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Basal area growth models for individual trees of Norway spruce, Scots pine, birch and other broadleaves in Norway

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Abstract

Distance-independent individual tree growth models based on about 30,000 observations from the National Forest Inventory and the Norwegian Forest Research Institute have been developed for the main tree species in Norway. The models predict 5-year basal area increment over bark for trees larger than 5 cm at breast height. Potential input variables were of four types: size of the tree, competition indices, site conditions, and stand variables including species, mixtures and layers. The squared correlation coefficient (R^2) varied from 0.26 to 0.55. The accuracy of the models was tested by comparing the individual tree models with Norwegian diameter increment models. The accuracy is similar, but individual tree models forecast diameter distributions directly. The inclusion of species mixture and layer as variables increases the reliability of the models in mixed and in uneven-aged stands.

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1. Introduction

Growth models are the most important components in long-term forest planning systems, together with algorithms for stand establishment and models for harvest regimes and natural mortality. For several decades, foresters and forest researchers in many countries have emphasised the value of more accurate and cost-effective planning tools (Clutter et al., 1983; Vanclay, 1994).

Recent developments with respect to environmentally oriented practices in forestry involve different selective cutting techniques (Schieler and Schadauer, 1993; Hart, 1995; Nyland, 1996). For ecologists, preserving biodiversity and increasing the number of stands with mixed species and uneven-aged structure is a major goal (Burton et al., 1992; United Nations, 1992; Otto, 1994). However, yield tables and growth models used in planning and prognosis have been developed mainly from even-aged forest, where mean values and the mean tree of a stand are the basic components of the forecast. This has been the situation in Norway for decades (Braastad, 1975, 1977, 1980; Blingsmo, 1984). The stand simulator BESTPROG (Blingsmo and Veidahl, 1994) is based on such forecasts. So far in Norway, there is only one growth and yield study of mixed stands (Strand, 1983). Only a limited number of individual tree models have

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been developed in Norway: this presents a challenge for future research.

Distance-dependent individual tree growth models have existed for many years (Ek and Monserud, 1974; Hegyi, 1974; Sterba, 1983; Pretsch, 1992; Hasenauer, 1994; Nagel, 1996). In general, these models have been shown to give accurate predictions in both even- and uneven-aged stands with one or several tree species, and they are very useful for research purposes. Distance-dependent models have a high potential for estimating the response of different treatments. However, in general investigations they have shown little improvement over distance-independent models (Munro, 1974; Vanclay, 1994).

With this background, practically oriented distance-independent growth models have been developed (Bella, 1970; Arney, 1974; Söderberg, 1986; Holte and Solberg, 1989; Monserud and Sterba, 1996; Hasenauer et al., 1998). If these models are based on data from even-aged stands with small variation of tree diameters they are probably not so suitable for uneven-aged stands (Schütz, 1989; Andreassen, 1994; Peng, 2000). However, many models have been developed on the basis of national inventories or large regions (Belcher et al., 1982; Wykoff and Crookston, 1982; Söderberg, 1986; Wykoff, 1990; Ojansuu et al., 1991; Jonsson et al., 1993; Monserud and Sterba, 1996) and these might also be suitable for both even- and uneven-aged stands.

Many of the growth models include age as a variable (e.g. Söderberg, 1986; Pukkala, 1989). Age can easily be described in even-aged stands, but is obviously not straightforward to sample in uneven-aged forests in which the trees differ considerably in age.

The main goal of this study was to develop distance-independent individual tree growth models for uneven- and even-aged forest that can be used both for research purposes and for practical planning. The models were developed both with and without site index.

2. Materials and methods

2.1. Materials

The main body of data was obtained from permanent plots of the Norwegian Institute of Land Inventory, with additional observations from the long-term experimental plots of the Norwegian Forest Research Institute. The main data set consisted of about 3500 plots systematically distributed in a 3 km × 3 km grid including most of the forest in Norway, from latitude 58°N up to 70°N. Every county except Finnmark (the northernmost county) was represented. Plots divided by marsh, water, roads or large variation in site class (more than two site classes) were excluded from the data. The plots are circular with a radius of 5.64 m (100 m^2), and the trees were measured twice in the period from 1986 to 1997 with a mean inventory period of 5.4 years. In this investigation the growth were weighted with the length of the growth period so all observations could be compared with 5-year periods. In early tests the length of the growth period was discovered not to be a significant variable. An angle and a distance from the plot centre identified the trees. Together with the previous diameter, this provided a check of the re-measurement. The measuring height was marked by a dot 1.3 m above ground level. From the database of the Norwegian Forest Research Institute, 20 plots with large trees were added to the sample to ensure enough material in the larger diameter classes. Initial tests have shown that models could be sensitive at the end of or outside the diameter distribution of the base material. The diameter of all trees was measured in millimetres with a calliper and the height was measured on sample trees in decimetres with a vertex. A quarter of the trees were sampled by a relascope selection for height measurements. The sample of plots should represent a cross-section of forest conditions and forest management practices in Norway.

