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Forest Ecology and Management xxx (2003) xxx–xxx

Forest Ecology and Management

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Modeling spatial variation in tree diameter–height relationships

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Received 13 May 2003; received in revised form 10 July 2003; accepted 1 September 2003

Abstract

Traditionally the relationship between tree diameter and total height in a forest stand is investigated by using linear or non-linear regression models, in which the spatial heterogeneity in the relationship is largely ignored. The objective of this study is to explore and model the spatial variations in the tree diameter–height relationship in an eucalypt stand using geographically weighted regression (GWR). GWR attempts to capture spatial variations by calibrating a multiple regression model fitted at each tree, weighting all neighboring trees by a function of distance from the subject tree. GWR produces a set of parameter estimates and model statistics (e.g. model R^2) for each tree in the stand. The results indicate that GWR significantly improves model fitting over ordinary least squares (OLS). The GWR model produces smaller model residuals across diameter classes than the traditional OLS model does. The overall average (0.074) of the model absolute residual from the GWR model is 43% smaller than that (0.129) from the OLS model. Furthermore, the parameter estimates and model statistics of the GWR model can be mapped using visualization tools such as geographical information system (GIS) to illustrate local spatial variations in the regression relationship under study.

Keywords: Spatial autocorrelation and heterogeneity; Spatial statistics; Forest modeling

1. Introduction

Spatial autocorrelation (i.e. spatial dependency) and heterogeneity (i.e. spatial non-stationarity) are two aspects of spatial variability (Anselin and Griffith, 1988). Spatial autocorrelation represents the correlation between the value of a random variable at a location *i* and the values of the same variable at "neighboring" locations *j*. It is generally known as Tobler's law of geography, where "everything is related to everything, but near things are more related than distant things" (Anselin, 1988). The spatial autocorrelation in the error terms of a regression model causes biased estimation of error variance, while regression coefficients remain unbiased. Thus, significance tests and measures of model fit may be misleading (Anselin and Griffith, 1988; Anselin, 1990). On the other hand, spatial heterogeneity is defined as structural instability in the form of systematically varying model parameters or different response functions (Anselin and Griffith, 1988; Anselin, 1990). It is related to locations in space, missing variables, and functional mis-specification

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doi:10.1016/j.foreco.2003.09.004

(Anselin, 1988; Fotheringham, 1997). Ignoring spatial heterogeneity causes biased parameter estimates, misleading significance tests, and sub-optimal prediction (Anselin and Griffith, 1988).

The spatial variations of a tree variable (e.g. growth, total height, etc.) in a forest stand result from the complex historical and environmental mosaic imposed by competition and systematic environmental heterogeneity. Forest researchers have realized that spatial patterns of tree locations strongly affect (1) competition among neighboring trees, (2) size variability and distribution, (3) growth and mortality, and (4) crown structure (Kuuluvainen and Pukkala, 1987; Kenkel, 1988; Miller and Weiner, 1989; Kenkel et al., 1989; Moeur, 1993; Rouvinen and Kuuluvainen, 1997; Newton and Jolliffe, 1998; Dovciak et al., 2001). Positive spatial dependency (i.e. trees are surrounded by similar-sized trees) among neighboring trees is mainly due to the effects of micro-site. It is often observed in young, pre-canopy closure stands, and older, senescent stands. Inter-tree competition tends to create a negative autocorrelation (i.e. larger-sized trees are surrounded by smaller trees and vice versa) among neighboring trees (Garcia, 1992, 1994; Magnussen, 1994). Although forest modelers have used a variety of statistical techniques to study spatial autocorrelations (e.g. Reed and Burkhart, 1985; Schoonderwoerd and Mohren, 1988; Kenkel et al., 1989; Moeur, 1993; Biondi et al., 1994; Liu and Burkhart, 1994; Frohlich and Quednau, 1995; Kohl and Gertner, 1997; Fox et al., 2001; Cressie and Collins, 2001), the influence of micro-site variation or spatial heterogeneity has been largely neglected and has just received attention in recent years (Garcia, 1994; Fox et al., 2001).

In the field of spatial modeling, a number of statistical techniques have been developed to model local variations on complex relationships between random variables over space (i.e. in a regression context). These include the spatial expansion method (Casetti, 1997; Jones and Casetti, 1992), the random coefficient model (Fotheringham and Brunsdon, 1999), the spatial adaptive filtering method (Gorr and Olligschlaeger, 1994), the multilevel modeling method (Jones, 1997; Duncan, 1997), the spatial autoregressive model (Cliff and Ord, 1981), the moving average model (Haining, 1978), and the spatial error model (Anselin, 1988). In recent years, geographically weighted regression (GWR) has become popular to explore spatial heterogeneity (Brunsdon et al., 1996; Fotheringham et al., 2000, 2002). GWR attempts to capture spatial variations by calibrating a multiple regression model that allows different relationships between variables to exist at different points in space. The basic idea is that a regression model is fitted at each point in the data, weighting all observations by a function of distance from that point. The neighbors near to the point have more influence on the resulting regression coefficients than the observations further away. GWR produces a set of parameter estimates at each point in the defined geographical area. These parameter estimates and model statistics (e.g. model R^2) can be mapped using visualization tools such as geographical information system (GIS) to investigate local spatial variation in the regression relationships under study (e.g. Brunsdon et al., 1996; Fotheringham and Brunsdon, 1999; Fotheringham et al., 2000, 2002).

In this study we applied the GWR methodology for modeling the spatial variations in tree diameter–height relationship in an eucalypt stand. We attempted (1) to fit a regression model to the relationship by both OLS and GWR methods, (2) to test and compare the improvement of GWR over OLS on model fitting, (3) to illustrate the GWR model coefficients and R^2 over space using graphical visualization tools, and (4) to discuss the possible applications of GWR methodology in forest modeling and management.

