplication” on page 3-13 for details on multiplication of LTI models.

`sys = series(sys1,sys2,outputs1,inputs2)` forms the more general series connection.



The index vectors `outputs1` and `inputs2` indicate which outputs y_1 of `sys1` and which inputs u_2 of `sys2` should be connected. The resulting model `sys` has u as input and y as output.

Example

Consider a state-space system `sys1` with five inputs and four outputs and another system `sys2` with two inputs and three outputs. Connect the two systems in series by connecting outputs 2 and 4 of `sys1` with inputs 1 and 2 of `sys2`.

```
outputs1 = [2 4];
inputs2 = [1 2];
sys = series(sys1,sys2,outputs2,inputs1)
```

See Also

`append`	Append LTI systems
`feedback`	Feedback connection
`parallel`	Parallel connection

Purpose

Set or modify LTI model properties

Syntax

```
set(sys,'Property',Value)
set(sys,'Property1',Value1,'Property2',Value2,...)

set(sys,'Property')
set(sys)
```

Description

`set` is used to set or modify the properties of an LTI model (see "LTI Properties" on page 2-26 for background on LTI properties). Like its Handle Graphics counterpart, `set` uses property name/property value pairs to update property values.

`set(sys,'Property',Value)` assigns the value `Value` to the property of the LTI model `sys` specified by the string `'Property'`. This string can be the full property name (for example, `'UserData'`) or any unambiguous case-insensitive abbreviation (for example, `'user'`). The specified property must be compatible with the model type. For example, if `sys` is a transfer function, `Variable` is a valid property but `StateName` is not (see "Model-Specific Properties" on page 2-28 for details).

`set(sys,'Property1',Value1,'Property2',Value2,...)` sets multiple property values with a single statement. Each property name/property value pair updates one particular property.

`set(sys,'Property')` displays admissible values for the property specified by `'Property'`. See "Property Values" below for an overview of legitimate LTI property values.

`set(sys)` displays all assignable properties of `sys` and their admissible values.

Example

Consider the SISO state-space model created by

```
sys = ss(1,2,3,4);
```

You can add an input delay of 0.1 second, label the input as `torque`, reset the D matrix to zero, and store its DC gain in the `'Userdata'` property by

```
set(sys,'inputd',0.1,'inputn','torque','d',0,'user',dcgain(sys))
```

Note that set does not require any output argument. Check the result with get by typing

```
get(sys)
```

```
    a: 1
    b: 2
    c: 3
    d: 0
    e: []
    StateName: {''}
    Ts: 0
    InputDelay: 0.1
    OutputDelay: 0
    ioDelayMatrix: 0
    InputName: {'torque'}
    OutputName: {''}
    InputGroup: {0x2 cell}
    OutputGroup: {0x2 cell}
    Notes: {}
    UserData: -2
```

Property Values

The following table lists the admissible values for each LTI property. N_u and N_y denotes the number of inputs and outputs of the underlying LTI model. For K-dimensional LTI arrays, let S_1, S_2, ..., S_K denote the array dimensions.

Table 11-15: LTI Properties

Property Name	Admissible Property Values
`Ts`	• 0 (zero) for continuous-time systems • Sample time in seconds for discrete-time systems • –1 or [ ] for discrete systems with unspecified sample time **Note:** Resetting the sample time property does not alter the model data. Use `c2d`, `d2c`, or `d2d` for discrete/continuous and discrete/discrete conversions.
`ioDelayMatrix`	Input/Output delays specified with • Nonnegative real numbers for continuous-time models (seconds) • Integers for discrete-time models (number of sample periods) • Scalar when all I/O pairs have the same delay • N_y-by-N_u matrix to specify independent delay times for each I/O pair • Array of size N_y-by-N_u-by-S_1-by-...-by-S_n to specify different I/O delays for each model in an LTI array.
`InputDelay`	Input delays specified with • Nonnegative real numbers for continuous-time models (seconds) • Integers for discrete-time models (number of sample periods) • Scalar when $N_u = 1$ or system has uniform input delay • Vector of length N_u to specify independent delay times for each input channel • Array of size N_y-by-N_u-by-S_1-by-...-by-S_n to specify different input delays for each model in an LTI array.

Table 11-15: LTI Properties (Continued)

Property Name	Admissible Property Values
`OutputDelay`	Output delays specified with • Nonnegative real numbers for continuous-time models (seconds) • Integers for discrete-time models (number of sample periods) • Scalar when $N_y = 1$ or system has uniform output delay • Vector of length N_y to specify independent delay times for each output channel • Array of size N_y-by-N_u-by-S_1-by-. . .-by-S_n to specify different output delays for each model in an LTI array.
`Notes`	String, array of strings, or cell array of strings
`UserData`	Arbitrary MATLAB variable
`InputName`	• String for single-input systems, for example, `'thrust'` • Cell vector of strings for multi-input systems (with as many cells as inputs), for example, `{'u';'w'}` for a two-input system • Padded array of strings with as many rows as inputs, for example, `['rudder ' ; 'aileron']`
`OutputName`	Same as `InputName` (with "input" replaced by "output")
`InputGroup`	Cell array. See "Input Groups and Output Groups" on page 2-37.
`OutputGroup`	Same as `InputGroup`

Table 11-16: State-Space Model Properties

Property Name	Admissible Property Values
`StateName`	Same as `InputName` (with `Input` replaced by `State`)
`a`, `b`, `c`, `d`, `e`	Real-valued state-space matrices (multidimensional arrays, in the case of LTI arrays) with compatible dimensions for the number of states, inputs, and outputs. See "The Size of LTI Array Data for SS Models" on page 4-18.

Table 11-17: TF Model Properties

Property Name	Admissible Property Values
`num`, `den`	• Real-valued row vectors for the coefficients of the numerator or denominator polynomials in the SISO case. List the coefficients in *descending* powers of the variable s or z by default, and in *ascending* powers of $q = z^{-1}$ when the `Variable` property is set to `'q'` or `'z^-1'` (see note below). • N_y-by-N_u cell arrays of real-valued row vectors in the MIMO case, for example, `{[1 2];[1 0 3]}` for a two-output/one-input transfer function • N_y-by-N_u-by-S_1-by-…-by-S_K-dimensional real-valued cell arrays for MIMO LTI arrays
`Variable`	• String `'s'` (default) or `'p'` for continuous-time systems • String `'z'` (default), `'q'`, or `'z^-1'` for discrete-time systems

Table 11-18: ZPK Model Properties

Property Name	Admissible Property Values
`z`, `p`	• Vectors of zeros and poles (either real-valued or complex conjugate pairs of them) in SISO case • N_y-by-N_u cell arrays of vectors (entries are real-valued or in complex conjugate pairs) in MIMO case, for example, `z = {[],[-1 0]}` for a model with two inputs and one output • N_y-by-N_u-by-S_1-by-…-by-S_K-dimensional cell arrays for MIMO LTI arrays
`Variable`	• String `'s'` (default) or `'p'` for continuous-time systems • String `'z'` (default), `'q'`, or `'z^-1'` for discrete-time systems

Table 11-19: FRD Model Properties

Property Name	Admissible Property Values
`Frequency`	Real-valued vector of length N_f-by-1, where N_f is the number of frequencies
`Response`	• N_y-by-N_u-by-N_f-dimensional array of complex data for single LTI models • N_y-by-N_u-by-N_f-by-S_1-by-…-by-S_K-dimensional array for LTI arrays
`Units`	String `'rad/s'` (default), or `'Hz'`

Remark

For discrete-time transfer functions, the convention used to represent the numerator and denominator depends on the choice of variable (see the `tf` entry for details). Like `tf`, the syntax for `set` changes to remain consistent with the choice of variable. For example, if the `Variable` property is set to `'z'` (the default),

```
set(h,'num',[1 2],'den',[1 3 4])
```

produces the transfer function

$$h(z) = \frac{z+2}{z^2+3z+4}$$

However, if you change the `Variable` to `'z^-1'` (or `'q'`) by

```
set(h,'Variable','z^-1'),
```

the same command

```
set(h,'num',[1 2],'den',[1 3 4])
```

now interprets the row vectors `[1 2]` and `[1 3 4]` as the polynomials $1+2z^{-1}$ and $1+3z^{-1}+4z^{-2}$ and produces:

$$\bar{h}(z^{-1}) = \frac{1 + 2z^{-1}}{1 + 3z^{-1} + 4z^{-2}} = zh(z)$$

Because the resulting transfer functions are different, make sure to use the convention consistent with your choice of variable.