About 30,000 trees of the most common tree species in Norway were represented (Table 1). Norway spruce (*Picea abies*), Scots pine (*Pinus sylvestris*) and birch (*Betula pendula* and *Betula pubescens*) were analysed separately. The remaining tree species were analysed in the group “other broadleaves”. The species in this group (in descending order by number) were: grey alder (*Alnus incana*), rowan (*Sorbus aucuparia*), aspen (*Populus tremula*), sallow (*Salix caprea*), oak (*Quercus robur* and *Quercus petraea*), bird cherry (*Prunus padus*), ash (*Fraxinus excelsior*), common alder (*Alnus glutinosa*), beech (*Fagus silvatica*), Norway maple (*Acer platanoides*), wych elm (*Ulmus glabra*), and small-leafed lime (*Tilia cordata*). Other

Table 1
Distribution of tree observations by tree species and stand structure

Species	One layer	Two or more layers	Sum
Norway spruce	10,268	3121	13,389
Scots pine	4,805	1064	5,869
Birch	6,158	2582	8,740
Other broadleaves	2,627	1009	3,636
Sum	23,858	7776	31,634

conifers (planted exotic species) are excluded from the data.

A total of 2626 plots were classified having 1 layer, and 904 plots had 2 or more layers. The diameter distribution of Scots pine, and even more of Norway spruce, had a wide range (Table 2). Birch and other broadleaves had a narrower range. The range and distribution of different stand characteristics are listed in Table 3. A wide range of forest site quality classes in Norway was represented.

Site index was calculated for the dominant tree species in the 0.1 ha surrounding stand according to the Norwegian site index system (Tveite and Braastad, 1981). Often the site index of the minor tree species was estimated as well as that of the dominant species, according to the field instructions (NIJOS, 1996). To avoid confusion about site index and tree species in this analysis, the site index was estimated only for the two conifers. The site index was estimated only to the nearest 3 m classes. Stand age was estimated for the same trees as the site index. The plots are represented by cutting classes III–V (NIJOS, 1996) which include stand age above about 30 years.

Table 2
Diameter distribution for trees larger than 5 cm

Class (cm)	Norway spruce	Scots pine	Birch	Other broadleaves
10	8,093	2511	7619	3070
20	3,390	1955	1020	499
30	1,026	902	87	56
40	410	273	10	8
50	265	189	3	2
60	134	36	1	0
70	52	3	0	0
80	16	0	0	1
90	3	0	0	0
Sum	13,389	5869	8740	3636

2.2. *Statistical methods*

Multiple least-square estimation was used to elaborate the models. A combination of methods were used to select the variables, either plain or transformed: first a stepwise regression with different combinations of variables to get knowledge about the independent variables. Then a selection where several aspects were considered: (i) desired variables (tree size, competition, site index or latitude/altitude, and stand descriptions); (ii) reasonable variables with reasonable sign of the estimates; and (iii) available variables in Norwegian surveys. The use of repeats of the same base variable transformed may result in intercorrelation. To depress the intercorrelation, a regression procedure using the Yule–Walker estimates in the ‘‘autoreg processes’’ of the SAS system (SAS, 1989) was applied. The bias of the logarithmic dependent variable was corrected by adjusting the intercept term to harmonise the average of the residuals equal to 0 for the transformed models.

2.3. *The model*

The logarithm of the basal area increment was chosen as the dependent variable because this is usually one of the best variables reflecting the nonlinear curve of tree growth (Cole and Stage, 1972; Wykoff, 1990). The variables could be divided into four main groups and the model had the following form:

$$\ln(\mathrm{bai5}) = a + b_i \times \mathrm{SIZE} + c_i \times \mathrm{COMPETITION} + d_i \times \mathrm{SITE} + e_i \times \mathrm{STAND} \qquad (1)$$

The dependent variable, bai5, is the 5-year basal area increment of a tree (cm^2 per 5-year increment).

Variables in these *groups* are further described. In the model, a is the intercept, and b–e are vectors of the coefficients. The basal area of the individual tree in Eq. (1.1) describes SIZE:

$$\mathrm{SIZE} = b_1 \times \ln(\mathrm{ba}) + b_2 \times \mathrm{ba} + b_3 \times (\mathrm{ba})^2 \qquad (1.1)$$

where ‘‘ba’’ is the basal area of the individual tree at breast height (cm^2).

Table 3
Distribution of the plots in stand characteristic classes

Site index for main tree species			Stand basal area		Mean diameter		Mean height		Altitude		Latitude		Species proportion			
Site index class (m)	Norway spruce	Scots pine	Class (m^2 ha^{-1})	Number	Class (cm)	Number	Class (m)	Number	Class (m)	Number	Class (°N)	Number	Class	Norway spruce	Scots pine	Broadleaves
5	150	192	<10	1077	10	1906	<5	21	<200	1018	58	294	<0.1	1657	2019	1763
8	372	551	20	1951	20	1374	10	2714	400	1873	60	1466	0.2	264	164	560
11	450	393	40	433	30	226	20	780	800	638	62	886	0.4	213	160	252
14	483	190	60	63	40	21	30	15	1200	1	64	445	0.6	254	189	165
17	355	59	80	6	50	2					66	122	0.8	378	296	124
20	200	12			60	1					68	173	1	764	645	666
23	87	0									70	144				
26	9	0														

In early tests we experienced a bias for small (underestimated) and large (overestimated) trees if only one or two variables of the individual tree basal area were included. If three or more transformed variables of the basal area were included, most of this bias was eliminated. We decided to use three size variables if we found that these variables gave significant contributions.