2. Theoretical background

Suppose we have a set of n observations $\{X_{ij}\}$, $i = 1, 2, \ldots, n$, on p independent or predictor variables, $j = 1, 2, \ldots, p$, and a set of n observations on a dependent or response variable $\{y_i\}$. The relationship between the dependent variable y and X_s can be regressed using OLS as follows:

$$y_i = \beta_0 + \sum_{j=1}^{p} X_{ij}\beta_j + \varepsilon_i \quad (1)$$

where β_0 is an intercept coefficient, β_j the slope coefficient for the jth independent variable X_j, and ε_i the random error term whose distribution is $N(0, \sigma^2 I)$, with I denoting an $n \times n$ identity matrix.

Thus, the OLS estimate of β, the vector of regression coefficients, is obtained by the least-squares method as

$$\hat{\beta} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}y \tag{2}$$

where superscript T denotes the transpose of a matrix. The relationship represented by Eq. (1) is assumed to be universal or constant across the geographical study area.

Now suppose that we have a set of location coordinates $\{(u_i, v_i)\}$ for each observation i. The underlying model for GWR is

$$y_i = \beta_0(u_i, v_i) + \sum_{j=1}^{p} X_{ij}\beta_j(u_i, v_i) + \varepsilon_i \tag{3}$$

where $\{\beta_0(u_i, v_i), \beta_1(u_i, v_i), \ldots, \beta_p(u_i, v_i)\}$ are $p+1$ continuous functions of the location (u_i, v_i) in the geographical study area. Again, ε_i is the random error term with a distribution $N(0, \sigma^2 I)$. The aim of GWR is to obtain non-parametric estimates of these functions for each independent variable X_j and each geographical location i (i.e. each tree in this study). This can be achieved by using data near location i. The estimation procedure of GWR is as follows: (1) find a point at one particular location i, (2) compute a weight (w_{ik}) for each neighboring observation k according to the distance (d_{ik}) between the location k and the point i, and (3) estimate the model coefficients using weighted least-squares regression such that

$$\hat{\beta}_i = (X^{\mathrm{T}}W_iX)^{-1}X^{\mathrm{T}}W_iy \tag{4}$$

where the weight matrix W_i is

$$W_i = \begin{pmatrix} w_{i1} & 0 & \cdots & 0 \\ 0 & w_{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{in} \end{pmatrix} \tag{5}$$

If $W_i = I$ (identity matrix), i.e. each observation in the data has a weight of unity, the GWR model is equivalent to the OLS model. Eq. (4) is not a single equation, but an array of equations, with each β_i corresponding to a column of the matrix whose elements are β_{ij}. Once each of the w_{ik} has been calculated, the coefficient matrix β can be computed column by column by repeated application of Eq. (4). Therefore, we obtain a set of estimates of spatially varying parameters without specifying a function form for the spatial variation. Essentially, GWR lets the data speak for themselves when estimating each regression coefficient β_{ij} for each independent variable and each geographical location (Brunsdon et al., 1998).

In this study we used a Gaussian distance-decay-based kernel function for computing the weight w_{ik} as follows:

$$w_{ik} = \mathrm{e}^{-(d_{ik}/h)^2} \tag{6}$$

where h is referred to as the bandwidth. This kernel function assumes that the bandwidth at each point i is a constant across the study area (i.e. fixed kernel). If the locations i and k coincide (i.e. $d_{ik} = 0$), then w_{ik} equals unity; while w_{ik} decreases according to a Gaussian curve as the distance d_{ik} increases. However, the weights are non-zero for all data points, no matter how far they are from the point i (Brunsdon et al., 1998; Fotheringham et al., 2000, 2002).

Given the greater flexibility of the GWR coefficients over space, GWR always provides a better fit in terms of the residual sum of squares. It is important to test whether the local model (i.e. GWR) offers an improvement over the global model (i.e. OLS). The null hypothesis that the parameter estimates are constant for all locations in the study area can be tested as follows (Brunsdon et al., 1999; Leung et al., 2000). Under OLS, the parameter estimates are given by Eq. (2), and the fitted values of y are given by

$$\hat{y} = X(X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}y = S_0 \cdot y \tag{7}$$

where S_0 is called the hat matrix of OLS (Montgomery and Peck, 1992). The residual sum of squares of the OLS model is

$$\mathrm{RSS}_0 = y^{\mathrm{T}}(I - S_0)^{\mathrm{T}}(I - S_0)y = y^{\mathrm{T}}R_0y \tag{8}$$

and RSS_0/σ^2 is exactly distributed as a χ^2 distribution with $(n-p-1)$ degrees of freedom where σ^2 is the common variance of the error terms ε_i.

Under GWR, the parameter estimates are given by Eq. (4). For any given location i, the fitted value of y_i is estimated by

$$\hat{y}_i = x_i^{\mathrm{T}} \cdot \hat{\beta}_i = x_i(X^{\mathrm{T}}W_iX)^{-1}X^{\mathrm{T}}W_iy \tag{9}$$

where $x_i^{\mathrm{T}} = [1 x_{i1}, x_{i2}, \ldots, x_{ip}]$ is the ith row of X $(i = 1, 2, \ldots, n)$, and $\hat{\beta}_i$ the estimated parameter vector at the location i. Let $\hat{y} = [\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_n]^{\mathrm{T}}$ be the vector of the fitted values for all available observations. Then

$$\hat{y} = X(X^{\mathrm{T}}W_iX)^{-1}X^{\mathrm{T}}W_iy = S_1 \cdot y \quad (10)$$

where S_1 is considered the hat matrix of GWR since it also transforms the observed values of y into the fitted ones. However, the hat matrix S_0 in OLS is symmetric and idempotent, but S_1 in GWR is generally not because of the weighting matrix W_i (it is different at each location i) (Leung et al., 2000). The residual sum of squares of the GWR model can be expressed as

$$\mathrm{RSS}_1 = y^{\mathrm{T}}(I - S_1)^{\mathrm{T}}(I - S_1)y = y^{\mathrm{T}}R_1y \quad (11)$$

But RSS_1/σ^2 is generally not distributed as an exact χ^2 distribution due to the complexity of the W_i matrix. However, $\delta_1\,\mathrm{RSS}_1/\delta_2\sigma^2$ can well be approximated by a χ^2 distribution with δ_1^2/δ_2 degrees of freedom, where $\delta_1 = \mathrm{tr}(R_1)$, and $\delta_2 = \mathrm{tr}(R_1^2)$, with tr denoting the trace of a square matrix (Leung et al., 2000).