See Also

`get`	Access/query LTI model properties
`frd`	Specify a frequency response data model
`ss`	Specify a state-space model
`tf`	Specify a transfer function
`zpk`	Specify a zero-pole-gain model

sgrid

Purpose

Generate an *s*-plane grid of constant damping factors and natural frequencies

Syntax

```
sgrid
sgrid(z,wn)
```

Description

`sgrid` generates a grid of constant damping factors from zero to one in steps of 0.1 and natural frequencies from zero to 10 rad/sec in steps of one rad/sec, and plots the grid over the current axis. If the current axis contains a continuous *s*-plane root locus diagram or pole-zero map, `sgrid` draws the grid over the plot.

`sgrid(z,wn)` plots a grid of constant damping factor and natural frequency lines for the damping factors and natural frequencies in the vectors `z` and `wn`, respectively. If the current axis contains a continuous *s*-plane root locus diagram or pole-zero map, `sgrid(z,wn)` draws the grid over the plot.

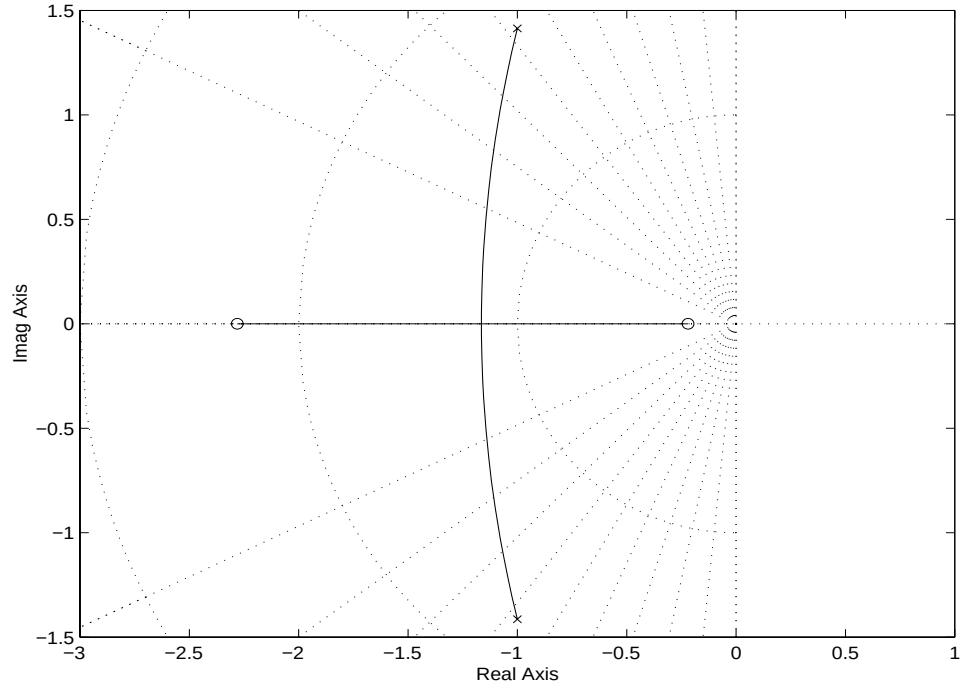
Example

Plot *s*-plane grid lines on the root locus for the following system.

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

You can do this by typing

```
H = tf([2 5 1],[1 2 3])

Transfer function:
2 s^2 + 5 s + 1
---------------
 s^2 + 2 s + 3

rlocus(H)
sgrid
```



Limitations

`sgrid` plots the grid over the current axis regardless of whether the axis contains a root locus diagram or pole-zero map. Therefore, if the current axes contains, for example, a step response, you may superimpose a meaningless *s*-plane grid over the plot. In addition, `sgrid` does not rescale the axis limits of the current axis. Therefore, the *s*-plane grid may not appear in the plot if the left half of the *s*-plane is not encompassed by the axis limits.

See Also

`pzmap`	Plot pole-zero map
`rlocus`	Plot root locus
`zgrid`	Generate *z*-plane grid lines

sigma

Purpose

Singular values of the frequency response of LTI models

Syntax

```
sigma(sys)
sigma(sys,w)
sigma(sys,w,type)

sigma(sys1,sys2,...,sysN)
sigma(sys1,sys2,...,sysN,w)
sigma(sys1,sys2,...,sysN,w,type)
sigma(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[sv,w] = sigma(sys)
sv = sigma(sys,w)
```

Description

`sigma` calculates the singular values of the frequency response of an LTI model. For an FRD model, `sys`, `sigma` computes the singular values of `sys.Response` at the frequencies, `sys.frequency`. For continuous-time TF, SS, or ZPK models with transfer function $H(s)$, `sigma` computes the singular values of $H(j\omega)$ as a function of the frequency ω. For discrete-time TF, SS, or ZPK models with transfer function $H(z)$ and sample time T_s, `sigma` computes the singular values of

$$H(e^{j\omega T_s})$$

for frequencies ω between 0 and the Nyquist frequency $\omega_N = \pi/T_s$.

The singular values of the frequency response extend the Bode magnitude response for MIMO systems and are useful in robustness analysis. The singular value response of a SISO system is identical to its Bode magnitude response. When invoked without output arguments, `sigma` produces a singular value plot on the screen.

`sigma(sys)` plots the singular values of the frequency response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. The frequency points are chosen automatically based on the system poles and zeros, or from `sys.frequency` if `sys` is an FRD.

`sigma(sys,w)` explicitly specifies the frequency range or frequency points to be used for the plot. To focus on a particular frequency interval `[wmin,wmax]`, set

`w = {wmin,wmax}`. To use particular frequency points, set `w` to the corresponding vector of frequencies. Use `logspace` to generate logarithmically spaced frequency vectors. The frequencies must be specified in rad/sec.

`sigma(sys,[],type)` or `sigma(sys,w,type)` plots the following modified singular value responses:

`type = 1`	Singular values of the frequency response H^{-1}, where H is the frequency response of `sys`.
`type = 2`	Singular values of the frequency response $I + H$.
`type = 3`	Singular values of the frequency response $I + H^{-1}$.

These options are available only for square systems, that is, with the same number of inputs and outputs.

To superimpose the singular value plots of several LTI models on a single figure, use

```
sigma(sys1,sys2,...,sysN)
sigma(sys1,sys2,...,sysN,[],type)  % modified SV plot
sigma(sys1,sys2,...,sysN,w)        % specify frequency range/grid
```

The models `sys1,sys2,...,sysN` need not have the same number of inputs and outputs. Each model can be either continuous- or discrete-time. You can also specify a distinctive color, linestyle, and/or marker for each system plot with the syntax

```
sigma(sys1,'PlotStyle1',...,sysN,'PlotStyleN')
```

See `bode` for an example.

When invoked with output arguments,

```
[sv,w] = sigma(sys)
sv = sigma(sys,w)
```

return the singular values `sv` of the frequency response at the frequencies `w`. For a system with `Nu` input and `Ny` outputs, the array `sv` has `min(Nu,Ny)` rows and as many columns as frequency points (length of `w`). The singular values at the frequency `w(k)` are given by `sv(:,k)`.

Remark

If `sys` is an FRD model, `sigma(sys,w)`, `w` can only include frequencies in `sys.frequency`.