COMPETITION is a general description of all trees in the stand summarised as stand basal area, BA ($m^2\ ha^{-1}$), mean height weighted by the basal area of the trees, H_L (dm), basal area mean diameter, D_g (mm), number of trees per hectare, N, and the quotient dbh/D_g (DQ) in the equation:

$$\text{COMPETITION} = c_1 \times \text{BA} + c_2 \times H_L + c_3 \times D_g + c_4 \times N + c_5 \times \text{DQ} \qquad (1.2)$$

The quotient DQ indicates the social ranking between the subject tree and the other trees, and reflects how large this tree is compared to its neighbour. To describe this status, Stage (1973) suggested using the percentile in the distribution of the tree basal area; Monserud and Sterba (1996) suggested the tree size in relation to the basal area of the larger trees; and Söderberg (1986) the tree diameter in relation to the diameter of the largest tree. Wykoff (1990) combined several effects. We decided to use the tree diameter in relation to the mean diameter to indicate the social ranking. However, all the variables in Eq. (1.2) give some measure of competition.

SITE describes the productivity of the plot. In Eq. (1.3), $G40$ is the site index of spruce (m), $F40$ the site index of pine, LAT the latitude in degrees, multiplied by 10, plus minutes as percentage of a degree (e.g. $60^\circ 30' \geq 650$), and ALT the altitude (m).

$$\text{SITE} = d_1 \times G40 + d_2 \times F40 + d_3 \times \text{LAT} + d_4 \times \text{ALT} \qquad (1.3)$$

However, only one of these site index variables can be used in the model, either for spruce or for pine. Subsequently, if the site index of the second species is set to 0, then the site index behaves like a dummy variable. In the model the component of the second tree species will automatically be deleted when multiplying by 0. Regarding SITE, only model type “a” (Table 4) includes the conventional forest site index $G40$ and $F40$. Model type “b” includes the topographic effects of latitude and altitude instead of site index. The distribution of SITE variables is listed in Table 3.

STAND characteristics could be divided into the three categories, Species, Mixtures and LAY, according to the equation:

$$\text{STAND} = e_{1-3} \times \text{Species} + e_{4-6} \times \text{Mixtures} + e_7 \times \text{LAY} \qquad (1.4)$$

The stand structure of the forest contains information about its past history as well as its present state. To increase the number of observations, and to simplify the model, tree species were included as dummy variables (S_1: Norway spruce, S_2: Scots pine, S_3: birch) in models 5 and 6. If the tree species occurs the dummy equals 1, otherwise 0.

$$\text{Species} = e_1 \times S_1 + e_2 \times S_2 + e_3 \times S_3 \qquad (1.4.1)$$

For “other broadleaves”, all these dummies equal 0. Some models of the minor species were based on few observations. However, the Species Mixture is included in the equation:

$$\text{Mixture} = e_4 \times P_1 + e_5 \times P_2 + e_6 \times P_3 \qquad (1.4.2)$$

where P_1 is the proportion of Norway spruce multiplied by 10, and P_2 and P_3 are the proportions of Scots pine and broadleaves (including birch), respectively.

The sum of the species proportions (P_1, P_2 and P_3) is constantly 10 in each plot. Species distribution and species mixture of the data are listed in Tables 1 and 3.

LAY describes the structure of the stand and is characterised by one (even-aged, LAY = 1) or more than one layer (uneven-aged, LAY = 0). We are aware that the layer effect might also be included in the competition variables. Still, to be able to indicate possible effects of treatment we found it convenient to separate the forest into one or more layers. Variables in these groups were included only if they made a significant contribution to the model with α less than 0.10. However, in compliance with the goals of the investigation, some variables were excluded in order to limit the input data.

3. Results

3.1. Selection of models

Several models were set up and compared with the basic data through an iterative process by combining

Table 4
Individual tree growth models of Norway spruce, Scots pine, birch, and other broadleaves