Therefore, the partial F-test can be formed as

$$F = \frac{\mathrm{RSS}_1/\delta_1}{\mathrm{RSS}_0/(n-p-1)} \quad (12)$$

and it may reasonably follow an F-distribution with δ_1^2/δ_2 degrees of freedom in the numerator and $n-p-1$ degrees of freedom in the denominator. This approximation is based on the fact that both the numerator and denominator in Eq. (12) are quadratic forms of normal variates and are well approximated by χ^2 distributions (Leung et al., 2000). Another possibility to test the same null hypothesis is to use the method of analysis of variance. Suppose

$$\mathrm{DSS} = \mathrm{RSS}_0 - \mathrm{RSS}_1 \quad (13)$$

This quantity measures the difference of the residual sums of squares, and the distribution of $(v_1\,\mathrm{DSS}/v_2\sigma^2)$ can be approximated by a χ^2 distribution with v_1^2/v_2 degrees of freedom, where $v_1 = \mathrm{tr}(R_0 - R_1)$, and $v_2 = \mathrm{tr}[(R_0 - R_1)^2]$. Using this fact, a test statistic can be constructed as follows:

$$F = \frac{\mathrm{DSS}/v_1}{\mathrm{RSS}/\delta_1} \quad (14)$$

which has an approximate F-distribution with degrees of freedom v_1^2/v_2 and δ_1^2/δ_2.

In summary, GWR provides a set of localized parameters for the multivariate relationship in a regression model. Since the parameter estimates, standard errors of the parameters, and model R^2 are all associated with specific locations, they can be mapped in a visualization tool such as GIS to explore spatial heterogeneity or non-stationary across the study area. More details on GWR estimation procedure and related issues can be found in Brunsdon et al. (1996, 1998) and Fotheringham et al. (1998, 2000, 2002).

3. Data and methods

3.1. Data

One sample plot was selected from the database of a research project on the growth and yield of the regrowth *Eucalyptus fastigata* forests in Glenbog State Forest in southeast New South Wales, Australia (Bi, 1994; Bi and Jurskis, 1996a). The study area lies within latitude 36°30′ to 37°10′ S and longitude 149°20′ to 129°35′ E. Rising from a coastal area of 20–650 m a.s.l., the area covers a plateau with elevation ranging from 800 to more than 1100 m and mean annual rainfall ranging from about 700 to 1200 mm. The mean maximum temperature for the hottest month is in the range of 23–28 °C and the mean minimum for the coldest month is −4 to 3 °C. Regrowth forests constitute a substantial proportion of the tableland forests where sawlogs and pulpwood have been harvested for many years. *E. fastigata* is the most important commercial timber species on the tablelands. It can occur in pure stands, but often grows in association with other eucalypts such as *E. obliqua*, *E. nitens*, *E. viminalis*, *E. elata*, *E. cypellocarpa* and *E. radiata*. Acacias and other species of shrub or small trees are also present in the forests (Bi, 1994; Bi and Jurskis, 1996a,b).

The sample plot was a circular plot of 101 spatially mapped trees. With a radius of 40 m, it was the largest one among a number of plots sampled in the database. This plot belonged to forest type 154 as defined by Baur (1965), where *E. fastigata* was the only dominant species and may form pure stands at favorable sites. The subordinate associate species in this plot included *E. obliqua*, *E. badjensis* and *E. nitens*. Besides these tall tree species, stems of *Acacia dealbata* were also present. Due to the history of fire in the stand, double stems, coppicing and double leaders occur rather frequently. The tree diameter at breast height overbark

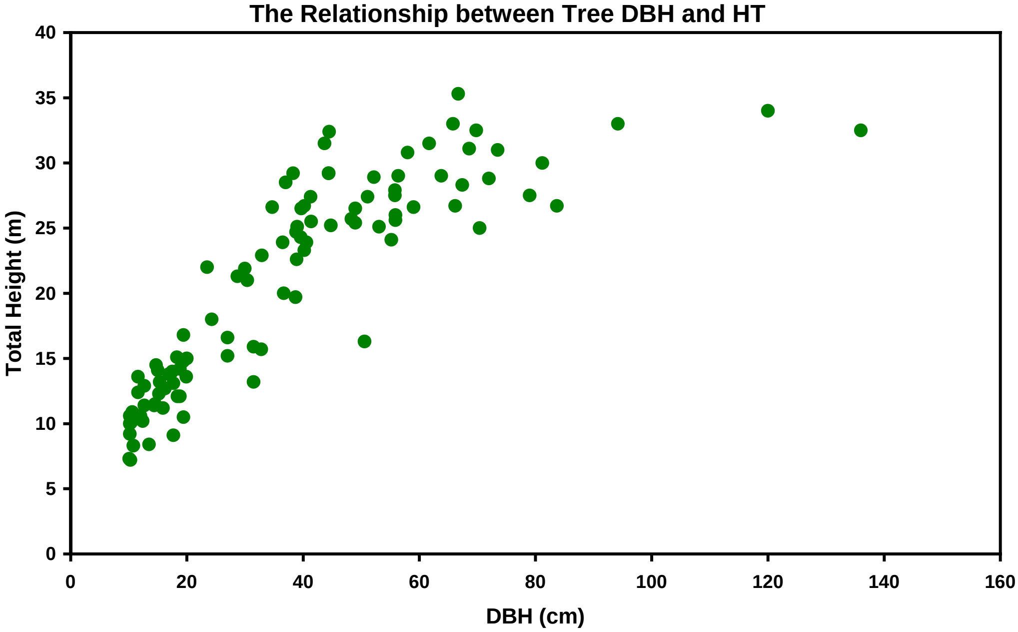

Fig. 1. The scatterplot of tree height against DBH for the sample plot.