Example

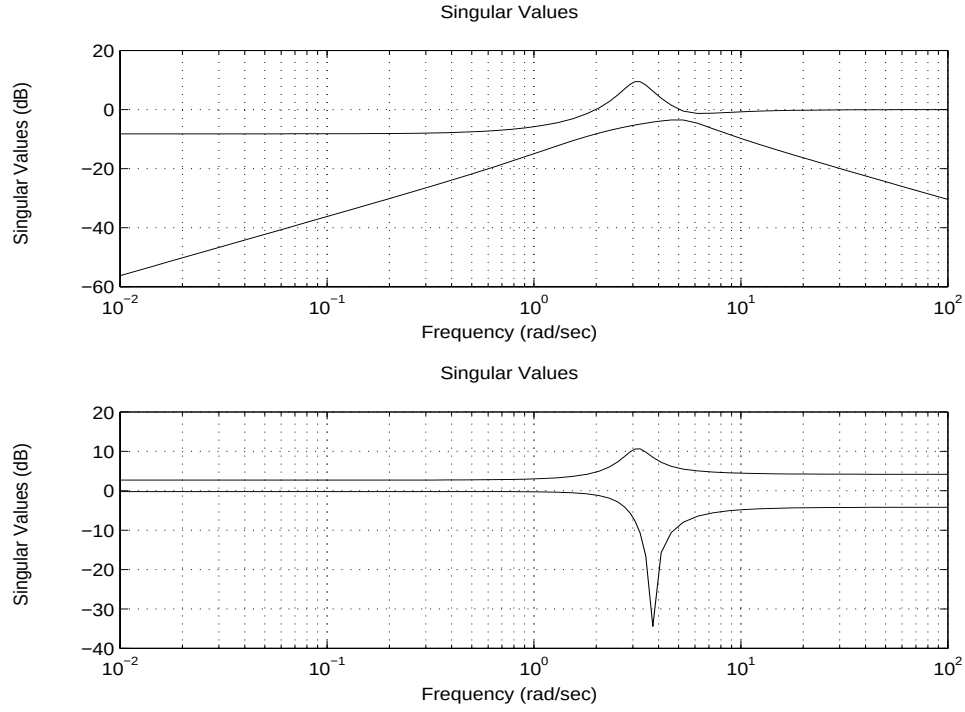
Plot the singular value responses of

$$H(s) = \begin{bmatrix} 0 & \dfrac{3s}{s^2+s+10} \\ \dfrac{s+1}{s+5} & \dfrac{2}{s+6} \end{bmatrix}$$

and $I + H(s)$.

You can do this by typing

```
H = [0 tf([3 0],[1 1 10]) ; tf([1 1],[1 5]) tf(2,[1 6])]

subplot(211)
sigma(H)
subplot(212)
sigma(H,[],2)
```



Algorithm

`sigma` uses the `svd` function in MATLAB to compute the singular values of a complex matrix.

See Also

`bode`	Bode plot
`evalfr`	Response at single complex frequency
`freqresp`	Frequency response computation
`ltiview`	LTI system viewer

`nichols`	Nichols plot
`nyquist`	Nyquist plot

Purpose

Provide the output/input/array dimensions of LTI models, the model order of TF, SS, and ZPK models, and the number of frequencies of FRD models

Syntax

```
size(sys)
d = size(sys)
Ny = size(sys,1)
Nu = size(sys,2)
Sk = size(sys,2+k)
Ns = size(sys,'order')
Nf = size(sys,'frequency')
```

Description

When invoked without output arguments, `size(sys)` returns a vector of the number of outputs and inputs for a single LTI model. The lengths of the array dimensions are also included in the response to `size` when `sys` is an LTI array. `size` is the overloaded version of the MATLAB function `size` for LTI objects.

`d = size(sys)` returns:

- The row vector `d = [Ny Nu]` for a single LTI model `sys` with `Ny` outputs and `Nu` inputs
- The row vector `d = [Ny Nu S1 S2 ... Sp]` for an `S1`-by-`S2`-by-...-by-`Sp` array of LTI models with `Ny` outputs and `Nu` inputs

`Ny = size(sys,1)` returns the number of outputs of `sys`.

`Nu = size(sys,2)` returns the number of inputs of `sys`.

`Sk = size(sys,2+k)` returns the length of the `k`-th array dimension when `sys` is an LTI array.

`Ns = size(sys,'order')` returns the model order of a TF, SS, or ZPK model. This is the same as the number of states for state-space models. When `sys` is an LTI array of SS models with differing numbers of states in each model:

- `Ns` is the multidimensional array of the orders of each of the models in the LTI array.
- The dimensions of `Ns` are given by the array dimensions of the LTI array `sys`.

`Nf = size(sys,'frequency')` returns the number of frequencies when `sys` is an FRD. This is the same as the length of `sys.frequency`.

Example

Consider the random LTI array of state-space models

```
sys = rss(5,3,2,3);
```

Its dimensions are obtained by typing

```
size(sys)
```

```
3x1 array of state-space models
Each model has 3 outputs, 2 inputs, and 5 states.
```

See Also

`isempty`	Test if LTI model is empty
`issiso`	Test if LTI model is SISO
`ndims`	Number of dimensions of an LTI array

Purpose

Perform model reduction based on structure

Syntax

```
msys = sminreal(sys)
```

Description

`msys = sminreal(sys)` eliminates the states of the state-space model `sys` that don't affect the input/output response. All of the states of the resulting state-space model `msys` are also states of `sys` and the input/output response of `msys` is equivalent to that of `sys`.

`sminreal` eliminates only structurally non minimal states, i.e., states that can be discarded by looking only at hard zero entries in the *A*, *B*, and *C* matrices. Such structurally nonminimal states arise, for example, when linearizing a Simulink model that includes some unconnected state-space or transfer function blocks.

Remark

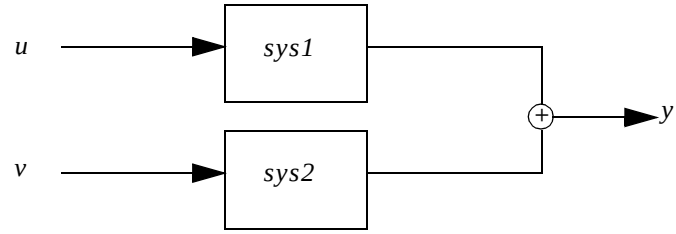
The model resulting from `sminreal(sys)` is not necessarily minimal, and may have a higher order than one resulting from `minreal(sys)`. However, `sminreal(sys)` retains the state structure of `sys`, while, in general, `minreal(sys)` does not.

Example

Suppose you concatenate two SS models, `sys1` and `sys2`.

```
sys = [sys1,sys2];
```

This operation is depicted in the diagram below.



If you extract the subsystem `sys1` from `sys`, with

```
sys(1,1)
```

all of the states of sys, including those of sys2 are retained. To eliminate the unobservable states from sys2, while retaining the states of sys1, type

```
sminreal(sys(1,1))
```

See Also

minreal	Model reduction by removing unobservable/ uncontrollable states or cancelling pole/zero pairs

Purpose

Specify state-space models or convert an LTI model to state space

Syntax

```
sys = ss(a,b,c,d)
sys = ss(a,b,c,d,Ts)
sys = ss(d)
sys = ss(a,b,c,d,ltisys)

sys = ss(a,b,c,d,'Property1',Value1,...,'PropertyN',ValueN)
sys = ss(a,b,c,d,Ts,'Property1',Value1,...,'PropertyN',ValueN)

sys_ss = ss(sys)
sys_ss = ss(sys,'minimal')
```

Description

`ss` is used to create real-valued state-space models (SS objects) or to convert transfer function or zero-pole-gain models to state space.

Creation of State-Space Models

`sys = ss(a,b,c,d)` creates the continuous-time state-space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

For a model with `Nx` states, `Ny` outputs, and `Nu` inputs:

- `a` is an `Nx`-by-`Nx` real-valued matrix.
- `b` is an `Nx`-by-`Nu` real-valued matrix.
- `c` is an `Ny`-by-`Nx` real-valued matrix.
- `d` is an `Ny`-by-`Nu` real-valued matrix.

The output `sys` is an SS model that stores the model data (see "State-Space Models" on page 2-14). If $D = 0$, you can simply set `d` to the scalar `0` (zero), regardless of the dimension.