Variables	Species, model number, and structure											
	Norway spruce		Scots pine		Birch		Other broadleaves		All species			
	1a	1b	2a	2b	3a	3b	4a	4b	5a (E)	5b (E)	6a (U)	6b (U)
a (corrected)	−0.8706	2.2388	−0.8407	0.4391	−0.5651	−0.00043	−0.6194	−0.1527	−26.6658	0.1302	−63.7544	−44.1262
ln(ba)	0.7069	0.7263	0.7541	0.7155	0.4889	0.5123	0.6695	0.6450	0.5352	0.5923	0.5596	0.5428
ba × 10^{-3}	−0.0975	−0.0792	−0.3463	−0.4691	0.3650	0.3469	−0.3145	−0.2169	−0.2689	−0.2212	−0.4820	−0.3113
$(\mathrm{ba})^2 \times 10^{-6}$	0.0405	0.0533	0.0564	0.1173	–	–	–	–	0.0873	0.0754	0.1220	0.0995
DQ	0.2232	0.1676	0.0849	0.1863	0.3798	0.3637	0.4070	0.3927	0.6063	0.4636	0.4550	0.4294
$D_g \times 10^{-2}$	0.302	–	–	–	–	–	–	–	0.168	0.106	0.279	0.252
$H_L \times 10^{-2}$	−0.750	–	−0.427	–	–	–	–	–	–	–	–	–
BA	−0.0106	−0.0118	−0.0115	−0.0161	−0.0110	−0.0080	−0.0155	−0.0131	−0.0151	−0.0109	−0.0175	−0.0131
$N \times 10^{-3}$	−0.123	–	−0.194	–	–	–	–	–	–	–	–	–
*G*40	0.09337	–	0.08253	–	0.03473	–	0.02478	–	0.05957	–	0.05435	–
*F*40	0.08841	–	0.08899	–	0.03332	–	0.02285	–	0.05960	–	0.04838	–
LAT × 10^{-3}	–	−4.311	–	−0.974	–	−0.398	–	.	–	−0.611	–	−1.526
ALT × 10^{-3}	–	−0.7422	–	−0.3893	–	−0.3373	–	−0.3115	–	−0.5660	–	−0.3483
P_1	−0.0122	0.0236	0.0378	–	−0.0094	–	–	−0.0121	2.5658	0.0227	6.2599	4.4872
P_2	–	–	–	–	–	–	–	–	2.5612	–	6.2629	4.4661
P_3	0.0280	0.0594	0.0451	0.0337	−0.0088	–	−0.0079	–	2.5607	0.0174	6.2797	4.5122
S_1	–	–	–	–	–	–	–	–	0.2201	0.1569	0.3467	0.2331
S_2	–	–	–	–	–	–	–	–	0.04583	0.02132	0.3065	0.2074
S_3	–	–	–	–	–	–	–	–	−0.3709	−0.4852	−0.3773	−0.5166
LAY	0.0043	0.0620	−0.0328	−0.0376	0.0595	0.0797	0.0569	0.0713	–	–	–	–
R^2	0.55	0.49	0.48	0.42	0.28	0.26	0.36	0.35	0.43	0.44	0.46	0.52
CV	27	28	23	24	41	40	35	36	30	30	35	36
R.M.S.E.	0.76	0.80	0.73	0.77	0.90	0.89	0.84	0.87	0.83	0.82	0.86	0.88
d.f.	13,374	13,377	5847	5850	8729	8731	3575	3576	23,679	23,680	7769	7769

The dependent variable is the logarithm of 5-year basal area increment in cm^2 per 5-year increment. E, even-aged; U, uneven-aged; R^2, squared multiple correlation coefficient; CV, coefficient of variation; R.M.S.E., root mean square error of the logarithmic model; d.f., degrees of freedom.

many objectives and by analysing the residuals in relation to the input variables. In addition, RMSE and R^2 for the models were compared. In this process the variables were combined and transformed, and also included and excluded in compliance with the goals and on the basis of silvicultural knowledge. A total of 12 models were selected (Table 4). Two models were developed for each of the species Norway spruce, Scots pine, birch, and ''other broadleaves''. In addition, four models with species as dummy variables were developed; these include separate models for each layer. Tests showed no significant correlation between the length of the growth period and residuals of the models (R^2 varied from 0.0001 to 0.004), which indicate we have succeeded in harmonising the length of the growth period.

The squared multiple correlation coefficient (R^2) of Norway spruce and Scots pine varied from 0.26 to 0.55 (Table 4). For birch and other broadleaves the R^2 was low, less than 0.36. Models for uneven-aged forest had a similar fit to those for even-aged forest. The coefficient of variation was in the range from 0.23 to 0.41 (Table 4), while the root mean square error was about the same for all models (about 0.8 cm^2 per 5-year increment).

3.2. Evaluation and comparison of models

The residuals of the new models displayed, in general, a good fit for most of the diameter classes (Fig. 1). Some heteroscedastic tendencies appear in Fig. 1 with small residuals for small trees. In a test to reduce heteroscedasticity by using a weighted least squares method and weighting with the inverse of the variance, the overall mean of the parameter estimates changed with 0.3%.

No specific data set was selected to be used for validation. Optimal parameter estimates were of more interest.