(DBH) and total height (HT) of trees with DBH greater than 10 cm were measured (Bi and Jurskis, 1996b). The descriptive statistics of tree DBH and HT are listed in Table 1. The scatterplot of tree DBH against HT is illustrated in Fig. 1. The location of trees is shown in Fig. 2 with the circle diameter proportional to tree DBH.

3.2. Regression model

Forest researchers have developed many models for the relationships between tree diameter and total height for various tree species over the past several decades. In this study we did not intend to develop a predictive model for the DBH–HT relationship for the sample plot. Rather, we would use the following linear regression model, based on Fig. 1, to explore spatial heterogeneity in the relationship in the sample plot:

$$\ln(\mathrm{HT}) = \beta_0 + \beta_1 \ln(\mathrm{DBH}) + \varepsilon \qquad (15)$$

where HT is the tree total height (m), DBH the tree DBH (cm), ln the natural logarithm, β_0 and β_1 the regression coefficients to be estimated, and ε the model random error.

Eq. (15) was fitted to the sample plot by both OLS and GWR methods, using a computer software GWR 2.0.[1] The improvement of the GWR model over the OLS model was tested using Eq. (14). Model residual and absolute residual from both models were examined and investigated by locally weighted scatterplot smoothing (loess) (Cleveland, 1979, 1993). The model residual was defined as the difference between the observed and predicted HT, and the absolute residual was calculated by taking the absolute value of the model residual. The averages of the model residual and absolute residual were then calculated respectively for the sample plot. To examine the model

Table 1
Descriptive statistics of tree diameter (DBH) and total height (HT) for the 101 trees in the sample plot

Variable	Mean	S.D.	Minimum	Maximum
DBH (cm)	38.2	24.6	10.1	136.0
HT (m)	20.8	8.0	7.2	35.3

[1] The detailed information on the GWR 2.0 is available at the website http://www.ncl.ac.uk/geography/GWR or by e-mail to stewart.fotheringham@ncl.ac.uk (Fotheringham et al., 2002).

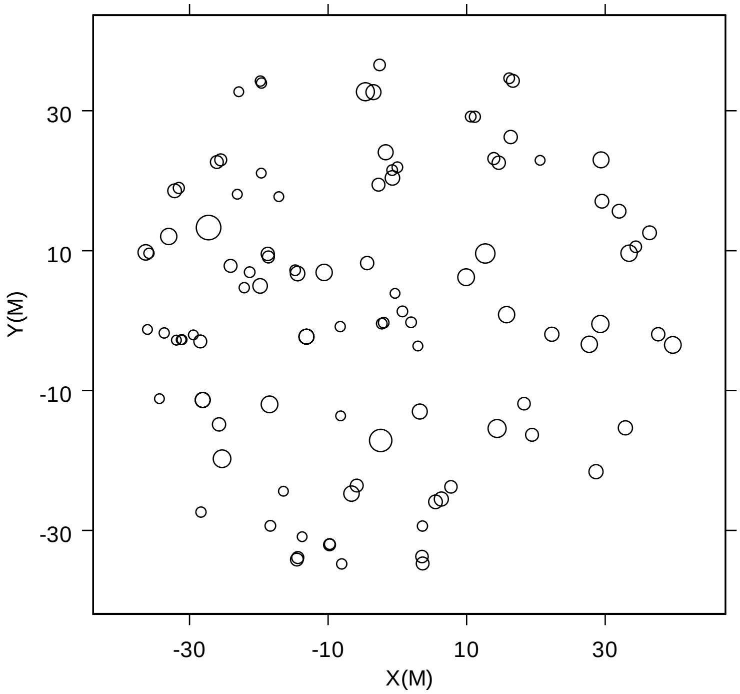

Fig. 2. The location of trees in the sample plot. The circle diameter is proportional to tree DBH.

residuals across tree sizes, all trees in the sample plot were grouped into 20 cm diameter classes. The averages of the model residual and absolute residual were respectively computed for each diameter class. Lastly, the GWR model coefficients and R^2 over space were illustrated using graphical visualization tools.

4. Results and discussion

The Moran coefficient (MC) (Moran, 1950) was computed for the sample plot based on the tree DBH. The global MC was +0.33, indicating that trees were surrounded by similar-sized neighbors. Fig. 2 illustrates the clusters of trees in the sample plot. It seems that there are a few large-sized trees and several groups of small-sized trees. The 3D diagram of the tree height shows the vertical profile of stand structure across the sample plot (Fig. 3). In general, eucalypt trees can grow from seedlings or coppices. In the former case, trees are more or less separate individuals in a stand. In the latter case, two or more trees can grow together sharing a common base. When measured at breast height, they represent a small cluster often at the scale of less than 1 m. *E. fastigata* can throw out new shoots from the bases of saplings that have collars of thick bark at the base, even after severe fires. The tops of the trees are killed, but the bases are not (Jacob, 1955). The previous analysis of this plot using Ripley's *K*-function detected clustering at 0.5–0.6, 5–7 m and larger spatial scales. The clustering at 0.5–0.6 m was most likely due to double leaders and trees grown from coppices sharing the same stump.

The Gaussian distance-decay-based kernel function (Eq. (6)) was used to fit the GWR version of Eq. (15). The bandwidth was pre-defined for the kernel function. We preferred to use a bandwidth that included significant competitive neighbors. A variogram was used to determine the distance for selecting influential neighboring trees in the sample plots. The result indicated that the variogram curve tended to flatten out after 7 m (i.e. range = 7 m), meaning there was no spatial autocorrelation among trees beyond this distance (Isaaks and Srivastava, 1989; Kohl and Gertner, 1997). Therefore, we used a bandwidth of $h = 7$ m.