`sys = ss(a,b,c,d,Ts)` creates the discrete-time model

$$x[n+1] = Ax[n] + Bu[n]$$
$$y[n] = Cx[n] + Du[n]$$

with sample time Ts (in seconds). Set Ts = –1 or Ts = [] to leave the sample time unspecified.

sys = ss(d) specifies a static gain matrix D and is equivalent to

```
sys = ss([],[],[],d)
```

sys = ss(a,b,c,d,ltisys) creates a state-space model with generic LTI properties inherited from the LTI model ltisys (including the sample time). See "Generic Properties" on page 2-26 for an overview of generic LTI properties.

See "Building LTI Arrays" on page 4-12 for information on how to build arrays of state-space models.

Any of the previous syntaxes can be followed by property name/property value pairs.

```
'PropertyName',PropertyValue
```

Each pair specifies a particular LTI property of the model, for example, the input names or some notes on the model history. See the set entry and the example below for details. Note that

```
sys = ss(a,b,c,d,'Property1',Value1,...,'PropertyN',ValueN)
```

is equivalent to the sequence of commands.

```
sys = ss(a,b,c,d)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

Conversion to State Space

sys_ss = ss(sys) converts an arbitrary TF or ZPK model sys to state space. The output sys_ss is an equivalent state-space model (SS object). This operation is known as *state-space realization*.

sys_ss = ss(sys,'minimal') produces a state-space realization with no uncontrollable or unobservable states. This is equivalent to sys_ss = minreal(ss(sys)).

Examples

Example 1

The command

```
sys = ss(A,B,C,D,0.05,'statename',{'position' 'velocity'},...
                      'inputname','force',...
                      'notes','Created 10/15/96')
```

creates a discrete-time model with matrices A, B, C, D and sample time 0.05 second. This model has two states labeled `position` and `velocity`, and one input labeled `force` (the dimensions of A, B, C, D should be consistent with these numbers of states and inputs). Finally, a note is attached with the date of creation of the model.

Example 2

Compute a state-space realization of the transfer function

$$H(s) = \begin{bmatrix} \dfrac{s+1}{s^3+3s^2+3s+2} \\ \dfrac{s^2+3}{s^2+s+1} \end{bmatrix}$$

by typing

```
H = [tf([1 1],[1 3 3 2]) ; tf([1 0 3],[1 1 1])];
sys = ss(H);
size(sys)

State-space model with 2 outputs, 1 input, and 5 states.
```

Note that the number of states is equal to the cumulative order of the SISO entries of $H(s)$.

To obtain a minimal realization of $H(s)$, type

```
sys = ss(H,'min');
size(sys)

State-space model with 2 outputs, 1 input, and 3 states.
```

The resulting state-space model order has order three, the minimum number of states needed to represent $H(s)$. This can be seen directly by factoring $H(s)$ as the product of a first order system with a second order one.

$$H(s) = \begin{bmatrix} \frac{1}{s+2} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{s+1}{s^2+s+1} \\ \frac{s^2+3}{s^2+s+1} \end{bmatrix}$$

See Also

`dss`	Specify descriptor state-space models.
`frd`	Specify FRD models or convert to an FRD.
`get`	Get properties of LTI models.
`set`	Set properties of LTI models.
`ssdata`	Retrieve the A, B, C, D matrices of state-space model.
`tf`	Specify transfer functions or convert to TF.
`zpk`	Specify zero-pole-gain models or convert to ZPK.

Purpose

State coordinate transformation for state-space models

Syntax

```
sysT = ss2ss(sys,T)
```

Description

Given a state-space model sys with equations

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

(or their discrete-time counterpart), ss2ss performs the similarity transformation $\bar{x} = Tx$ on the state vector x and produces the equivalent state-space model sysT with equations.

$$\dot{\bar{x}} = TAT^{-1}\bar{x} + TBu$$
$$y = CT^{-1}\bar{x} + Du$$

sysT = ss2ss(sys,T) returns the transformed state-space model sysT given sys and the state coordinate transformation T. The model sys must be in state-space form and the matrix T must be invertible. ss2ss is applicable to both continuous- and discrete-time models.

Example

Perform a similarity transform to improve the conditioning of the A matrix.

```
T = balance(sys.a)
sysb = ss2ss(sys,inv(T))
```

See ssbal for a more direct approach.

See Also

balreal	Gramian-based I/O balancing
canon	Canonical state-space realizations
ssbal	Balancing of state-space models using diagonal similarity transformations

ssbal

Purpose

Balance state-space models using a diagonal similarity transformation

Syntax

```
[sysb,T] = ssbal(sys)
[sysb,T] = ssbal(sys,condT)
```

Description

Given a state-space model `sys` with matrices (A, B, C, D),

```
[sysb,T] = ssbal(sys)
```

computes a diagonal similarity transformation T and a scalar α such that

$$\begin{bmatrix} TAT^{-1} & TB/\alpha \\ \alpha CT^{-1} & 0 \end{bmatrix}$$

has approximately equal row and column norms. `ssbal` returns the balanced model `sysb` with matrices

$$(TAT^{-1}, TB/\alpha, \alpha CT^{-1}, D)$$

and the state transformation $\bar{x} = Tx$ where $\bar{x}$ is the new state.

`[sysb,T] = ssbal(sys,condT)` specifies an upper bound `condT` on the condition number of T. Since balancing with ill-conditioned T can inadvertently magnify rounding errors, `condT` gives control over the worst-case roundoff amplification factor. The default value is `condT=1/eps`.

Example

Consider the continuous-time state-space model with the following data.

$$A = \begin{bmatrix} 1 & 10^4 & 10^2 \\ 0 & 10^2 & 10^5 \\ 10 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0.1 & 10 & 100 \end{bmatrix}$$

```
a = [1 1e4 1e2;0 1e2 1e5;10 1 0];
b = [1;1;1];
c = [0.1 10 1e2];
sys = ss(a,b,c,0)
```

Balance this model with ssbal by typing

```
ssbal(sys)
```

```
a =
                    x1          x2          x3
           x1        1        2500     0.39063
           x2        0         100      1562.5
           x3     2560          64           0


b =
                    u1
           x1      0.125
           x2        0.5
           x3         32


c =
                    x1          x2          x3
           y1       0.8          20       3.125


d =
                    u1
           y1         0

Continuous-time system.
```

Direct inspection shows that the range of numerical values has been compressed by a factor 100 and that the B and C matrices now have nearly equal norms.

Algorithm

ssbal uses the MATLAB function balance to compute T and α.

See Also

balreal	Gramian-based I/O balancing
ss2ss	State coordinate transformation

ssdata

Purpose

Quick access to state-space model data

Syntax

```
[a,b,c,d] = ssdata(sys)
[a,b,c,d,Ts] = ssdata(sys)
```

Description

`[a,b,c,d] = ssdata(sys)` extracts the matrix (or multidimensional array) data (A, B, C, D) from the state-space model (LTI array) `sys`. If `sys` is a transfer function or zero-pole-gain model (LTI array), it is first converted to state space. See Table 11-16, "State-Space Model Properties," on page 11-196 for more information on the format of state-space model data.

`[a,b,c,d,Ts] = ssdata(sys)` returns the sample time `Ts` in addition to `a, b, c`, and `d`.

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.statename
```

See Also

`dssdata`	Quick access to descriptor state-space data
`get`	Get properties of LTI models
`set`	Set model properties
`ss`	Specify state-space models
`tfdata`	Quick access to transfer function data
`zpkdata`	Quick access to zero-pole-gain data

Purpose

Build an LTI array by stacking LTI models or LTI arrays along array dimensions of an LTI array

Syntax

```
sys = stack(arraydim,sys1,sys2,...)
```

Description

`sys = stack(arraydim,sys1,sys2,...)` produces an array of LTI models `sys` by stacking (concatenating) the LTI models (or LTI arrays) `sys1`,`sys2`,... along the array dimension `arraydim`. All models must have the same number of inputs and outputs (the same I/O dimensions). The I/O dimensions are not counted in the array dimensions. See "Dimensions, Size, and Shape of an LTI Array" on page 4-7, and "Building LTI Arrays Using the stack Function" on page 4-15 for more information.