One approach for testing was to make a comparison of the new models with other models against the basic data. This evaluation involved comparing the accuracy (standard deviation and residuals) of new versus old models (developed from other data). In this test, we

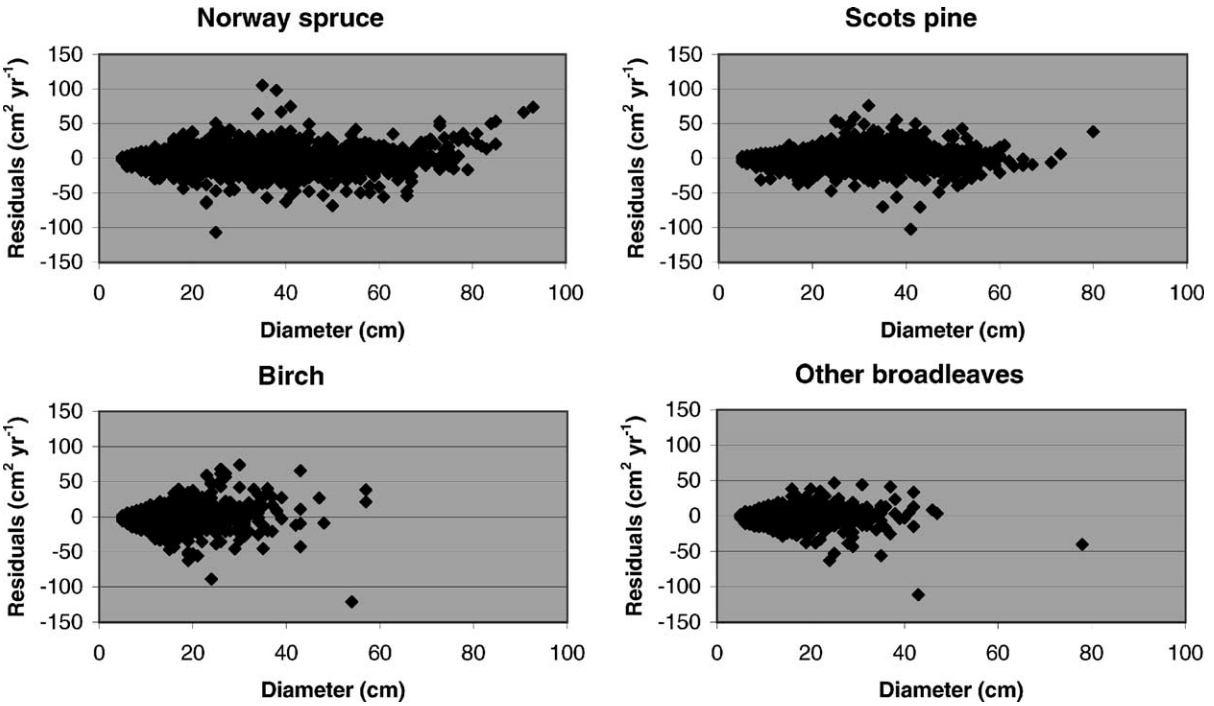

Fig. 1. Residuals of basal area growth models of individual trees (type a).

Table 5
Comparison of stand residuals (predicted minus observed mean tree basal area increment, cm^2 per year)

Species	Model	Mean	Minimum	Maximum	Standard deviation	Skewness	Kurtosis
Norway spruce	1a	−0.01	−68.54	34.77	5.40	−2.07	28.75
	1b	0.12	−60.44	30.32	5.79	−1.64	15.14
	B8	1.26*	−43.50	37.03	3.65	0.82	34.24
	Br16	1.75*	−36.68	40.37	4.45	0.51	22.90
	HS1	−0.78*	−74.14	29.97	6.64	−2.84	20.36
	HS2	−2.88*	−70.64	28.52	7.29	−2.65	14.87
Scots pine	2a	0.29	−54.98	33.48	6.03	−0.52	10.51
	2b	0.53	−62.48	33.05	6.37	−1.57	15.31
	B5	0.34*	−30.70	26.26	3.76	−1.17	13.74
Birch	3a	0.03	−45.32	61.61	4.82	2.44	45.09
	3b	−0.02	−46.32	62.51	4.81	2.36	47.09
	B2	−0.21	−56.99	37.93	4.09	−1.04	44.74
Other broadleaves	4a	0.16	−19.96	28.54	3.83	0.49	10.85
	4b	0.26	−17.86	29.34	3.86	0.75	10.94
	5a	−0.04	−43.31	37.84	4.39	−0.56	14.12
All species	5b	0.05	−55.41	31.07	4.75	−1.55	16.14
	6a	−0.02	−22.27	21.49	3.18	−0.01	10.05
	6b	−0.06	−28.37	19.67	3.40	−0.71	11.57

B2, B5 and B8 are models after Blingsmo (1984), Br16 is a model after Braastad (1974) and HS1 and HS2 are models after Holte and Solberg, 1989.

* $P < 0.01$ (values were significantly different from 0).

used four Norwegian stand models (Braastad, 1974; Blingsmo, 1984) and two individual tree models from mid-Sweden (Holte and Solberg, 1989).

The residuals of the mean tree basal area increment of both new and test models were compared on a stand level (Table 5). With a total fit, the mean value of the residuals approached 0. Additionally, minimum, maximum, standard deviation, skewness and kurtosis of the residuals are listed for each model. The individual tree basal area increment was summarised to stand level, and observed mean tree basal area increment was 7.4, 7.2, 2.4 and 3.5 (cm^2 per year) for Norway spruce, Scots pine, birch, and other broadleaves, respectively. Some other aspects of accuracy could be illuminated: the stand models overestimated the growth of Norway spruce by 25–35%, while the individual tree models of Holte and Solberg (1989) underestimated it by 10–30%; the models of Blingsmo (1984) overestimated the growth of Scots pine by 9%, while birch was underestimated by 8%; the standard deviation was slightly larger for the individual tree models compared with stand models.