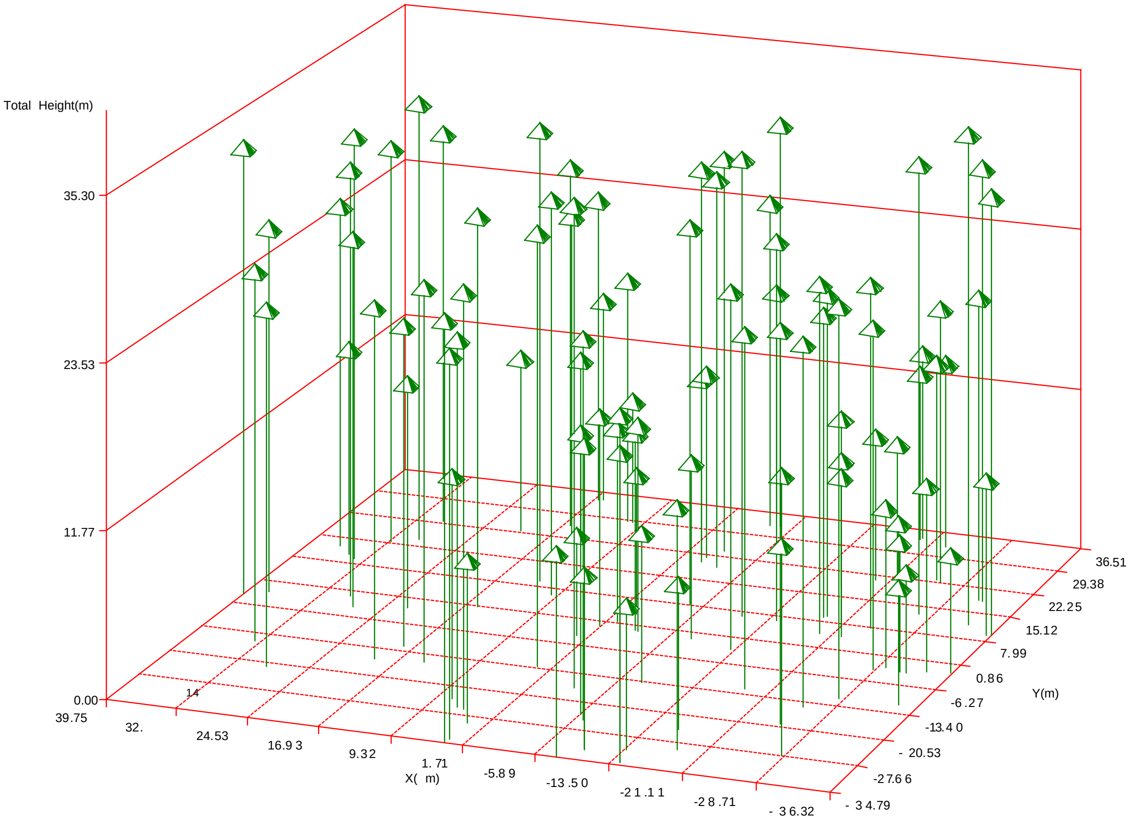

Fig. 3. The 3D diagram of the tree height.

The weight for each neighboring tree within the circle was computed by Eq. (6), and the weight equals zero for trees beyond the bandwidth.

Eq. (15) was fitted to all trees in the sample plot using both OLS and GWR methods. The partial F-test (Eq. (14)) was utilized to test the null hypothesis that the GWR model represented no improvement over a global OLS model. The result indicated a rejection of the null hypothesis (P-value $= 0.0036$ for the partial F-test) (Table 2), suggesting that GWR model significantly improved model fitting over the OLS model. It was evident that the linear relationship between the logarithmic transformed tree DBH and HT was not constant across the geographical area of the sample plot. Table 3 showed the descriptive statistics of the model parameter estimates from both OLS and GWR models. Fotheringham et al. (2002) suggests comparing the range of the GWR local parameter estimates with a confidence interval (CI) around the OLS global estimate of the equivalent parameter. For example, 50% of the GWR local parameter estimates should fall between the 25 and 75% quartiles, while approximately 68% of values in a normal distribution are within ±1 standard deviation. If the inter-quartile range of the local estimates is greater than that of ±1 standard deviation of the equivalent global parameter, this suggests the relationship under study might be non-stationary (Fotheringham et al., 2002).

Table 2
Improvement of the GWR model over the OLS model[a]

Source	SS	d.f.	MS	F	P-value
OLS residuals	2.74	2.00			
GWR improvement	1.71	43.10	0.0396		
GWR residuals	1.03	55.90	0.0184	2.1521	0.0036

[a] Where GWR improvement sum of squares (SS) was the difference between the OLS residuals sum of squares and the GWR residuals sum of squares (Fotheringham et al., 2002).

Table 3
Descriptive statistics of the parameter estimates for the OLS and GWR models

Statistics	$\hat{\beta}_0$	$\hat{\beta}_1$
OLS model		
Estimate	0.9085	0.5944
Standard error	0.0866	0.0248
Lower limit of 95% CI	0.7366	0.5452
Upper limit of 95% CI	1.0803	0.6435
$\beta - 1$ S.D.	0.8219	0.5696
$\beta + 1$ S.D.	0.9951	0.6192
GWR model		
Mean	0.8450	0.6160
Minimum	−1.0128	0.1429
25% quartile	0.5026	0.5348
Median (50% quartile)	0.6790	0.6626
75% quartile	1.0649	0.7116
Maximum	2.6335	1.0123

Table 3 indicated that (1) for the intercept coefficient $\hat{\beta}_0$, the inter-quartile range 0.5026–1.0649 of the GWR local parameter estimates was outside the range 0.8219–0.9951 of ± 1 standard error of the OLS parameter estimate and (2) similarly, for the slope coefficient $\hat{\beta}_1$, the inter-quartile range 0.5348–0.7116 of the GWR local parameter estimates was also outside the range 0.5696–0.6192 of ± 1 standard error of the OLS parameter estimate. Furthermore, the 95% CI (0.7366–1.0803) of the OLS $\hat{\beta}_0$ approximately fell in the range 0.6790–1.0649 between the median and 75% quartile of the GWR $\hat{\beta}_0$, indicating that the local $\hat{\beta}_0$ values were relatively smaller than the OLS $\hat{\beta}_0$ value. The 95% CI (0.5452–0.6435) of the OLS $\hat{\beta}_1$ was within the range 0.5348–0.6626 between the 25% quartile and median of the GWR $\hat{\beta}_1$, and thus, the local $\hat{\beta}_1$ values were relatively larger than the OLS $\hat{\beta}_1$ value. In each case above, only about 25% of all GWR parameter estimates fell within the 95% CIs of the two OLS parameters. In other words, at least 75% of the GWR parameter estimates were statistically different from the OLS parameter estimates. Therefore, the model parameters of Eq. (15) indeed varied from sub-areas to sub-areas within the plot.