Example

If `sys1` and `sys2` are two LTI models with the same I/O dimensions:

- `stack(1,sys1,sys2)` produces a 2-by-1 LTI array.
- `stack(2,sys1,sys2)` produces a 1-by-2 LTI array.
- `stack(3,sys1,sys2)` produces a 1-by-1-by-2 LTI array.

Purpose

Step response of LTI systems

Syntax

```
step(sys)
step(sys,t)

step(sys1,sys2,...,sysN)
step(sys1,sys2,...,sysN,t)
step(sys1,'PlotStyle1',...,sysN,'PlotStyleN')

[y,t,x] = step(sys)
```

Description

`step` calculates the unit step response of a linear system. Zero initial state is assumed in the state-space case. When invoked with no output arguments, this function plots the step response on the screen.

`step(sys)` plots the step response of an arbitrary LTI model `sys`. This model can be continuous or discrete, and SISO or MIMO. The step response of multi-input systems is the collection of step responses for each input channel. The duration of simulation is determined automatically based on the system poles and zeros.

`step(sys,t)` sets the simulation horizon explicitly. You can specify either a final time `t = Tfinal` (in seconds), or a vector of evenly spaced time samples of the form

```
t = 0:dt:Tfinal
```

For discrete systems, the spacing `dt` should match the sample period. For continuous systems, `dt` becomes the sample time of the discretized simulation model (see “Algorithm”), so make sure to choose `dt` small enough to capture transient phenomena.

To plot the step responses of several LTI models `sys1`,..., `sysN` on a single figure, use

```
step(sys1,sys2,...,sysN)
step(sys1,sys2,...,sysN,t)
```

All systems must have the same number of inputs and outputs but may otherwise be a mix of continuous- and discrete-time systems. This syntax is useful to compare the step responses of multiple systems.

You can also specify a distinctive color, linestyle, and/or marker for each system. For example,

```
step(sys1,'y:',sys2,'g--')
```

plots the step response of `sys1` with a dotted yellow line and the step response of `sys2` with a green dashed line.

When invoked with output arguments,

```
[y,t] = step(sys)
[y,t,x] = step(sys)       % for state-space models only
y = step(sys,t)
```

return the output response `y`, the time vector `t` used for simulation, and the state trajectories `x` (for state-space models only). No plot is drawn on the screen. For single-input systems, `y` has as many rows as time samples (length of `t`), and as many columns as outputs. In the multi-input case, the step responses of each input channel are stacked up along the third dimension of `y`. The dimensions of `y` are then

(length of `t`) × (number of outputs) × (number of inputs)

and `y(:,:,j)` gives the response to a unit step command injected in the `j`th input channel. Similarly, the dimensions of `x` are

(length of `t`) × (number of states) × (number of inputs)

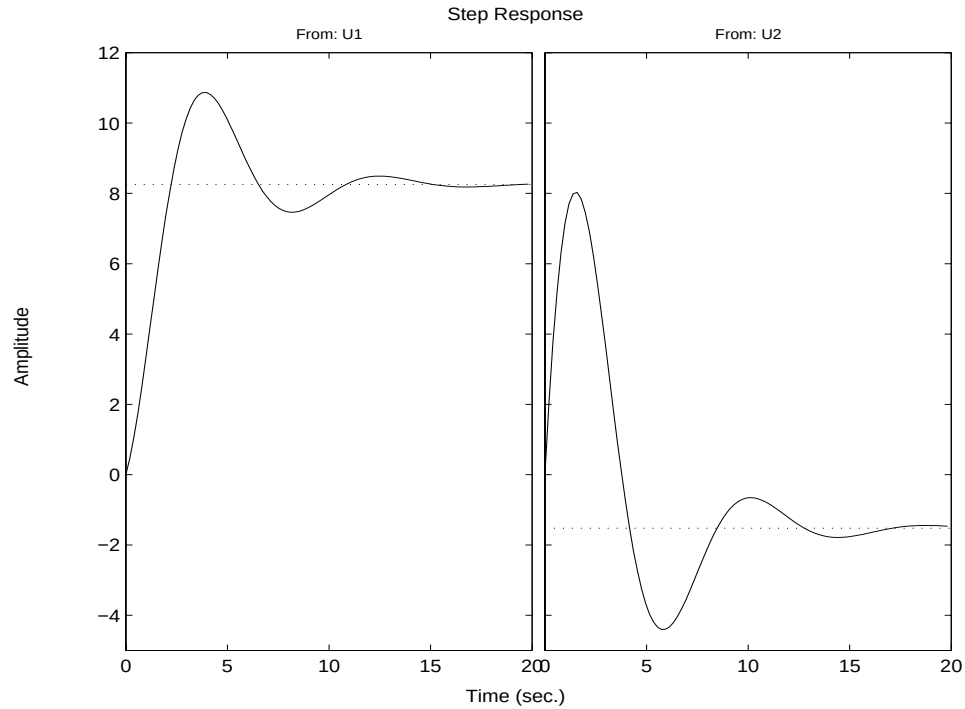
Example

Plot the step response of the following second-order state-space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.5572 & -0.7814 \\ 0.7814 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1.9691 & 6.4493 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

```
a = [-0.5572   -0.7814;0.7814   0];
b = [1 -1;0 2];
c = [1.9691   6.4493];
sys = ss(a,b,c,0);
step(sys)
```



The left plot shows the step response of the first input channel, and the right plot shows the step response of the second input channel.

Algorithm

Continuous-time models are converted to state space and discretized using zero-order hold on the inputs. The sampling period is chosen automatically based on the system dynamics, except when a time vector `t = 0:dt:Tf` is supplied (`dt` is then used as sampling period).

See Also

`impulse`	Impulse response
`initial`	Free response to initial condition
`lsim`	Simulate response to arbitrary inputs
`ltiview`	LTI system viewer

Purpose

Specify transfer functions or convert LTI model to transfer function form

Syntax

```
sys = tf(num,den)
sys = tf(num,den,Ts)
sys = tf(M)
sys = tf(num,den,ltisys)

sys = tf(num,den,'Property1',Value1,...,'PropertyN',ValueN)
sys = tf(num,den,Ts,'Property1',Value1,...,'PropertyN',ValueN)

sys = tf('s')
sys = tf('z')

tfsys = tf(sys)
tfsys = tf(sys,'inv')   % for state-space sys only
```

Description

`tf` is used to create real-valued transfer function models (TF objects) or to convert state-space or zero-pole-gain models to transfer function form.

Creation of Transfer Functions

`sys = tf(num,den)` creates a continuous-time transfer function with numerator(s) and denominator(s) specified by `num` and `den`. The output `sys` is a TF object storing the transfer function data (see "Transfer Function Models" on page 2-8).

In the SISO case, `num` and `den` are the real-valued row vectors of numerator and denominator coefficients ordered in *descending* powers of s. These two vectors need not have equal length and the transfer function need not be proper. For example, `h = tf([1 0],1)` specifies the pure derivative $h(s) = s$.

To create MIMO transfer functions, specify the numerator and denominator of each SISO entry. In this case:

- `num` and `den` are cell arrays of row vectors with as many rows as outputs and as many columns as inputs.
- The row vectors `num{i,j}` and `den{i,j}` specify the numerator and denominator of the transfer function from input `j` to output `i` (with the SISO convention).

If all SISO entries of a MIMO transfer function have the same denominator, you can set `den` to the row vector representation of this common denominator. See "Examples" for more details.

`sys = tf(num,den,Ts)` creates a discrete-time transfer function with sample time `Ts` (in seconds). Set `Ts = -1` or `Ts = []` to leave the sample time unspecified. The input arguments `num` and `den` are as in the continuous-time case and must list the numerator and denominator coefficients in *descending* powers of z.

`sys = tf(M)` creates a static gain `M` (scalar or matrix).

`sys = tf(num,den,ltisys)` creates a transfer function with generic LTI properties inherited from the LTI model `ltisys` (including the sample time). See "Generic Properties" on page 2-26 for an overview of generic LTI properties.