One way to evaluate the models is by comparing the residuals with stand characteristics (Table 6). A correlation matrix highlights how well the models describe the growth at different sites. It must be emphasised that a low value of correlation indicates a good fit in this table. By summarising these correlation numbers, a ranking system for the models was developed. However, it must be stressed that a high correlation might emphasise the importance of this particular stand characteristic. Most stand variables in this test had already been included in the models, but not consistently. The models including site index gave the best total ranking. The Scots pine stand model B5 of Blingsmo (1984) had the highest ranking in the test. The new individual tree models had a high ranking, and the deviations from the 0 line were small.

The site index had the highest mean squared residual correlation. However, a high coefficient of correlation was also identified for models not including this variable. The stand age, which is not included in the new models, had the second highest squared residual correlation in most of the models.

Table 6
Pearson correlation matrix (R) between stand growth residuals and stand characteristics

Model	Stand characteristics								Ranking of squared correlation
	$H40$	BA	D_g	H_L	LAT	ALT	Stand age	R^2 all characteristics	
1a	−0.13[a]	−0.02[a]	−0.04[a]	−0.13[a]	0.12	0.03	0.15[a]	0.09	4
1b	−0.37[a]	−0.03[a]	−0.06[a]	−0.08	0.11[a]	0.06[a]	0.37[a]	0.22	10
2a	0.07[a]	0.04	0.04[a]	−0.03[a]	0.05	0.01[a]	0.13[a]	0.07	3
2b	−0.25[a]	0.02[a]	0.03	−0.04	0.05	0.02	0.24[a]	0.11	5
3a	−0.01	0.01[a]	0.15[a]	0.07	0.01	0.03	0.11[a]	0.04	2
3b	−0.07	−0.01	0.13[a]	0.05	0.03	0.02	0.13[a]	0.04	2
4a	−0.10	0.01	−0.09[a]	−0.03	−0.01	0.13[a]	0.05[a]	0.04	2
4b	−0.19[a]	−0.01	−0.08[a]	−0.04	0.02	0.10	0.08[a]	0.06	3
5a	−0.06[a]	0.04[a]	0.05	0.02[a]	0.04[a]	0.06[a]	0.22[a]	0.07	3
5b	−0.29[a]	−0.00[a]	−0.01[a]	−0.08[a]	0.07	0.06[a]	0.30[a]	0.15	7
6a	−0.10	0.07	0.14[a]	0.05[a]	0.08[a]	−0.01	0.26[a]	0.09	4
6b	−0.29[a]	0.06[a]	0.06	−0.04[a]	0.10	0.02[a]	0.32[a]	0.18	8
HS1	−0.47[a]	−0.27	−0.33[a]	−0.32	0.17[a]	0.20	0.29[a]	0.31	11
HS2	−0.52[a]	−0.27[a]	−0.39[a]	−0.41[a]	0.20[a]	0.24	0.28[a]	0.38	12
Br16	−0.19[a]	−0.37[a]	−0.15[a]	−0.22[a]	0.13[a]	0.09[a]	0.11[a]	0.19	9
B8	−0.25[a]	−0.19[a]	−0.04[a]	−0.16[a]	0.14[a]	0.13	0.05[a]	0.11	5
B5	−0.03	−0.15[a]	−0.06	−0.07	0.06	0.06[a]	−0.11[a]	0.03	1
B2	−0.10[a]	−0.03[a]	0.01	0.03[a]	−0.06	0.09[a]	−0.19[a]	0.12	6
Mean R^2	0.05	0.02	0.02	0.02	0.01	0.01	0.04	0.12	

B2, B5, B8, Br16, HS1 and HS2 are residuals of models according to Blingsmo (1984), Braastad (1974) and Holte and Solberg (1989). R^2 is the squared residual correlation with all stand characteristics.

[a] $\alpha < 0.1$.

On the basis of the evaluation, the following major trends could be outlined. The new individual tree models work best at "medium" sites. For very low or high site indices they over- or underestimate the growth by 10–50% for single plots. Most of the models overestimate the increment by 10–30% for the lowest site indices ($H40 < 8$ m). For a wide range of densities, for altitudes below 700 m, and for plots with a mean diameter greater than 10 cm, the models behaved fairly well. In open stands with stand basal

Table 7
Sensitivity analysis

Change of input variable	Model number												
	1a	1b	2a	2b	3a	3b	4a	4b	5a	5b	6a	6b	Mean
dbh	146	154	128	134	134	134	136	135	175	160	134	135	140
D_g	110	97	98	96	93	94	93	93	96	96	99	99	96
Site index	133	–	130	–	111	–	108	–	120	–	118	–	118
BA	93	92	95	93	96	97	93	94	92	94	94	95	94
Properties of broadleaves	109	107	101	107	–	–	98	102	99	99	104	106	104
ALT	–	96	–	98	–	98	–	99	–	97	–	98	98
LAT	–	59	–	89	–	95	–	–	–	93	–	83	84
From even- to uneven-aged	100	90	103	97	94	92	94	93	–	–	90	94	96
From uneven- to even-aged	100	112	97	104	106	108	106	107	103	110	–	–	105
Mean	113	101	107	102	106	103	104	103	114	107	110	101	106

Predicted 5-year individual tree basal area increment in percent of observed by adding 20% extra of the input variable or changing dummy variables.

area below 5 m^2 ha^{-2}, and in stands with small-sized trees, the models underestimated the growth by 10–30%. At high altitudes (700–1000 m) the increment was overestimated by 10–30%. However, the stand models and the Swedish models had a similar pattern of deviation.