The variation in the GWR estimates of $\hat{\beta}_0$ and $\hat{\beta}_1$ may reflect the composition of trees in terms of species and age classes in a neighborhood defined by a radius of 7 m. The stand had been disturbed by fire and logging, which resulted in spatial heterogeneity in species composition and age structure in the regrowth stand. Although *E. fastigata* was the dominant species, other eucalypt species were also present. There were 17 *A. dealbata* trees distributed mainly in two patches in the plot. Being a small tree species, *A. dealbata* has a different DBH–HT pattern to the tall tree species of eucalypts (Bi et al., 2000). The variation in age and species across the plot would lead to different values of $\hat{\beta}_0$ and $\hat{\beta}_1$ estimated by GWR. A local neighborhood of smaller *A. dealbata* would have different parameters to another neighborhood of larger eucalypts. For locations surrounded by big old trees, one can expect flatter DBH–HT relationships. For those surrounded by a mix of small and big trees, a steeper relationship may result.

Locally weighted scatterplot smoothing (loess) was fitted to the residual plots from both OLS and GWR models using the same smoothing parameter (0.65). The residual sum of squares and residual standard error for the OLS model were 2.225 and 0.152, and for the GWR model were 0.952 and 0.099, respectively. Fig. 4(a) shows that the OLS model produces negative residuals for small and large tree heights and positive residuals for intermediate tree heights. In contrast, the loess curve showed nearly no trend in the residual plot of the GWR model (Fig. 4(b)). Fig. 5 illustrates the performance evaluation of both OLS and GWR models across tree DBH classes. Although the overall average of the model residual from the OLS model is zero, it appears that the OLS model produces larger negative biases (over-estimation) for large-sized trees (e.g. >60 cm in diameter) than does the GWR model (Fig. 5(a)). Fig. 5(b) shows the model absolute residuals from both OLS and GWR models. It is clear that the GWR model consistently produces smaller model errors across the diameter classes than the OLS model. The overall average (0.074) of the model absolute residual from the GWR model is 43% smaller than that (0.129) from the OLS model.

The OLS produced one regression model, and the model coefficients were referred as the global coefficients because they were assumed constant across the sample plot (Table 3). The OLS model was

$$\ln(\hat{H}T) = 0.9085 + 0.5944 \ln(\mathrm{DBH}) \quad (16)$$

with the model $R^2 = 0.85$ and RMSE = 0.1663. On the other hand, the overall model R^2 was 0.94 and RMSE was 0.1357 for the GWR model. In addition, the GWR model produced localized parameter estimates