There are several ways to create LTI arrays of transfer functions. To create arrays of SISO or MIMO TF models, either specify the numerator and denominator of each SISO entry using multidimensional cell arrays, or use a `for` loop to successively assign each TF model in the array. See "Building LTI Arrays" on page 4-12 for more information.

Any of the previous syntaxes can be followed by property name/property value pairs

```
'Property',Value
```

Each pair specifies a particular LTI property of the model, for example, the input names or the transfer function variable. See `set` entry and the example below for details. Note that

```
sys = tf(num,den,'Property1',Value1,...,'PropertyN',ValueN)
```

is a shortcut for

```
sys = tf(num,den)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

Transfer Functions as Rational Expressions in s or z

You can also use real-valued rational expressions to create a TF model. To do so, first type either:

- `s = tf('s')` to specify a TF model using a rational function in the Laplace variable, `s`.
- `z = tf('z',Ts)` to specify a TF model with sample time `Ts` using a rational function in the discrete-time variable, `z`.

Once you specify either of these variables, you can specify TF models directly as rational expressions in the variable `s` or `z` by entering your transfer function as a rational expression in either `s` or `z`.

Conversion to Transfer Function

`tfsys = tf(sys)` converts an arbitrary SS or ZPK LTI model `sys` to transfer function form. The output `tfsys` (TF object) is the transfer function of `sys`. By default, `tf` uses `zero` to compute the numerators when converting a state-space model to transfer function form. Alternatively,

```
tfsys = tf(sys,'inv')
```

uses inversion formulas for state-space models to derive the numerators. This algorithm is faster but less accurate for high-order models with low gain at $s = 0$.

Examples

Example 1

Create the two-output/one-input transfer function

$$H(p) = \begin{bmatrix} \dfrac{p+1}{p^2+2p+2} \\ \dfrac{1}{p} \end{bmatrix}$$

with input `current` and outputs `torque` and `ang velocity`.

To do this, type

```
num = {[1 1] ; 1}
den = {[1 2 2] ; [1 0]}
H = tf(num,den,'inputn','current',...
                'outputn',{'torque' 'ang. velocity'},...
                'variable','p')

Transfer function from input "current" to output...
              p + 1
 torque:  -------------
          p^2 + 2 p + 2

                1
 ang. velocity: -
                p
```

Note how setting the `'variable'` property to `'p'` causes the result to be displayed as a transfer function of the variable *p* .

Example 2

To use a rational expression to create a SISO TF model, type

```
s = tf('s');
H = s/(s^2 + 2*s +10);
```

This produces the same transfer function as

```
h = tf([1 0],[1 2 10]);
```

Example 3

Specify the discrete MIMO transfer function

$$H(z) = \begin{bmatrix} \frac{1}{z+0.3} & \frac{z}{z+0.3} \\ \frac{-z+2}{z+0.3} & \frac{3}{z+0.3} \end{bmatrix}$$

with common denominator $d(z) = z + 0.3$ and sample time of 0.2 seconds.

```
nums = {1 [1 0];[−1 2] 3}
Ts = 0.2
H = tf(nums,[1 0.3],Ts)     % Note: row vector for common den. d(z)
```

Example 4

Compute the transfer function of the state-space model with the following data.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

To do this, type

```
sys = ss([−2 −1;1 −2],[1 1;2 −1],[1 0],[0 1])
tf(sys)

Transfer function from input 1 to output:
      s
-------------
s^2 + 4 s + 5

Transfer function from input 2 to output:
s^2 + 5 s + 8
-------------
s^2 + 4 s + 5
```

Example 5

You can use a `for` loop to specify a 10-by-1 array of SISO TF models.

```
s = tf('s')
H = tf(zeros(1,1,10));
   for k=1:10,
       H(:,:,k) = k/(s^2+s+k);
   end
```

The first statement pre-allocates the TF array and fills it with zero transfer functions.

Discrete-Time Conventions

The control and digital signal processing (DSP) communities tend to use different conventions to specify discrete transfer functions. Most control

engineers use the z variable and order the numerator and denominator terms in descending powers of z, for example,

$$h(z) = \frac{z^2}{z^2 + 2z + 3}$$

The polynomials z^2 and $z^2 + 2z + 3$ are then specified by the row vectors `[1 0 0]` and `[1 2 3]`, respectively. By contrast, DSP engineers prefer to write this transfer function as

$$h(z^{-1}) = \frac{1}{1 + 2z^{-1} + 3z^{-2}}$$

and specify its numerator as `1` (instead of `[1 0 0]`) and its denominator as `[1 2 3]`.

`tf` switches convention based on your choice of variable (value of the `'Variable'` property).

Variable	Convention
`'z'` (default)	Use the row vector `[ak ... a1 a0]` to specify the polynomial $a_k z^k + ... + a_1 z + a_0$ (coefficients ordered in *descending* powers of z).
`'z^-1'`, `'q'`	Use the row vector `[b0 b1 ... bk]` to specify the polynomial $b_0 + b_1 z^{-1} + ... + b_k z^{-k}$ (coefficients in *ascending* powers of z^{-1} or q).

For example,

```
g = tf([1 1],[1 2 3],0.1)
```

specifies the discrete transfer function

$$g(z) = \frac{z + 1}{z^2 + 2z + 3}$$

because z is the default variable. In contrast,

```
h = tf([1 1],[1 2 3],0.1,'variable','z^-1')
```

uses the DSP convention and creates

$$h(z^{-1}) = \frac{1 + z^{-1}}{1 + 2z^{-1} + 3z^{-2}} = zg(z)$$

See also `filt` for direct specification of discrete transfer functions using the DSP convention.

Note that `tf` stores data so that the numerator and denominator lengths are made equal. Specifically, `tf` stores the values

```
num = [0 1 1]; den = [1 2 3]
```

for `g` (the numerator is padded with zeros on the left) and the values

```
num = [1 1 0]; den = [1 2 3]
```

for `h` (the numerator is padded with zeros on the right).

Algorithm

`tf` uses the MATLAB function `poly` to convert zero-pole-gain models, and the functions `zero` and `pole` to convert state-space models.

See Also

`filt`	Specify discrete transfer functions in DSP format
`frd`	Specify a frequency response data model
`get`	Get properties of LTI models
`set`	Set properties of LTI models
`ss`	Specify state-space models or convert to state space
`tfdata`	Retrieve transfer function data
`zpk`	Specify zero-pole-gain models or convert to ZPK

Purpose

Quick access to transfer function data

Syntax

```
[num,den] = tfdata(sys)
[num,den] = tfdata(sys,'v')
[num,den,Ts] = tfdata(sys)
```

Description

`[num,den] = tfdata(sys)` returns the numerator(s) and denominator(s) of the transfer function for the TF, SS or ZPK model (or LTI array of TF, SS or ZPK models) `sys`. For single LTI models, the outputs `num` and `den` of `tfdata` are cell arrays with the following characteristics:

- `num` and `den` have as many rows as outputs and as many columns as inputs.
- The `(i,j)` entries `num{i,j}` and `den{i,j}` are row vectors specifying the numerator and denominator coefficients of the transfer function from input `j` to output `i`. These coefficients are ordered in *descending* powers of s or z.

For arrays `sys` of LTI models, `num` and `den` are multidimensional cell arrays with the same sizes as `sys`.

If `sys` is a state-space or zero-pole-gain model, it is first converted to transfer function form using `tf`. See Table 11-15, "LTI Properties," on page 11-195 for more information on the format of transfer function model data.

For SISO transfer functions, the syntax

```
[num,den] = tfdata(sys,'v')
```

forces `tfdata` to return the numerator and denominator directly as row vectors rather than as cell arrays (see example below).

`[num,den,Ts] = tfdata(sys)` also returns the sample time `Ts`.

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.Ts
sys.variable
```

Example

Given the SISO transfer function

```
h = tf([1 1],[1 2 5])
```

you can extract the numerator and denominator coefficients by typing

```
[num,den] = tfdata(h,'v')

num =
     0     1     1

den =
     1     2     5
```

This syntax returns two row vectors.