A simple sensitivity analysis was performed in which the main independent variables were manipulated by adding 20% to the observed value. This approach indicates the effect of wrong input values or the relative importance of different input variables. The most sensitive independent variable was tree size. Size increased the increment by 40% when the diameter was increased by 20% (Table 7). The next most important variables were site index and latitude. The other variables had a minor effect.

4. Discussion

4.1. Model development

In this investigation, 12 new growth models were developed. These models are among the first individual tree models developed in Norway and cover important forest regions and districts from 58 to 70°N. In addition, most growth conditions with respect to site index, density, competition, stand age and species are represented.

It is important, when the accuracy of the models is discussed, to address the many sources of uncertainties related to growth analysis, e.g. related to different input variables and to model errors of the predictions (Kangas and Kangas, 1999). In this study we have developed deterministic growth models. Despite arising problems of multicollinearity, similar models have successfully been applied in other parts of the world, and seem to have an appropriate level of reliability (Söderberg, 1986; Wykoff, 1990; Monserud and Sterba, 1996).

The length of the growth period had no influence upon the accuracy. The Norway spruce models had the highest squared correlation coefficient ($R^2 = 0.49-0.55$), while the Scots pine models had the lowest coefficient of variation, about 23% (Table 4). The variance that could be explained by these models is comparable with that in similar investigations (Söderberg, 1986; Wykoff, 1990; Monserud and Sterba, 1996). However, most of these include additional independent variables such as crown width, soil type, etc. which presuppose detailed and high cost inventory. Monserud and Sterba (1996) recognised a particularly poor fit for Scots pine. The same pattern was identified in our study. This phenomenon may have a genetic explanation attributable both to the wider distribution of pine compared with Norway spruce and also to the longer time lapse since its post-glacial arrival. These factors may have led to a larger number of local races. Also, pine occupies a larger proportion of the low and irregular site indices and thereby produces a small and variable increment, which may lead to a lower correlation coefficient.

Birch had the lowest coefficient of correlation ($R^2 = 0.26$). This may be attributable to a mix of the two species, *B. pubescens* and *B. pendula*, which are rarely distinguished when recording the data. However, for other broadleaves, which include several species, the coefficient was somewhat higher ($R^2 = 0.36$). This difference is hard to explain, but we identified more outliers in birch (Fig. 1).

One of the goals in this investigation was to include both layers and species mixture in the models. Individual tree models can be used to consider the consequences of different silviculture treatments in detail. In these data, an increased proportion of broadleaves increased the increment in conifer-dominated stands. This effect is also reflected in the models.

The different growth of trees in even- and uneven-aged forest is emphasised in many investigations (Schütz, 1989; Lundqvist, 1989; Andreassen, 1994; Lädhe et al., 1994; O'Hara, 1996; Valappil, 1997). But despite numerous studies of the relative productivity of even-aged versus uneven-aged forest, there is no clear consensus. Consequently, most studies advise avoiding mixing of even- and uneven-aged forest in growth models. We developed both combined models, including layer (models 1–4), and separate models for even- and uneven-aged forest (models 5 and 6).

The multi-layered forest structure is less represented in the data. By including layer as a dummy variable, the different growth conditions of an uneven-aged stand may be undervalued, since most of the observations in the model are based on even-aged stands.

4.1.1. Comparison and evaluation

For the test models we have used in the comparison, we are well aware that different data, inventory periods, variables, transformations, structures etc. are involved. Most of the old Norwegian stand models were developed using data from even-aged experimental plots and they represent the mean growth over a much longer period of time. It is not straightforward to compare different models and treatments. One drawback is the fact that the material was sampled in a relatively short period of time, from 1986–1997. Solberg and Tveite (2000) found the actual diameter growth from 1991–1996 to be 17% lower than simulated with the same test stand models. Other authors also report lower growth rates in this particular period (Solberg et al., 2000). However, these reports are based on data from non-experimental plots and not from experiments in which the test models are developed. Spiecker et al. (1996) report no specific increase of productivity in Norway.

Compared with stand models, the new individual tree models have a lower correlation between residuals and stand characteristics (Table 6). This indicates a better fit outside an "average" stand. Age is not included in the new models but is included in the test models, except for model HS2. It is interesting that the Scots pine model, B5, of Blingsmo (1984) seems to include the stand characteristics better than all the new individual tree models for Scots pine. This might be related to the current status of Norwegian Scots pine stands, the majority of which are homogenous and even-aged. For birch and other broadleaves, the correlation of the residuals with stand characteristics is low, which indicates a generally good fit.