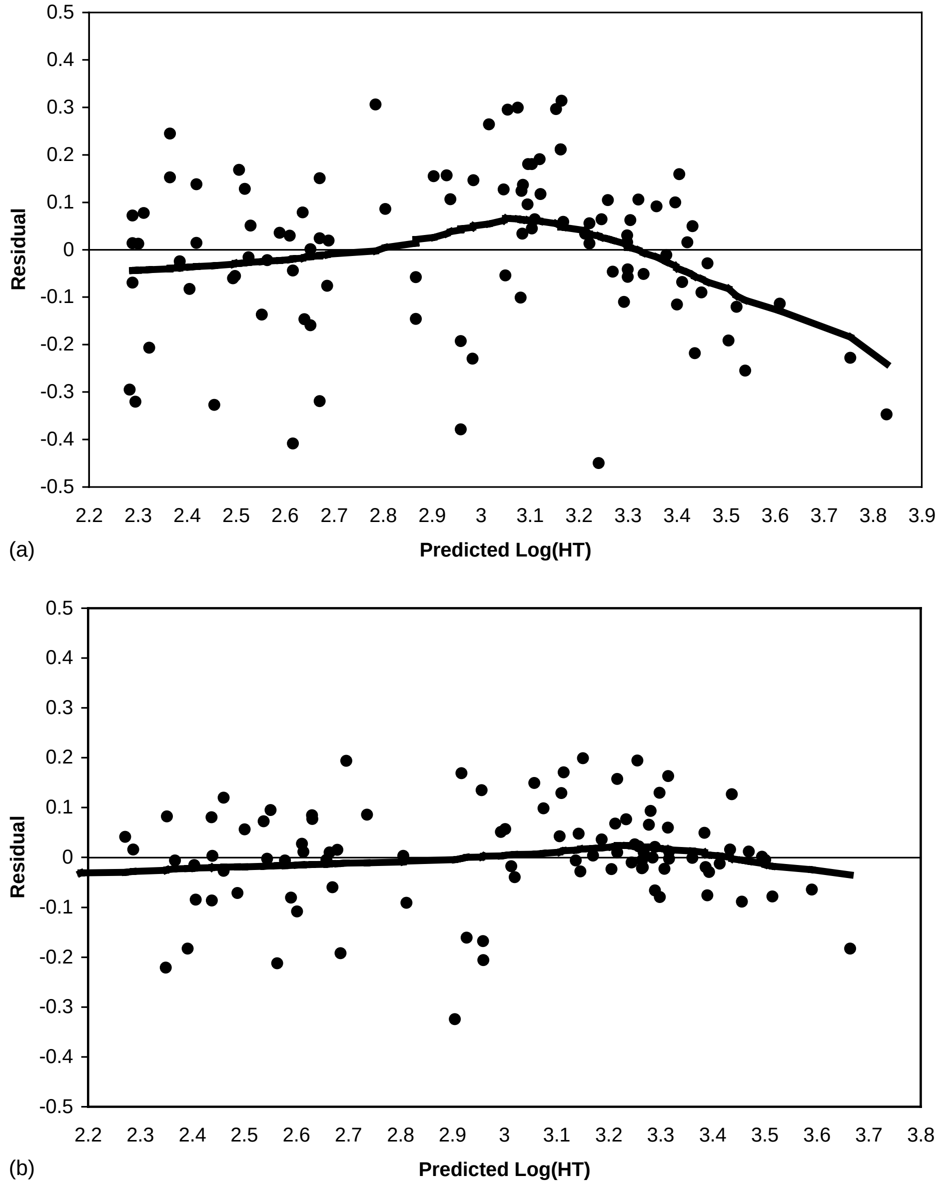

Fig. 4. Loess fit for the model residuals: (a) OLS model and (b) GWR model.

and model R^2 for each tree in the sample plot, which were mapped in contour plots (Fig. 6) to illustrate the spatial heterogeneity of the relationships between tree DBH and HT. Fig. 6(a) indicates that the estimates of the local intercept coefficient ($\hat{\beta}_0$) range from −1.0128 to 2.6335, instead of a constant (0.9085) for the OLS $\hat{\beta}_0$. Similarly, the range of the local estimates of the slope coefficient ($\hat{\beta}_1$) was from 0.1429 to 1.0123, rather than a constant (0.5944 in the OLS model) (Fig. 6(b)). Since the spatial variation of local growth condition and competition were taken into account by the GWR model, it produced better fitting for the regression relationship between tree DBH and HT. The local R^2 of the GWR model ranged from 0.699 to 0.998 (Fig. 6(c)). Only 4% of the local R^2 was lower than the global R^2 (=0.85). Most (about 90%) of trees

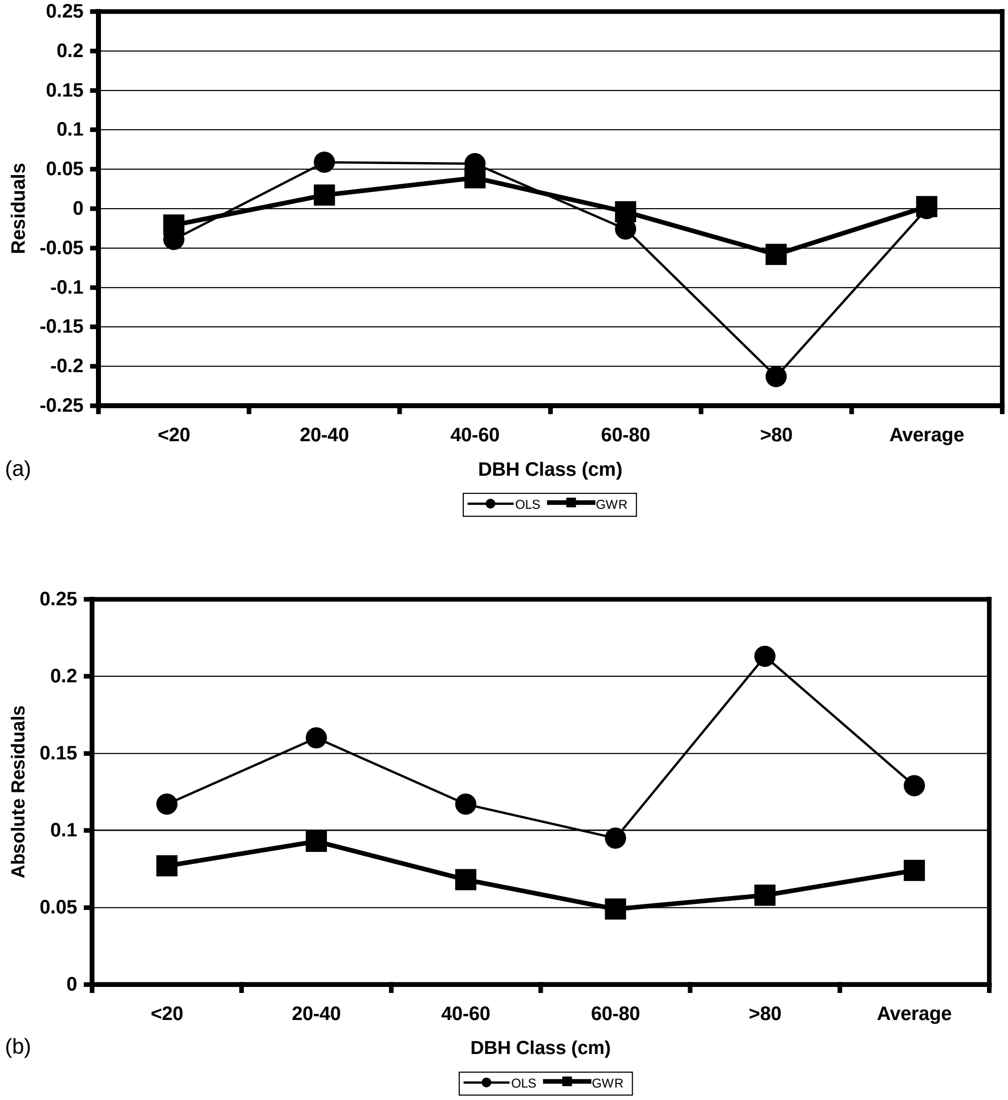

Fig. 5. Evaluation of the models across the diameter classes: (a) model residuals and (b) model absolute residuals.

were fitted with higher model R^2 (>0.90), and majority (about 70%) of trees were fitted with a R^2 larger than 0.95. Thus, the GWR model did not simply distribute the model's explanatory power spatially, but actually provided a better explanatory ability.