If you turn `h` into a MIMO transfer function by typing

```
H = [h ; tf(1,[1 1])]
```

the command

```
[num,den] = tfdata(H)
```

now returns two cell arrays with the numerator/denominator data for each SISO entry. Use `celldisp` to visualize this data. Type

```
celldisp(num)
```

and MATLAB returns the numerator vectors of the entries of `H`.

```
num{1} =
     0     1     1

num{2} =
     0     1
```

Similarly, for the denominators, type

```
celldisp(den)

den{1} =
     1     2     5

den{2} =
     1     1
```

See Also

`get`	Get properties of LTI models
`ssdata`	Quick access to state-space data

`tf`	Specify transfer functions
`zpkdata`	Quick access to zero-pole-gain data

totaldelay

Purpose

Return the total combined I/O delays for an LTI model

Syntax

```
td = totaldelay(sys)
```

Description

`td = totaldelay(sys)` returns the total combined I/O delays for an LTI model `sys`. The matrix `td` combines contributions from the `InputDelay`, `OutputDelay`, and `ioDelayMatrix` properties, (see `set` on page 11-193 or type `ltiprops` for details on these properties).

Delays are expressed in seconds for continuous-time models, and as integer multiples of the sample period for discrete-time models. To obtain the delay times in seconds, multiply `td` by the sample time `sys.Ts`.

Example

```
sys = tf(1,[1 0]);  % TF of 1/s
sys.inputd = 2;                    % 2 sec input delay
sys.outputd = 1.5;                 % 1.5 sec output delay
td = totaldelay(sys)

td =
    3.5000
```

The resulting I/O map is

$$e^{-2s} \times \frac{1}{s} e^{-1.5s} = e^{-3.5s} \frac{1}{s}$$

This is equivalent to assigning an I/O delay of 3.5 seconds to the original model `sys`.

See Also

`delay2z`	Change transfer functions of discrete-time LTI models with delays to rational functions or absorbs FRD delays into the frequency response phase information
`hasdelay`	True for LTI models with delays

Purpose

Transmission zeros of LTI models

Syntax

```
z = zero(sys)
[z,gain] = zero(sys)
```

Description

`zero` computes the zeros of SISO systems and the transmission zeros of MIMO systems. For a MIMO system with matrices (A, B, C, D), the transmission zeros are the complex values λ for which the normal rank of

$$\begin{bmatrix} A - \lambda I & B \\ C & D \end{bmatrix}$$

drops.

`z = zero(sys)` returns the (transmission) zeros of the LTI model `sys` as a column vector.

`[z,gain] = zero(sys)` also returns the gain (in the zero-pole-gain sense) if `sys` is a SISO system.

Algorithm

The transmission zeros are computed using the algorithm in [1].

See Also

`pole`	Compute the poles of an LTI model
`pzmap`	Compute the pole-zero map

References

[1] Emami-Naeini, A. and P. Van Dooren, "Computation of Zeros of Linear Multivariable Systems," *Automatica*, 18 (1982), pp. 415–430.

zgrid

Purpose

Generate a *z*-plane grid of constant damping factors and natural frequencies

Syntax

```
zgrid
zgrid(z,wn)
```

Description

`zgrid` generates a grid of constant damping factors from zero to one in steps of 0.1 and natural frequencies from zero to π in steps of $\pi/10$, and plots the grid over the current axis. If the current axis contains a discrete *z*-plane root locus diagram or pole-zero map, `zgrid` draws the grid over the plot without altering the current axis limits.

`zgrid(z,wn)` plots a grid of constant damping factor and natural frequency lines for the damping factors and normalized natural frequencies in the vectors `z` and `wn`, respectively. If the current axis contains a discrete *z*-plane root locus diagram or pole-zero map, `zgrid(z,wn)` draws the grid over the plot. The frequency lines for unnormalized (true) frequencies can be plotted using

```
zgrid(z,wn/Ts)
```

where `Ts` is the sample time.

`zgrid([],[])` draws the unit circle.

Example

Plot *z*-plane grid lines on the root locus for the system

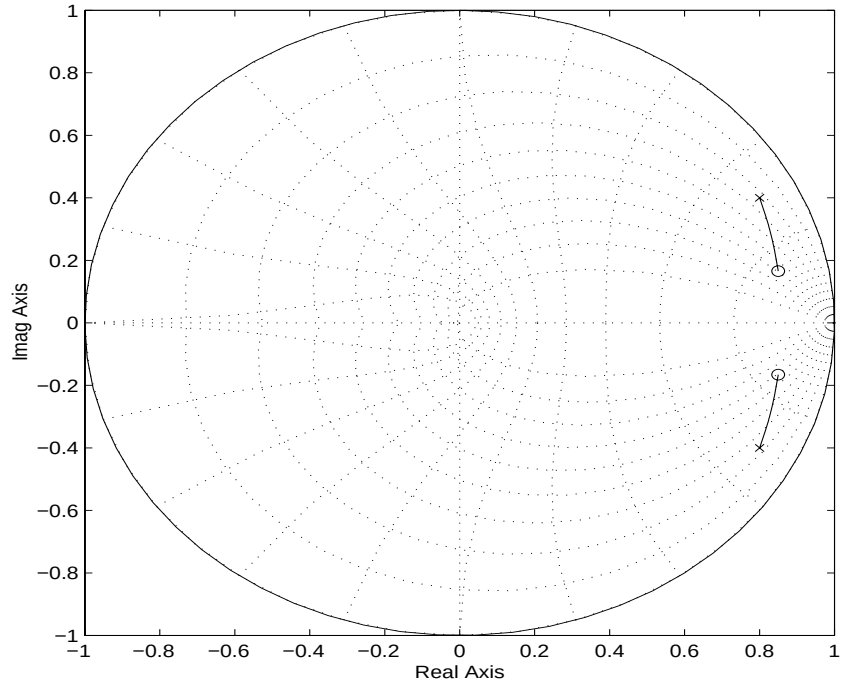
$$H(z) = \frac{2z^2 - 3.4z + 1.5}{z^2 - 1.6z + 0.8}$$

by typing

```
H = tf([2 -3.4 1.5],[1 -1.6 0.8],-1)

Transfer function:
2 z^2 - 3.4 z + 1.5
-------------------
 z^2 - 1.6 z + 0.8

Sampling time: unspecified
```

To see the z-plane grid on the root locus plot, type

```
rlocus(H)
zgrid
axis('square')
```



Limitations

`zgrid` plots the grid over the current axis regardless of whether the axis contains a root locus diagram or pole-zero map. Therefore, if the current axis contains, for example, a step response, you may superimpose a meaningless *z*-plane grid over the plot.

See Also

`pzmap`	Plot pole-zero map of LTI systems
`rlocus`	Plot root locus
`sgrid`	Generate *s*-plane grid lines

Purpose

Specify zero-pole-gain models or convert LTI model to zero-pole-gain form

Syntax

```
sys = zpk(z,p,k)
sys = zpk(z,p,k,Ts)
sys = zpk(M)
sys = zpk(z,p,k,ltisys)

sys = zpk(z,p,k,'Property1',Value1,...,'PropertyN',ValueN)
sys = zpk(z,p,k,Ts,'Property1',Value1,...,'PropertyN',ValueN)

sys = zpk('s')
sys = zpk('z')

zsys = zpk(sys)
zsys = zpk(sys,'inv')   % for state-space sys only
```

Description

`zpk` is used to create zero-pole-gain models (ZPK objects) or to convert TF or SS models to zero-pole-gain form.

Creation of Zero-Pole-Gain Models

`sys = zpk(z,p,k)` creates a continuous-time zero-pole-gain model with zeros `z`, poles `p`, and gain(s) `k`. The output `sys` is a ZPK object storing the model data (see “LTI Objects” on page 2-3).