In the evaluation process, we observed that the individual tree models work well on a wide range of sites. However, comparing the new models with stand models at medium and extreme sites, we found a bias for both new and old models. The Swedish individual tree models of Holte and Solberg (1989) had the largest deviation of the residuals.

In this study we have developed several new models for different species which can be used in both even- and uneven-aged forest. For some of the new models the mean accuracy is somewhat lower than that of stand models. The advantage of applying them in even-aged forest is then small. However, a major advantage with our new models is that the diameter distribution is calculated directly, which opens the way for many different possible analyses. For uneven-aged forest, no other national models are available and the advantage in using them is probably large in such stands.

Several Norwegian investigations of indigenous species involve validation or evaluation of the stand models (Braastad, 1974, 1975, 1977, 1980; Blingsmo, 1984; Opdahl, 1993). There have also been surveys on how well growth models work on a regional scale (e.g. Tveite, 1994; Øyen and Nes, 1997) or on the effects of silvicultural treatments (Blingsmo, 1988; Pettersen, 1997). In general these studies have found that (1) the old Norwegian diameter stand models perform quite well in dense forest, and mainly give a maximum value of what is possible to achieve. (2) It is uncertain how well the models behave in irregular-spaced or low-density stands (e.g. after heavy thinning), and for low site indices.

4.1.2. Accuracy

Compared with the Norwegian stand models of Blingsmo (1984) and Braastad (1974), the standard deviation of the new individual tree models was somewhat larger (Table 5). This indicates a limited potential for improving the accuracy in even-aged stands.

The error of the diameter measurements with callipers is not negligible. Prodan (1965) and McRoberts et al. (1995) indicated this error to be between 0.3 and 1.25%, which means an error of the basal area increment between 4 and 18%. The method of identifying the trees for remeasurement is also inaccurate since the trees are not numbered but only identified by an angle and a distance from the plot centre. This results in some outliers.

In addition, the variation in increment could also be ascribed to climate, soil, vegetation type and genetic components (Stage and Wykoff, 1993), which were not included in the model. The plot size, 100 m^2, might allow some edge effects. All these inaccuracies might reduce the correlation coefficient.

Residual plots (Fig. 1) underline the limitation of models applied to large trees. The use of the models for trees with diameters greater than 50 cm at breast height is not recommended. However, in Norwegian forestry, trees with diameters above 50 cm at breast height are both rare and difficult to sell, and the target cutting diameter is usually much less, normally around 30 cm. For trees below dbh 15 cm, particularly for broadleaves, Fig. 1 appears to show some heteroscedastic tendency. However, for small trees the residuals will be small on an arithmetic scale, while the coefficient of variation is large, mainly because of a larger deviation in competition and suppression. For bigger trees the largest residuals address the question of the quality of the data. However, if all outliers are excluded, some systematic error might appear. The presence of heteroscedasticity influences several of the statistical tests of the least squares method (White, 1980). To reduce heteroscedasticity, some writers recommend using a weighted least squares method, weighting with the inverse of the variance (e.g. Chatterjee and Price, 1977). However, if the importance of small trees is reduced, some of the effects of competition that we are trying to model may also be reduced. Also, these weights change the parameter estimates very little in analyses with some thousand observations and a symmetric distribution of the residuals. Furthermore, several scientists have used the logarithm of basal area as a dependent variable to stabilise the variance (Cole and Stage, 1972; Söderberg, 1986; Wykoff, 1990; Monserud and Sterba, 1996). Most of our independent variables are also selected in other investigations, and similar growth models have been successfully used for several years in many countries. Despite heteroscedasticity for small trees in this investigation, we think the influence on the model is very small.

Compared with the mean values, Norway spruce, Scots pine, and broadleaves, except for birch, have low values for skewness and kurtosis (Table 5). Øyen and Andreassen (2001) indicate skewness and kurtosis in the same range. For birch, the high skewness and kurtosis emphasise the tendency towards larger deviation for larger trees (Table 5 and Fig. 1).

4.1.3. Sensitivity analyses

The simplified sensitivity analysis indicated that the most sensitive input variable is the tree diameter. As discussed earlier, and to avoid skewness of increment, especially for small trees, it seems important to include several transformations of this variable in the models. Site index and latitude were also important. In the growth models of Blingsmo (1984), the site index and age are the most sensitive variables. The effect of other variables was small. This does not mean that these variables are negligible. We are well aware that changes in the input variables in the sensitivity analyses do not necessarily represent a realistic change. However, all the effects indicate reasonable responses of the model.

4.2. Model application and future use

One of the goals was to avoid variables not sampled in a normal forest inventory. This makes the model well suited to application in stands for which extensive forest inventory data is available. Age was excluded in the models, primarily because of the difficulties of age estimation in uneven-aged stands.

Individual tree models will probably make it easier to forecast changes in the structure, since the diameter distribution is estimated directly. If management decisions need information about tree dimensions, individual tree models will improve this estimate. The models might also improve the possibility of performing detailed analyses of timber quality, stand structure, production and economics in relation to species, layers and silvicultural methods.

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