Such improved explanatory ability will provide forest managers with more accurate yield estimates and will also enable scientists to gain greater insight in forest research. It has often been the case with experimental, growth and inventory plots to measure diameter for all trees and height for selected trees only. This practice comes of the desire to reduce measurement costs, and/or sometimes is due to poor visibility in the stand. In such cases, height–diameter equations are usually developed using data from those trees with complete measurements. The equations are then used to estimate tree height given the field measurements of tree diameter. In plots where individual trees are mapped and an enough number of trees are measured for height, GWR is clearly a preferred choice over

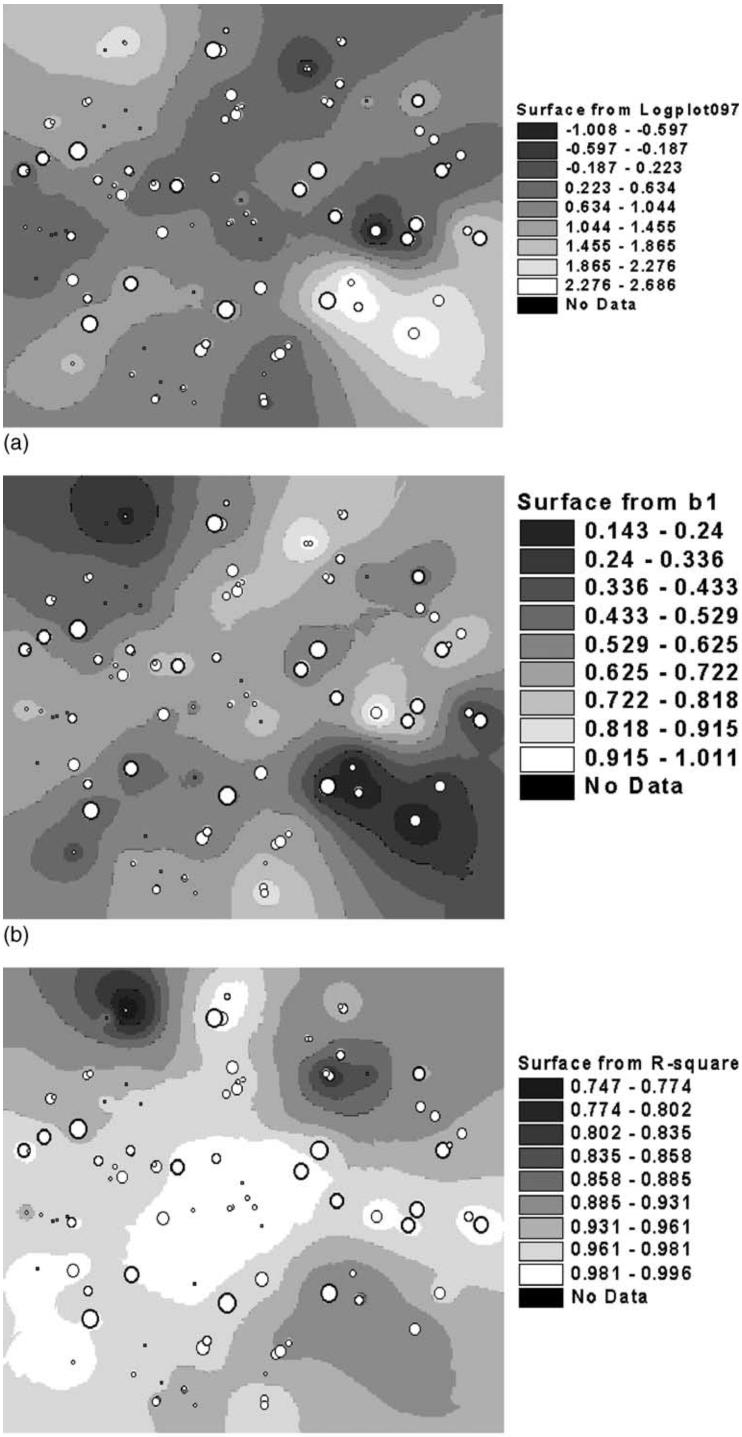

Fig. 6. The contour plots for the GWR model: (a) coefficient β_0, (b) coefficient β_1, and (c) model R^2.

OLS for estimating tree height. The improved height estimates would lead to more accurate stem volume calculations by volume equations. When input into taper functions, the improved height estimates would result in more precise estimation of product volume for a specified log length. The increased accuracy in volume estimates allows treatment response to be detected with greater precision in experiments because measurement error is reduced through the application of GWR.

More importantly, GWR can be used to develop site-specific growth and yield models over a forest landscape. The height–diameter relationship for individual trees in our example simply shows that GWR can generate a surface of functions from point-based information. Similarly height and age data from individual plots within a compartment can be used in GWR to generate a site quality surface for the compartment. The same would apply to other growth and yield functions. Such site-specific growth and yield functions will certainly aid and improve forest management in precision forestry.

5. Conclusion

Our results indicated that spatial heterogeneity in the relationship between tree DBH and total height in a forest stand could be explored and modeled using GWR. The GWR model produced better prediction of individual tree height than the traditional OLS model. The visualization of the GWR model coefficients and statistics highlighted the spatial distribution of the multivariate relationship under study.

Recently forest researchers have attempted to link forest growth and yield models to GIS to assist in forest inventory and management, as well as to estimate the impacts of selective timber harvesting (Berry et al., 1997; Bateman and Lovett, 1998; Anderson et al., 2000). In the future, when the spatial information of trees at a plot/stand level is available, the spatial modeling methodologies such as GWR can be applied to investigate the growth and yield relationships for each tree or sub-area in the plot/stand. Consequently, the influence of micro-site variation, competition status, growth potential, and the impacts of management activities on trees can be evaluated, tested, modeled, and readily displayed with GIS or statistical graphic packages. In addition, the GWR techniques can be readily extended to other study areas such as ecological modeling and geospatial analysis of landscape dynamics.

Acknowledgements

The authors would like to thank the editor and two anonymous reviewers for their valuable suggestions and helpful comments on the manuscript.

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