In the SISO case, `z` and `p` are the vectors of real or complex conjugate zeros and poles, and `k` is the real-valued scalar gain.

$$h(s) = k\,\frac{(s-z(1))(s-z(2))...(s-z(m))}{(s-p(1))(s-p(2))...(s-p(n))}$$

Set `z` or `p` to `[ ]` for systems without zeros or poles. These two vectors need not have equal length and the model need not be proper (that is, have an excess of poles).

You can also use rational expressions to create a ZPK model. To do so, use either:

- `s = zpk('s')` to specify a ZPK model from a rational transfer function of the Laplace variable, `s`.
- `z = zpk('z',Ts)` to specify a ZPK model with sample time `Ts` from a rational transfer function of the discrete-time variable, `z`.

Once you specify either of these variables, you can specify ZPK models directly as real-valued rational expressions in the variable `s` or `z`.

To create a MIMO zero-pole-gain model, specify the zeros, poles, and gain of each SISO entry of this model. In this case:

- `z` and `p` are cell arrays of vectors with as many rows as outputs and as many columns as inputs, and `k` is a matrix with as many rows as outputs and as many columns as inputs.
- The vectors `z{i,j}` and `p{i,j}` specify the zeros and poles of the transfer function from input `j` to output `i`.
- `k(i,j)` specifies the (scalar) gain of the transfer function from input `j` to output `i`.

See below for a MIMO example.

`sys = zpk(z,p,k,Ts)` creates a discrete-time zero-pole-gain model with sample time `Ts` (in seconds). Set `Ts = –1` or `Ts = []` to leave the sample time unspecified. The input arguments `z`, `p`, `k` are as in the continuous-time case.

`sys = zpk(M)` specifies a static gain `M`.

`sys = zpk(z,p,k,ltisys)` creates a zero-pole-gain model with generic LTI properties inherited from the LTI model `ltisys` (including the sample time). See “Generic Properties” on page 2-26 for an overview of generic LTI properties.

To create an array of ZPK models, use a `for` loop, or use multidimensional cell arrays for `z` and `p`, and a multidimensional array for `k`.

Any of the previous syntaxes can be followed by property name/property value pairs.

```
'PropertyName',PropertyValue
```

Each pair specifies a particular LTI property of the model, for example, the input names or the input delay time. See `set` entry and the example below for details. Note that

```
sys = zpk(z,p,k,'Property1',Value1,...,'PropertyN',ValueN)
```

is a shortcut for the following sequence of commands.

```
sys = zpk(z,p,k)
set(sys,'Property1',Value1,...,'PropertyN',ValueN)
```

Zero-Pole-Gain Models as Rational Expressions in s or z

You can also use rational expressions to create a ZPK model. To do so, first type either:

- `s = zpk('s')` to specify a ZPK model using a rational function in the Laplace variable, `s`.
- `z = zpk('z',Ts)` to specify a ZPK model with sample time `Ts` using a rational function in the discrete-time variable, `z`.

Once you specify either of these variables, you can specify ZPK models directly as rational expressions in the variable `s` or `z` by entering your transfer function as a rational expression in either `s` or `z`.

Conversion to Zero-Pole-Gain Form

`zsys = zpk(sys)` converts an arbitrary LTI model `sys` to zero-pole-gain form. The output `zsys` is a ZPK object. By default, `zpk` uses `zero` to compute the zeros when converting from state-space to zero-pole-gain. Alternatively,

```
zsys = zpk(sys,'inv')
```

uses inversion formulas for state-space models to compute the zeros. This algorithm is faster but less accurate for high-order models with low gain at $s = 0$.

Variable Selection

As for transfer functions, you can specify which variable to use in the display of zero-pole-gain models. Available choices include s (default) and p for continuous-time models, and z (default), z^{-1}, or $q = z^{-1}$ for discrete-time models. Reassign the `'Variable'` property to override the defaults. Changing the variable affects only the display of zero-pole-gain models.

Example

Example 1

Specify the following zero-pole-gain model.

$$H(z) = \begin{bmatrix} \dfrac{1}{z-0.3} \\ \dfrac{2(z+0.5)}{(z-0.1+j)(z-0.1-j)} \end{bmatrix}$$

To do this, type

```
z = {[] ; –0.5}
p = {0.3 ; [0.1+i 0.1–i]}
k = [1 ; 2]
H = zpk(z,p,k,–1)    % unspecified sample time
```

Example 2

Convert the transfer function

```
h = tf([–10 20 0],[1 7 20 28 19 5])

Transfer function:
             –10 s^2 + 20 s
---------------------------------------
s^5 + 7 s^4 + 20 s^3 + 28 s^2 + 19 s + 5
```

to zero-pole-gain form by typing

```
zpk(h)

Zero/pole/gain:
     –10 s (s–2)
----------------------
(s+1)^3 (s^2 + 4s + 5)
```

Example 3

Create a discrete-time ZPK model from a rational expression in the variable z, by typing

```
z = zpk('z',0.1);
H = (z+.1)*(z+.2)/(z^2+.6*z+.09)
```

```
Zero/pole/gain:
(z+0.1) (z+0.2)
---------------
   (z+0.3)^2

Sampling time: 0.1
```

Algorithm

zpk uses the MATLAB function roots to convert transfer functions and the functions zero and pole to convert state-space models.

See Also

frd	Convert to frequency response data models
get	Get properties of LTI models
set	Set properties of LTI models
ss	Convert to state-space models
tf	Convert to TF transfer function models
zpkdata	Retrieve zero-pole-gain data

Purpose

Quick access to zero-pole-gain data

Syntax

```
[z,p,k] = zpkdata(sys)
[z,p,k] = zpkdata(sys,'v')
[z,p,k,Ts,Td] = zpkdata(sys)
```

Description

`[z,p,k] = zpkdata(sys)` returns the zeros `z`, poles `p`, and gain(s) `k` of the zero-pole-gain model `sys`. The outputs `z` and `p` are cell arrays with the following characteristics:

- `z` and `p` have as many rows as outputs and as many columns as inputs.
- The `(i,j)` entries `z{i,j}` and `p{i,j}` are the (column) vectors of zeros and poles of the transfer function from input `j` to output `i`.

The output `k` is a matrix with as many rows as outputs and as many columns as inputs such that `k(i,j)` is the gain of the transfer function from input `j` to output `i`. If `sys` is a transfer function or state-space model, it is first converted to zero-pole-gain form using `zpk`. See Table 11-15, “LTI Properties,” on page 11-195 for more information on the format of state-space model data.

For SISO zero-pole-gain models, the syntax

```
[z,p,k] = zpkdata(sys,'v')
```

forces `zpkdata` to return the zeros and poles directly as column vectors rather than as cell arrays (see example below).

`[z,p,k,Ts,Td] = zpkdata(sys)` also returns the sample time `Ts` and the input delay data `Td`. For continuous-time models, `Td` is a row vector with one entry per input channel (`Td(j)` indicates by how many seconds the `j`th input is delayed). For discrete-time models, `Td` is the empty matrix `[]` (see `d2d` for delays in discrete systems).

You can access the remaining LTI properties of `sys` with `get` or by direct referencing, for example,

```
sys.Ts
sys.inputname
```

Example

Given a zero-pole-gain model with two outputs and one input

```
H = zpk({[0];[-0.5]},{[0.3];[0.1+i 0.1-i]},[1;2],-1)

Zero/pole/gain from input to output...
         1
 #1:  -------
      (z-0.3)

           2 (z+0.5)
 #2:  -------------------
      (z^2 - 0.2z + 1.01)

Sampling time: unspecified
```

you can extract the zero/pole/gain data embedded in `H` with

```
[z,p,k] = zpkdata(H)

z =
    [      0]
    [-0.5000]
p =
    [    0.3000]
    [2x1 double]
k =
     1
     2
```

To access the zeros and poles of the second output channel of `H`, get the content of the second cell in `z` and `p` by typing

```
z{2,1}

ans =
   -0.5000

p{2,1}

ans =
   0.1000+ 1.0000i
   0.1000- 1.0000i
```

See Also

`get`	Get properties of LTI models
`ssdata`	Quick access to state-space data
`tfdata`	Quick access to transfer function data
`zpk`	Specify zero-pole-gain models

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